Fuzzy Supra Topological Spaces

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Dedicated
To My Parents

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Finally, I would like to shoulder upon all the errors and shortcomings in the study if there be any, I am extremely sorry for that.

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STATEMENT OF ORGINALITY

I hereby declare that this thesis entitled "Fuzzy Supra Topological Spaces" submitted for the partial fulfillment for the degree of Master of Philosophy is done by myself under the supervision of ***Dr. Md. Bazlar Rahman*** *and* ***DR. Dewan Muslim Ali*** *as supervisorand co-supervisor respectively and is not submitted elsewhere for any other degree or diploma.*

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ABSTRACT

American Mathematician Lotfi A. Zadeh in 1965 first introduced the concept of fuzzy set. He interpreted a fuzzy set on a set as a mapping from the set into the unit interval I= [0.1], which is a generalization of the characteristic function of the set. Many mathematicians throughout the world used this set to fuzzify different areas of mathematics. Fuzzy supra topology is one of the outcomes of such fuzzification of the usual topology. This thesis is a collection with addition of several results on fuzzy supra topological spaces. Chapter one is a brief introduction of this thesis. Our main work is from second chapter where we have given some new examples and theorems on fuzzy supra topology. In a similar way we have studied third, fourth, fifth and sixth chapters.

CONTENT

INTRODUCTION

The concept of fuzzy set was first introduced by the American Mathematician Lotfi A. Zadeh in 1965. He interpreted a fuzzy set on a set as a mapping from the set into the unit interval, I = [o, 1], which is a generalization of the characteristic function of the set. Many mathematicians throughout the world used this set to fuzzify different areas of mathematics, e.g. L-fuzzy group, fuzzy ideals, fuzzy rings, fuzzy lattice, fuzzy topological spaces, L–fuzzy topological spaces, fuzzy supra topological space, fuzzy topological vector space etc. From the literature of fuzzy topological spaces, it is found that it is still developing in various directions. The field of knowledge on fuzzy supra topological spaces are not complete and we have observed some gaps there. Fuzzy topological spaces were, first introduced in the literature by C. L. Chang in 1968 who studied several basic concepts including fuzzy continuous map and compactness. Wong defined the same in 1975 and Lowen in 1976. In 1983 A .S. Mashhour introduced the Fuzzy supra topological space and studied S-continuous and continuous function. Some generalized from of semi-open and semi-closed sets are given by A.A. Allam and A.M. Zahran in 1986. In 1987, M.E. Abd El-Monsef introduced the Fuzzy Supra Topological Spaces and studied fuzzy supra continuous functions and characterized a number of basic concepts. In 1996 he introduced fuzzy S-continuous, fuzzy S-open, and fuzzy S-closed maps and established a number of characterizations. The concept of induced fuzzy supra topological spaces was introduced by Bhaumik and Mukherjee and Anjan Mukherjee introduced the concept of S-induced L-fuzzy supra topological spaces. The material of the thesis has been divided into six chapters. A brief scenario is as fallows:

Chapter one incorporates some of the basic definitions and examples on fuzzy sets, fuzzy topology, fuzzy supra topology and its related topics. They are stated without proof in different papers.

Our main work starts from the second chapter. We have introduced and studied some properties of fuzzy supra topological spaces and established relation and non relation among them.

In the third chapter we have introduced some R_0 and R_1 properties of fuzzy supra topological spaces and been studied.

In the forth chapter, we have introduced and studied some T_0 and T_1 properties of fuzzy supra topological spaces. Also the Cartesian product between two fuzzy supra topological spaces is shown with examples.

In the fifth chapter, we have introduced and studied some T_2 properties of fuzzy supra topological space and we have established relations among them.

In the sixth chapter, we have introduce and studied about the compactness and connectedness property of fuzzy supra topological spaces. We have established the relation between compactness property and continuity property of fuzzy supra topological spaces. In addition, we have studied some other properties of these concepts.

CHAPTER -1

Prerequisites

1.1. Introduction: This chapter incorporates concepts and results of the fuzzy sets, Fuzzy topological spaces and Fuzzy supra topological spaces which are to be used as ready references for understanding the subsequent chapters. Most of the results are quoted from various research papers. Through the sequel, we make use the following notations.

$[0, 1] = I$	: Closed unit interval
$J=$	: An index set.
$I_1 = [0, 1)$	: Right open unit interval.
$I_0 = (0, 1]$	: Left open unit interval.
$\lambda, \mu, \omega, u, v$	: Fuzzy set.
(X, t)	: Fuzzy topological space.
(X, t^*)	: Fuzzy supra topological space.
(X, T^*)	: Supra topological space.
$I(t^*) = \{u^{-1}(0, 1]; u \in t^*\}$	: Supra topology on X
$N_P^S =$	: Collection of supra S-neighborhood

1.2. Fuzzy Set: This thesis is a study of Fuzzy supra topological spaces .To present our work in a systematic way, we consider in this chapter various concepts and results from the theories of fuzzy sets, fuzzy topological spaces and Fuzzy supra topological spaces scattered in various research papers. For this, we start with.

Definition 1.2.1.: Let X be a non -empty set and $A \subseteq X$; now the characteristic function of A is a function, that declares which elements of X are members of the set A and which are not. It is denoted by κ_A or 1_A.The function κ_A or

$1_A: X \to [0, 1]$ is defined by κ_A or

$$1_A(x) = 1 \text{ if } x \in A$$
$$= 0 \text{ if } x \notin A$$

Throughout this work, we use, if needed, 1_A to denote the characteristic function of a set A.

Definition 1.2.2. Let X be a non - empty set and let I= [0, 1]. A fuzzy set in X is a function λ: X$\to$I which assigns to each, x$\in$X its grade of membership $\lambda(x) \in$I. **[54]**

Definition 1.2.3. Let I^X denote the set of all mappings λ: X$\to$I.A member of I^X is called fuzzy subset of X.

Definition 1.2.4. A fuzzy subset is empty if and only if its grade of membership is identically zero in X; It is denoted by 0. **[49]**

Definition 1.2.5. A fuzzy subset is whole if and only if its grade of membership is identically one on X it is denoted by 1. **.[49]**

Definition 1.2.6. For any two members $\lambda$and μ of I^X, $\lambda \geq \mu$ if $\lambda(x)\geq\mu(x)$ for each x $\in$X, and in this caseλ is said to contain μ and is denoted by$\lambda \supseteq \mu$ or μ is said to be contained in λ.**[54]**

Definition 1.2.7. Let λbe a fuzzy set; the complement $\lambda'$of $\lambda$with respect to the set X is 1-λ, It is defined by$\lambda'(x) = (1-\lambda)(x) = 1-\lambda$ (x) for each x$\in$X, it is also denoted byλ^C, obviously $(\lambda^C)^C=\lambda$.**[54]**

Definition 1.2.8. Let α and β be two fuzzy subsets of X, Then the union λ of α and β is a fuzzy subset of X, which written as $\lambda=\alpha\cap\beta$ and defined by,

$\lambda(x) = \max[\alpha(x), \beta(x)]\ \forall\ x\in X$.

In general, if J be an index set and A= $\cap\{u_i : I \in J\}$ be a family of fuzzy sets of X then $\cup u_i$ is defined by $(\cap u_i)(x) = \text{Sup}\{u_i : I \in J\}$ where x $\in$X. **[16]**

Definition 1.2.9. Let α and β be two fuzzy sets in X, Then the intersection $\lambda$of α and β is a fuzzy set of X, written as $\lambda=\alpha\cup\beta$, and defined by,

$\lambda(x) = \min[\alpha(x), \beta(x)]\ \forall\ x \in X$.

In general, if J be an index set and A= $\cup\{u_i : I \in J\}$ be a family of fuzzy sets of X then it is defined by $(\cup u_i)(x) = \text{Inf}\{u_i : I \in J\}\ \forall\ x \in X$. **[16]**

Definition 1.2.10. A fuzzy point in X is a special type of fuzzy set in X with membership function $\mu(y) = r$, for y=x and $\mu(y)= 0, \forall x \in X$ .and $y \neq x$ where $0 \leq r \leq 1$, this fuzzy point is said to have support x and value r is denoted by x_r or $r1_x$. **[49]**

Definition 1.2.11 A fuzzy point x_r is said to be contained in a fuzzy set λ in X, or belong to λ, denoted by $x_r \in \lambda$, if and only if $r < \lambda(x)$.

Evidently, every fuzzy set α can be express as the union of all the fuzzy points, which belong to α ..**[49]**

Definition 1.2.12. Let X be a set and u and v be two fuzzy subsets of X. Then the difference of u and v defined as $u- v =u \cup v^C$.

Laws of the algebra of fuzzy sets:

In ordinary set theory idempotent laws, Associative law, Commutative law, Distributive laws, Identity law, Demorgans laws are true. Analogously these laws are also true in fuzzy set theory. But in case of ordinary set, we have for any set A, $A \cap A' =X$, and $A \cup A' =\phi$. But these are not true in the case of fuzzy set.

For example, if X= {a, b, c} and λ is a fuzzy set defined by

$\lambda= \{(a, .2), (b, .7), (c, 1)\}$

$\lambda^C = \{(a, .8), (b, .3), (c, 0)\}$

So $\lambda \cap \lambda^C = \{(a, .8), (b, .7), (c, 1) \neq X$

And $\lambda \cup \lambda^C= \{(a, .2), (b, .3), (c, 0)\} \neq \phi$

Also as in ordinary set theory $A \cup B= \phi$ if and only if $A \subset B^c$ But in fuzzy subsets, the converse part is not necessarily true. For example if

$\beta= \{(a, .6), (b, .1), (c, 0)\}$ then

$\beta^C = \{(a, .4), (b, .9), (c, 1)\}$

Here $\lambda \subset \beta^C$ but $\lambda \cup \beta= \{(a, .2), (b, .1), (c, 0) \neq \phi$

1.3. Mapping and Fuzzy Subsets induced by mappings.

Definition 1.3.1. Let I = [0, 1], X be a set, and $t \subseteq I^X$ that is t is a collection of fuzzy sets in X. Then t is called a fuzzy topology on X if

(1) 0 and 1 belong to t;

(2) $\lambda, \mu \in t$ then $\lambda \wedge \mu \in t$ and

(3) $\lambda_i \in t$ $\forall$ $i \in J$ then $\vee\lambda_i \in t$ Where J is index set, then (X, t) is a called fuzzy topological space (in short fts) and the members of t are called t-open or simply open fuzzy sets. A fuzzy set λ is called a t-closed fuzzy set or closed fuzzy set if $1-\lambda \in t$. **[52]**

Examples of fuzzy topologies: The fuzzy topology {0, 1} on a set X is an indiscrete fuzzy topology on X. The discrete fuzzy topology on X containing all fuzzy sets in X .

Definition 1.3.2. Let f be a mapping from a set X into Y and λ be a fuzzy subset of Y. Then the inverse of λ written as $f^{-1}(\lambda)$ is a fuzzy subset of X and is defined by

$f^{-1}(\lambda)(x) = \lambda f(x)$, for $x \in X$. **[50]**

Here we mention some properties of fuzzy subsets induced by mappings.

Let f be a mapping from X into Y, λ be a fuzzy subset of X and μ be a fuzzy subset of Y then the following relations are true **[16].**

(a) $f^{-1}(\mu^C) = (f^{-1}(\mu))^C$ for any subset μ of Y.

(b) $f(\lambda^C) = (f(\lambda))^C$ for any subset λ of X.

(c) $\mu_1 \subset \mu_2 \Rightarrow f^{-1}(\mu_1) \subset f^{-1}(\mu_2)$, where μ_1 and μ_2 are two subsets of Y.

(d) $\lambda_1 \subset \lambda_2 \Rightarrow f(\lambda_1) \subset f(\lambda_2)$, where λ_1 and λ_2 are fuzzy subsets of X .

(e) $\mu \supset f(f^{-1}(\mu))$, for any subset μ of Y.

(f) $\lambda \subset f^{-1}(f(\lambda))$ for any subset λ of X.

(g) Let f be a function from X into Y and g is a function from Y into Z. Then $(g \circ f)^{-1}(w) = f^{-1}(g^{-1}(w))$, for any fuzzy set w in Z .Where $(g \circ f)$ is the composition of g and f.

Definition 1.3.3. Let f: $X \rightarrow Y$ be a mapping from X into Y, if λ is a fuzzy subset X and μbe a fuzzy set in Y then $f(\lambda)$ and $f^{-1}(\mu)$ are defined as follows.

$f(\lambda)(x) = \text{Sup}\ \lambda(x),\ x \in f^{-1}(y)$ if $f^{-1}(y) \neq \varphi$

$= 0$ otherwise.

$f^{-1}(\mu)(x) = \mu(f(x))$ for each $x \in X$. **[51]**

Definition 1.3.4.: The mapping f: $(X, t) \rightarrow (X, t)$ is called the identity mapping if $f(x) = x$ for each $x \in X$.

Definition 1.3.5 The function f: (X, t) $\rightarrow$(Y, s) is called continuous if for every$\lambda \in$ s, $f^{-1}(\lambda) \in$t, the function f is called fuzzy homeomorphic if and only if f is bijective and both f and f^{-1} are fuzzy continuous. **[50]**

Definition 1.3.6.: Let x_r be a fuzzy point in an fts (X, t). A fuzzy set α in X is called fuzzy neighborhood (in short nbd.) of x_r if and only if $\exists$ a t –open fuzzy set λ in X such that $x \in \lambda \subseteq \alpha$. If α is open (closed) then we call α is open (closed) nbd. of x_r .We denote the family of nbd. by N_x

Example 1.3.7.:- Let X = {a, b, c, x}, t = {1, 0, λ, μ}

where 1= {(a, 1), (b, 1), (c, 1), (x, 1)}

0= { (a, 0), (b, 0), (c, 0), (x, 0)}

λ= {(a, .2), (b, .5), (c, .7), (x, .9)}

α = {(a, .3), (b, .5), (c, .8), (x, .95)},

Here α is a neighborhood of x_r, r=.2 belongs to$\lambda$and $\lambda \subseteq \alpha$.

Definition 1.3.8. Let (X, t) is a fts. and λ be a fuzzy subset of X. The interior of $\lambda$is written as λ^0or int $\lambda$is defined by

$\lambda^0 = \vee \{\mu: \mu \leq \lambda \text{and} \mu \in t\}$

Example 1.3.9.: Let X= {a, b, c, x}, t= {1, 0, λ, μ}

Where 1 = {(a, 1), (b, 1), (c, 1), (x, 1)}; 0 = {(a, 0), (b, 0), (c, 0), (x, 0)}

λ= {(a, .2), (b, .5), (c, .7), (x, .9)}; μ= {(a, .3), (b, .5), (c, .8), (x, .95)}.

Here μ is a neighborhood of x_r, r=.2 belongs toλ and $\lambda \subseteq \mu$.

Hence x_r , r=.2 is the interior point.

Definition 1.3.10.: Let λ be a fuzzy set in a fts (X, t). Then the closure of λ, denoted by $\bar{\lambda} = \wedge \{\mu: \lambda \leq \mu$ and $\mu \in t^C \}$ or $\lambda^- = \text{Inf} \{\mu: \lambda \leq \mu$ and $\mu^c \in t\}$ it is the intersection of all closed containing λ and $\bar{\bar{\lambda}} = \bar{\lambda}$.

Example 1.3.11.: Let X= {a, b, c, x} t = {1, 0, λ, μ}

Where 1 = {(a, 1), (b, 1), (c, 1), (x, 1)}, 0 = {(a, 0), (b, 0), (c, 0), (x, 0)}

λ= {(a, 2), (b, .5), (c, .7), (x, .9)}, μ = {(a, .3), (b, .5), (c, .8), (x, .95)} Closed fuzzy sets are 0, 1,λ^C, μ^c therefore

0={(a, 0), (b, 0), (c, 0), (x, 0)}

$1=\{(a, 1), (b, 1), (c, 1), (x, 1)\}$

$\lambda^c = \{(a, .8), (b, .5), (c, .3), (x, .1)\}$

$\mu^c = \{(a, .7), (b, .5), (c, .2), (x, .05)\}$ the members of t^c are $\{0, 1, \lambda^c, \mu^c\}$.

Here intersection of all closed set containing λ is 1, hence closure of λ is i.e $\bar{\lambda} = 1$

Definition 1.3.12: Let f: (X, t) →(Y, s) be a function between two fts. Then f is called continuous iff and only if $f^{-1}(\mu) \in t$ for each $\mu \in s$.

Definition 1.3.13.: If (X, t) is an fts and $A \subseteq X$ then $t_A = (u \wedge A: u \in t)$ is a fuzzy topology on A, called the subspace fuzzy topology on A: (A, t_A) is referred to as a fuzzy subspace of (X, t).

Definition 1.3.14.: A fuzzy bitopological space (in short, fbts) is a triple (X, t_1, t_2) where X is a set and t_1, t_2 are two types of fuzzy topology on X.

Remark: We shall usually denote the union (supremum) and intersection (infimum) operation on fuzzy set by ∨ and ∧ respectively.

Definition 1.3.15: Let S be a subset of a topological space (X, T). Then S is called semi-open (Levine) if $S \subset S^{0-}$.

Let X={a, b, c}, T={x, φ, {b}, {a, b}, {b, c}}, and S={b, c} then $c \varepsilon \{b, c\} \subset \{b, c\}$ and there is a T open set containing b which is contained in {b, c}.that is $S^0 = S$. Complement of each T open set (closed set) is φ ,X,{a, c}, {c}, {a}.The set containing S^0 is X. Hence $S^{0-} = X$, and $S \subset S^{0-}$.

Definition 1.3.16: A fuzzy set λ of X, is called fuzzy semi -open if and only if $\lambda \leq \lambda^{0-}$ **.[48].**

Definition 1.3.17: A fuzzy set λ of a fuzzy topological space (X, t) is said to pre open iff $\lambda \leq \lambda^{0-}$. **[48]**

Definition 1.3.18.: Let X be a non empty set and $T^* \subset P(X)$. (Power set of X) is called a supra topology on X if $X \in T^*$ and T^* is closed under arbitrary union. The members of T^* are called supra open set. **[8]**

1.4. Fuzzy Supra Topological Space

Definition 1.4.1: Let X be a non empty set and I =[0.1]. A subfamily t* of I^X is said to be fuzzy supra topology on X, if

(1) $0, 1 \in t^*$

(2) $\alpha_i \in t^*$ for all $i \in J$ then $\vee \alpha_i \in t^*$.

(X, t^*) is called a fuzzy supra topological space. In short, a fuzzy supra topological space denoted by FSTS .The elements of t^* are called fuzzy supra open set in (X, t^*).A fuzzy set λ is supra closed if and only if complement of λ .i.e $\lambda' = 1 - \lambda$ is a fuzzy supra open set of (X, t*).:**[48]**

.**Example1.4.2:** Let X= {a, b, c, d} with a fuzzy supra topology.

t^*={1, 0, {(a, 0), (b, .5), (c, 1), (d, 0)}, {(a, .5), (b, .25), (c, 0), (d, 1)}, {(a, .5), (b, .5), (c, 1), (d, 1)}}

on X then the class of supra closed sets t^* are

t^{c*}={0, 1,{(a, 1),(b, .5),(c, 0),(d, 1)}, {(a,. 5),(b,.75), (c,1), (d,0)}, {(a,.5), (b,.5), (c,0),(d, 0)}

Note: It is clear that every fuzzy topological space is fuzzy supra topological space; but the converse may not true

Example: Let X= {a, b} and $\alpha, \beta, \gamma \in I^X$. Let α(a) =.2, α(b) =.3: β(a) =.4, β (b)=.1: γ(a) =.4, γ(b)=.3 then

t^*= {0, 1, α, β, γ}is a fuzzy supra topology on X but $t^*$is not fuzzy topology on X.

Definition 1.4.3. The supra closure of a fuzzy set λ is denoted by λ^{Scl} or Scl(λ)

Scl(λ) = ∧ {S: S is fuzzy supra closed set and λ≤S}**[53]** .

Definition 1.4.4. The supra interior of a fuzzy set λ is denoted by λ^{Si} or Si(λ)

Si (λ) = ∨ {S: S is fuzzy supra -open set and S≤λ}.**[53]**

Definition 1.4.5.: Let X be a set and t^* be the class of all fuzzy sets in X and t^*satisfies the axioms of fuzzy supra topological space on X. This fuzzy supra topology t^* on X is called the discrete fuzzy supra topology and the pair(X, t^*) is called the discrete fuzzy supra topological space.

Definition 1.4.6.: Let X be a set and t^* be the fuzzy supra topology on X consist of the fuzzy sets 0 and 1 alone. Then t^* is called the indiscrete fuzzy supra topology and the pair(X, t^*) is called the indiscrete fuzzy supra topological space.

Definition 1.4.7.: Let (X, t) be a fuzzy topological space and t^* be a fuzzy supra topology on X. We call t^* a fuzzy supra topology associated with t if $t \subset t^*$.**[53]**

Definition 1.4.8.: Let x_r be a pt. in an FSTS. (X, t^*).A fuzzy set α in X is called fuzzy supra neighborhood (in short nhd) of x_r iff $\exists$ a supra t^* –open fuzzy set β in X such that $x_r \in \beta \subseteq \alpha$ if $\alpha$is supra open then we call supra open nhd. of x_r.

Definition 1.4.9.: A fuzzy supra bi topological spaces (in short fsbts) is a triple (X, t_1^*, t_2^*) where X is a set and t_1^*, t_2^* are two fuzzy supra topologies on X.

Definition 1.4.10.: Let (X, t_1^*) and (Y, t_2^*) be two fuzzy supra topological spaces. A mapping f: $(X, t_1^*) \to (Y, t_2^*)$ is called fuzzy supra continuous if $f^{-1}(t_2^*) \subset t_1^*$.**[48]**

Definition 1.4.11.: Let (X, t_1^*) and (Y, t_2^*) be two fuzzy supra topological spaces. A mapping f: $(X, t_1^*) \to (Y, t_2^*)$ is called fuzzy supra continuous if the inverse image of each fuzzy supra open set in (Y, t_2^*) is t_1^* fuzzy supra open in X.

Definition 1.4.12. Let (X, t_1^*) and (Y, t_2^*) be two fuzzy supra topological spaces. A mapping f: $(X, t_1^*) \to (Y, t_2^*)$ is called s- continuous function if the inverse image of each fuzzy supra open set in (Y, t_2) is t_1^*fuzzy supra open in X . **[53]**

Definition 1.4.13 Let (X, t_1^*) and (Y, t_2^*) be two fuzzy supra topological spaces. A mapping $f: (X, t_1^*) \to (Y, t_2^*)$ is called fuzzy supra open map if the image of each fuzzy supra open in $t_1^*$is t_2^*fuzzy supra open in Y **.[53]**.

Definition 1.4.14.: Let (X, t_1^*) and (Y, t_2^*) be two fuzzy supra topological spaces. A mapping $f: (X, t_1^*) \to (Y, t_2^*)$ is called fuzzy supra closed map if the image of each fuzzy supra closed in $t_1^*$is t_2^* fuzzy supra closed in Y.

Definition 1.4.15.: Let (X, t^*) be a fuzzy supra topological space. A fuzzy point x_r is said to be an accumulation point of a subset S of X. If every nbd. of x_r of λ are quasi-coincident at some point different from the support X, where $x_r \in \lambda$.

Definition 1.4.16.: Let (X, t^*) be a fuzzy supra topological space. the union of all the accumulation points of a subset S of X is called the supra derived on S and is denoted by S^{Sd}.

1.5.: Coarser and Finer Topology

Definition 1.5.1.: If two fuzzy supra topologies $t_1^*$and t_2^*be such that $t_1^* \subset t_2^*$, we say that t_2^* is finer than $t_1^*$and $t_1^*$is coarser than t_2^*. $(t^*)'$ denotes the family of all t-Closed fuzzy supra sets.

1. 6: Base and Sub base

Definition 1.6.1.: Let (X, t^*) be a fuzzy supra topological space. A sub family B of t^* is a base for t^* if and only if each member of t^*can be expressed as the union of some members of B.

Definition 1.6.2.: Let (X, t^*) be a fuzzy supra topological space. A subfamily S of t^* is a sub base for t^* if and only if finite intersection of members of S form a base for t^*.

1.7. Compactness and Connectedness

Definition 1.7.1.: Let (X, t^*) be a fuzzy supra topological space. A family F of fuzzy supra open sets is a cover of a fuzzy supra open set μ if and only if $\mu \subset \{\cap \mu_\alpha : \mu_\alpha \in F\}$.

It is called a fuzzy supra open cover if each member μ_α is a fuzzy supra opens set. A sub cover of F is a subfamily of F which is also a cover of μ .It is called supra open cover of X if X= $\{\cap\mu_\alpha : \mu_\alpha \in F\}$.

Definition 1.7.2: A fuzzy supra topological space is supra compact if and only if every supra open cover has a finite sub cover.

Definition 1.7.3: Let (X, t*) be a fuzzy supra topological space and $\alpha \in [0, 1]$. A collection F of a fuzzy sets in X is called a α -shading of X if for each $x \in X$ $\exists$ a .g $\in F$ with g(x)> α and F is called α*- shading if g(x)$\geq \alpha$*.
A sub-collection of an α-shading (α*-shading) of X which is also an α - shading (α*-shading) is called an α-sub shading (α*-sub shading) of X.

Definition 1.7. 4: Two fuzzy set A_1 and A_2 in a fuzzy supra topological space (X, t^*) are said to be separated if and only if there exist $B_i \in t^*$ (i =1,2) such that $B_i \supset A_i$ (i=1, 2) and $B_1 \cap A_2 = \phi$, $B_2 \cap A_1 = \phi$.

CHAPTER-TWO

Fuzzy Supra Topological Spaces

2.1: **Introduction:** In this chapter we established certain fuzzy supra topological concept with new example and studied several properties of supra neighborhood.

2.2. Fuzzy Supra Topological Sets.

Theorem 2.2.1. Let τ be the collection of fuzzy supra closed sets in a fuzzy supra topological space (X, t*) satisfies: $0, 1 \in \tau$ and if $\lambda_s \in \tau^*$then $\bigcap_S \lambda_s \in \tau$ **[48]**

Proof: Since τ be the collection of fuzzy supra closed sets in a fuzzy supra topological spaces (X, t*). Then from definition τ^c is the collection of fuzzy supra open sets. Hence again from definition of fuzzy supra topological spaces $0, 1 \in \tau^c$ therefore there complement 1, 0 respectively members of τ . Now let $\lambda_s \in \tau$

then$\lambda_s' \in \tau^c$, where $s \in J$. Hence $\overset{S}{\cup} \lambda_s' \in \tau^c \Rightarrow \overset{S}{\cup} (1-\lambda_s) \in \tau^c \Rightarrow 1- \bigcap_S \lambda_s \in \tau^c$. Hence,

$\bigcap_S \lambda_s \in \tau$.This completes the proof.

Theorem2.2.2. [48] Let λ and μ are fuzzy supra open sets in the FSTS X, then

(1) λ is fuzzy supra open (fuzzy supra closed) if and only if $\lambda = \lambda^{SI}$ ($\lambda = \lambda^{SC}$)

(2) If $\lambda \leq \mu$, then $\lambda^{SI} \leq \mu^{SC}$ and $\lambda^{SC} \leq \mu^{SC}$.

(3) $\lambda^{SC} \cup \mu^{SC} \leq (\lambda \cap \mu)^{SC}$

(4) $\lambda^{SI} \cap \mu^{SI} \leq (\lambda \cap \mu)^{SI}$

(5) $\lambda^{SI} \cap \mu^{SI} \geq (\lambda \cup \mu)^{SI}$ (6) $\lambda^{SC} = 1-(1-\lambda)^{SI}$

Proof: (1) Firstly suppose $\lambda = \lambda^{SI}$ then from definition of fuzzy supra interior λ is the union of supra open set less than λ, so λ is fuzzy supra open.

Conversely suppose λ is fuzzy supra open in the FSTS X. Then λ is the union of fuzzy supra open set less than λ . Hence $\lambda = \lambda^{SI}$

(2) Since$\lambda \leq \mu$, Clearly from definition of fuzzy supra interior and fuzzy supra closure $\lambda^{SI} \leq \mu^{SI}$ and $\lambda^{SC} \leq \mu^{SC}$. The proofs for (3), (4), (5), (6) are similar. Now we give some examples

Example (2): Let X= {a, b, c, d}, with a fuzzy supra topology

$t^*= \{\{1, 0, \{(a, 0), (b, .5), (c, 1), (d, 0)\}, \{(a, .5), (b, .25), (c, 0), (d, 1)\}, \{(a, .5), (b, .5), (c, 1), (d, 1)\}$ on X.

Let $\lambda = \{(a, .5), (b, .5), (c, 1), (d, .25)\}$ and $\mu = \{(a, .75), (b, .5), (c, 1), (d, 1)\}$

$\lambda \leq \mu$ And $\lambda^{SI} = \{(a, 0), (b, .5), (c, 1), (d, 0)\}$

Again $\mu^{SI} = \vee\{\{(a, .5), (b, .5), (c, 1), (d, 1)\}, \{(a, .5), (b, .25), (c, 0), (d, 1)\}, \{(a, 0), (b, .5), (c, 1), (d, 1)\}$, therefore, it is clear that $\lambda^{SI} \leq \mu^{SI}$. Now the class of all fuzzy supra closed set of

$t^*= \{\{0, 1, \{(a, 1), (b, .5), (c, 0), (d, 1)\}, \{(a, .5), (b, .75), (c, 1), (d, 0)\}, \{(a, .5), (b, .5), (c, 0), (d, 0\}$ on X

Let $\lambda=\{(a,.25),(b,.5),(c,0),(d,.5)\}$ and $\mu=\{(a, .25), (b, .5), (c, .5), (d, .5)\}$ then $\lambda \leq \mu$ and $\lambda^{SC} =\{(a, 1), (b, .5), (c, 0), (d, 1)\}$ also $\mu^{SC} =\{(a, 1), (b, 1), (c, 1), (d, 1)\}$,Hence $\lambda^{SC} \leq \mu^{SC}$.

Example (3):-Let X= {a, b, c, d}, with a fuzzy supra topology

$t^*= \{\{1, 0, \{(a, 0), (b, .5), (c, 1), (d, 0)\}, \{(a, .5), (b, .25), (c, 0), (d, 1)\}, \{(a, .5), (b, .5), (c, 1), (d, 1)\}$ on X. Now the class of all fuzzy supra closed set of $t^*= \{\{0, 1, \{(a, 1), (b, .5), (c, 0), (d, 1)\}, \{(a, .5), (b, .75), (c, 1), (d, 0)\}, \{(a, .5), (b, .5), (c, 0), (d, 0\}$

Let $\lambda= \{(a, .25), (b, .5), (c, 0), (d, .5)\}$ and $\mu= \{(a, .25), (b, .5), (c, .5), (d, .5)\}$ then $\lambda^{SC}= \{(a, 1), (b, .5), (c, 0), (d, 1)\}$ and $\mu^{SC} = \{(a, 1), (b, 1), (c, 1), (d, 1)\}$

$\lambda^{SC} \cap \mu^{SC} =\{(a,1),(b,1),(c,1),(d,1)\}$ again $\lambda \cap \mu =\{(a,.25),(b,.5),(c,.5),(d,.5)\}$

$(\lambda \cap \mu)^{SC}=\{(a,1),(b,1),(c,1),(d,1)\}$ therefore $\lambda^{SC} \cap \mu^{SC} \leq (\lambda \cap \mu)^{SC}$

(4), (5) and (6) is very clear, we show only example.

Example (4): Let X= {a, b, c, d}, with a fuzzy supra topology

$t^*=\{\{1, 0, \{(a, 0), (b, .5), (c, 1), (d, 0)\}, \{(a, .5), (b, .25), (c, 0), (d, 1)\}, \{(a, .5), (b, .5), (c, 1), (d, 1)\}$ on X

Let $\lambda = \{(a, .25), (b, .5), (c, 1), (d, .25)\}$ and $\mu= \{(a, .75), (b, .5), (c, 1), (d, 1)\}$ then $\lambda^{SI} = \{(a, 0), (b, .5), (c, 1), (d, 0)\}$ also

$\mu^{SI} =\{(a,.5),(b,.5),(c,1),(d,1)\}$

Now $\lambda^{SI} \cap \mu^{SI} =\{(a,.5),(b,.5),(c, 1), (d, 1)\}$ and $\lambda \cap \mu=\{(a, .75), (b, .5), (c, 1), (d, 1)\}$, $(\lambda \cap \mu)^{SI}=\{(a, 1),(b, 1), (c, 1), (d, 1)\}$. Hence $\lambda^{SI} \cap \mu^{SI} \leq (\lambda \cap \mu)^{SI}$.

Example (5): Let X= {a, b, c, d}, with a fuzzy supra topology

$t^* = \{\{1, 0, \{(a, 0), (b, .5), (c, 1), (d, 0)\}, \{(a, .5), (b, .25), (c, 0), (d, 1)\},$

{(a, .5), (b, .5), (c, 1), (d, 1} on X.

Let λ= {(a, .5), (b, .5), (c, 1), (d, .25)} and μ= {(a, .75), (b, .5), (c, 1), (d, 1)} then λ^{SI} = {(a, 0), (b, .5), (c, 1), (d, 0)} also μ^{SI} = {(a, .5), (b, .5), (c, 1), (d, 1)} Now $\lambda \cup \mu$= {(a, .5), (b, .5), (c, 1), (d, 1)} and $(\lambda \cup \mu)^{SI}$={(a, 0), (b, .5), (c, 1), (d, 0)} $\lambda^{SI} \cup \mu^{SI}$ = {(a, 0), (b, .5), (c, 1), (d, 0)}, Hence $\lambda^{SI} \cup \mu^{SI} \geq (\lambda \cup \mu)^{SI}$.

Example (6): Let X= {a, b, c, d}, with a fuzzy supra topology

t^{*} = {{1, 0, {(a, 0), (b, .5), (c, 1), (d, 0)}, {(a, .5), (b, .25), (c, 0), (d, 1)},

{(a, .5), (b, .5), (c, 1), (d, 1} on X. Now the class of all fuzzy supra closed set of t*=

{{0, 1, {(a, 1), (b, .5), (c, 0), (d, 1)}, {(a, .5), (b, .75), (c, 1), (d, 0)},

{(a, .5), (b, .5), (c, 0), (d, 0)}

Let λ= {(a, .25), (b, .5), (c, 0), (d, .5)} then

λ^{SC} = {(a, 1), (b, .5), (c, 0), (d, 1)} now 1-λ= {(a, .75), (b, .5), (c, 1), (d, .5)}

$(1- \lambda)^{SI}$= {(a, .0), (b, .5), (c, 1), (d, .5)}

$1-(1-\lambda)^{SI}$={(a, 1), (b, .5), (c, 0), (d, 1)}

Hence$\lambda^{SC} = 1-(1-\lambda)^{SI}$ showed .

Implication 2.2.3: [48] We show that in a fuzzy topological space,

$\lambda^{-} \cup \mu^{-} = (\lambda \cap \mu)^{-}$ and $\lambda^{0} \cup \mu^{0} = (\lambda \cap \mu)^{0}$

but this is not satisfied in a fuzzy supra topological space, where λ and μ are fuzzy sets in the fsts .

Example Let X= {a, b, c, d}, with a fuzzy supra topology

t*={{0, 1, {(a, 0), (b, .5), (c, 1), (d, 0)}, {(a, .5), (b, .25), (c, 0), (d, 1)}, {(a, .5), (b, .5), (c, 1), (d, 1)on X. Then the class of all fuzzy supra closed set of t* is

$(t^*)^C$={{1, 0, {(a, 1), (b, .5), (c, 0), (d, 1)}, {(a, .5), (b, .75), (c, 1), (d, 0)},

{(a, .5), (b, .5), (c, 0), (d, 0)}}

λ ={(a,.25), (b, .5), (c, 0), (d,.5)}

μ= {(a, .25), (b, .5), (c, .5), (d, 0)}

λ^{-} = {(a, 1), (b, .5), (c, 0), (d, 1)}

μ^{-}= {(a,.5),(b,.75),(c,1),(d,0)} ,

$\lambda^{-} \cap \mu^{-}$ = {(a, 1), (b, .75), (c, 1), (d, 1)} but $\lambda \cap \mu$= {(a, .25), (b, .5), (c, .5), (d, .5)} so $(\lambda \cap \mu)^{-}$= {(a, 1), (b, 1), (c, 1), (d, 1)} therefore $\lambda^{-} \cap \mu^{-} \neq (\lambda \cap \mu)^{-}$.

Again Let X= {a, b, c, x} and t = {1, 0, λ, μ}

$1 = \{(a, 1), (b, 1), (c, 1),(x, 1)\}$

$0=\{(a, 0), (b, 0), (c, 0), (x, 0)\}$

$\lambda = \{(a, .2), (b, .5), (c, .7), (x, .9)\}$

$\mu = \{(a, 3), (b, .5), (c, .8), (x, .95)\}$

Closed fuzzy sets are 0, 1, $\lambda^{C,}$ μ^{C}

$0= \{(a, 0), (b, 0), (c, 0), (x, 0)\}$

$1= \{(a, 1), (b, 1), (c, 1), (x, 1)\}$

$\lambda^{C} =\{(a,.8), (b,.5), (c,.3),(x,.1),$

$\mu^{C}= \{(a, .7), (b, .5), (c, .2), (x, .05)\}$ the member t^{c} of are $\{0, 1, \lambda^{c}, \mu^{C}\}$ .Here intersection of all closed sets containing $\lambda$is 1 hence closure of λ is i.e $\lambda^{-} =1$ again $\mu^{-} =\{(a, 1), (b, 1), (c, 1), (x, 1)\}$, $\lambda^{-}\cap\mu^{-}=1$,

$\lambda\cap\mu= \{(a, .3), (b, .5), (c, .8), (d, .95)\}$ and $(\lambda\cap\mu)^{-}= 1$,

So $\lambda^{-}\cap\mu^{-}= (\lambda\cap\mu)^{-}$

Again let if $\lambda= \{(a, .5), (b, .5), (c, 1), (d, .25)\}$, and $\mu = \{(a, 1), (b, .5), (c, .5), (d, 0)\}$, then the union of supra open sets contained in λ is $\{(a, 0), (b, .5), (c, 1), (d, 0)$; hence $\lambda^{SI}= \{(a, 0), (b, .5), (c, 1), (d, 0)\}$ similarly

$\mu^{SI}= \{(a, .5), (b, .25), (c, 0), (d, 1)\}$, so

$\lambda^{SI} \cup\mu^{SI} = \{(a, 0), (b, 25), (c, 0), (d, 0)\}$

$\lambda\cup\mu= \{(a, 5), (b, 5), (c, 1),(d, .25)\}$,

$(\lambda\cup\mu)^{SI}= 0$, so $\lambda^{SI} \cup\mu^{SI} \neq (\lambda\cup\mu)^{SI}$ showed .

Now from above

Let $X= \{a, b, c, x\}$ and $t = \{1, 0, \alpha, \beta\}$where (X ,t) is fts

Where $1= \{(a, 1), (b, 1), (c, 1), (x, 1)\}$

$0= \{(a, 0), (b, 0), (c, 0), (x, 0)\}$

$\alpha= \{(a, .2), (b, .5), (c, .7), (x, .9)\}$

$\beta= \{(a, .3), (b, .5), (c, .8), (x, .95)\}$

Let $\lambda= \{(a, .5), (b, .5), (c, 1), (d, .25)\}$,

$\mu= \{(a, 1), (b, .5), (c, .5), (d, 0)\}$, $\lambda^{0}=0$ and $\mu^{0} =0$ and $\lambda^{0}\cap\mu^{0}=0$

$\lambda\cup\mu = \{(a, .5), (b, .5), (c, .5), (d, .0)\}$, $(\lambda\cup\mu)^{0}=0$

So $\lambda^{0}\cap\mu^{0}=0= (\lambda\cup\mu)^{0}$

2.3. Fuzzy Supra Neighborhood System.

Definition: 2.3.1 Let (X, t*) be a supra fuzzy topological space and u be a fuzzy set in X, then the supra s-neighborhood of a point p in the fuzzy set $w \in t^*$ such that $p \in u \leq w$. The collection of all neighborhood of a point is denoted by $N_P{}^S$ and called by supra neighborhood system of a point P.

Example: Let X={x, y, z} and $t^* = \{1, 0, u= \{(x, .5), (y, .25), (z, 0)\},$ $w = \{(x, .5), (y, .25), (z, .5)\}\}$.

Consider the supra fuzzy topology t^* on X is generated by {0, u, w, 1} observe that x_r=.25 $\in u \leq w$. Here w is the neighborhood of x_r.

Theorem 2.3.2.: Let λ be a fuzzy set in a supra fuzzy topological space(X, t*), then λ is fuzzy supra open iff for each point $p \in \lambda$ then $\lambda \in N_P{}^S$ where $N_P{}^S$ is the fuzzy supra neighborhood system of the point p.

Proof: Let λ be fuzzy supra open and a fuzzy point $x_r \in \lambda$. Then we have $x_r \in \lambda \subseteq \lambda$ therefore is λ a nhd of x_r.

Conversely, let λ is a neighborhood of each point contained in λ. Let { x_r } be the set of fuzzy points in λ and $\{y^r\}$ be the supra open fuzzy sets containing x_r such that $x_r \in y^r(x) \subseteq \lambda$ (such y^r exist as λ is an nhd of x_r). Now $x_r \in y_r \subseteq \lambda \Rightarrow$ the family $\{N_p^S$ Supra s-nbd. of a fuzzy point p in X} has the following properties

$x_r \subseteq y_r(x) \leq \lambda(x)\ \forall x \in X$

$\Rightarrow x_r \leq \sup y_r(x) \leq \lambda(x)\ \forall x \in X$

$\Rightarrow \sup x_r \leq \sup y^r(x) \leq \lambda(x)\ \forall\ x \in X$

$\Rightarrow \cup x_r(x) \leq \cup\ y_r(x) \subset \lambda.$

Also since $\cup\ x_r(x) = \lambda$. So $\lambda = \cup\ y_r(x) \Rightarrow \lambda$ is supra open.

Theorem 2.3.3 Let (X, t*) be a fuzzy supra topological space. Then

(1)$1 \in N_P{}^S$;

(2) $\lambda \in N_P{}^S \Rightarrow p \in \lambda$

(3) If$\lambda \geq \mu \in N_p^S \Rightarrow \lambda \in N_P{}^S$

(4) If x is the support of a fuzzy point p then ∃ fuzzy points q in X such that

$N_P^S = \cap \{N_q^s : 0<q(x) < p(\in x)\}$

(5) $\alpha_j \in N_P^S$ then $\cup \alpha_j \in N_P^S$

(6) If $\alpha \in N_P^S$ then $\exists\ \beta \in N_P^S$ such that $\beta \subseteq \alpha$ and $\beta \in N_q^S$ **[48]**

Proof:(1) $1 \in t^* \Rightarrow 1$ is supra open and hence 1 is a supra s-nbd of each of its fuzzy points $\Rightarrow 1$ is an nbd. of $p \in X \Rightarrow 1 \in N_P^S$,

(2) If $\lambda \in N_P^S$ then from definition $\exists\ \beta \in t^*$ such that $p \in \beta \subseteq \lambda \Rightarrow p \in \lambda$

(3) Given $\mu \in N_P^S$, then $\exists\ \gamma \in t^*$ such that $p \in \gamma \subseteq \lambda \Rightarrow \lambda \in N_P^S$.

(4)Since x is the support of a fuzzy point p then for $0<\delta<1$ then $p(x) = \delta$, let the value of the fuzzy point q are $\delta \leq \gamma \leq 1$ then $p(x) \subseteq q(x)$, so the value of N_P^S is infimum, So $N_P^S = \cap \{ N_q^s : 0< q(x)<p(x)\}$

(5)If $\alpha_j \in N_P^S$ then $\beta_j \in N_P^S \Rightarrow p \in \cup \beta_j \subseteq \cup \alpha_j \Rightarrow \cup \alpha_j \in N_P^S$ for $j \in J$

(6) Finally if $\alpha \in N_P^S$ then $\exists\ \beta \in t^*$such that $p \in \beta \subseteq \alpha$ so $\beta$is supra open . Thenβ is an nhd .of $p \Rightarrow \beta \in N_P^S \Rightarrow p \in \beta \subseteq \alpha$

Theorem 2.3.4.: Let (X, t^*) be a fuzzy supra topological space. Then the family $\{N_P^S$: supra s-nbd. of a fuzzy point p in X$\}$ has the following properties.

(1)$1 \in N_P^S$

(2) $\lambda \in N_P^S \Rightarrow p \in \lambda$

(3) If $\lambda \geq \mu \in N_P^S \Rightarrow \lambda \in N_P^S$

(4) If x is the support of a fuzzy point p then $\exists$ fuzzy points q in X such that $N_P^S = \cap \{ N_P^S :0< q(x)< p(x)\}$

(5) $\alpha_j \in N_P^S$ then $\cup \alpha_j \in N_P^S$

(6) If $\alpha \in N_P^S$ then $\exists\ \beta \in N_P^S$ such that $\beta \subseteq \alpha$ and $\beta \in N_q^s$ then there exist a supra topology t^* on X and with the property (1)- (5) if N_q^s is family of s-nbd. of a point q then $N_P^S = N_q^s$.

Proof: We define t^* as follows a fuzzy set $\mu \in t^*$, iff $\mu \in N_P^S$, $\forall\ p \in \mu$, then t^* is a fuzzy supra topology on X.

(1) $0 \in t^*$, since $0 \in N_P^S$ as $p \in 0 \leq \mu\ \forall$ fuzzy set μ in X, $\forall\ \mu \in t^*$

(2) By the property (1) $1 \in N_P^S$ and since any $p \in 1 \Rightarrow 1 \in t^*$,

(3) Suppose $\alpha_j \in t^*$, for $j \in J \Rightarrow \alpha_j \in N_P^S\ \forall\ p \in \alpha_j$.

$\Rightarrow \alpha_j \in N_P^S \in \forall\ p \in \cup\{\alpha_j : j \in J\}$

$\Rightarrow \cup\{\alpha_j : j \in J\} \in N_P^S\ \forall\ p \in \cup\{\alpha_j : j \in J\}$

$\Rightarrow \cup\{\alpha_j : j \in J\} \in t^*$. Thus t^* is a supra topology on X. Next we show that

$N_P^S = N_q^s$. Let $\alpha \in N_P^S$ by property (6) $\exists \beta \in N_P^S$ such that $\beta \subseteq \alpha$ and $\beta \in N_q^s$ for all $q \in \beta$

$\Rightarrow \beta \in t^*$, β is the t^* supra open set s.t $p \in \beta \subseteq \alpha$

$\Rightarrow \alpha$ is the t^* nbd of p . $\Rightarrow \alpha \in N_q^s \Rightarrow N_P^S \subseteq N_q^s$ (A)

Conversely let $\alpha \in N_q^s$ Let α is the t^* nbd. of q . So $\exists$ supra open set β s.t $p \in \beta \subseteq \alpha$.

Now $p \in \beta \subseteq \alpha \Rightarrow \beta \in N_P^S$; $\forall\ \beta$ But $\beta \in N_P^S$ and $\beta \subseteq \alpha \Rightarrow \alpha \in N_P^S$; hence $N_q^s \subseteq N_P^S$

From (A) and (B) we get $N_P^S = N_q^s$.

2.4. <u>Fuzzy Product</u>

The concepts of fuzzy products have already appeared in the research work of Hutton [**42**], Mashhour et al [**9**].

Definition2.4.1: If λ_1 and λ_2 be two supra subsets of X and Y respectively then the Cartesian product $\lambda_1 \times \lambda_2$ of fuzzy supra subsets of λ_1 and λ_2 is a fuzzy subset of X×Y defined by $(\lambda_1 \times \lambda_2)(x, y) = \min\{\lambda_1(x), \lambda_2(y)\}$ for each pair $(x, y) \in X \times Y$.

Definition 2.4.2: Let $(x_1, t_1^*)$and (x_2, t_2^*)be two fuzzy supra fuzzy topological spaces the product fuzzy supra topology is the collection $u_1 \times u_2$ where $u_1 \in t_1^*$and $u_2 \in t_2^*$, it is denoted by $x_1 \times x_2$

Proposition 2.4.3: If u_1 is a fuzzy supra subset of a fuzzy supra topological space (X, t*).and u_2 is a fuzzy supra subset of a fuzzy supra topological spaces (Y, s*) then show that $\overline{u_1 \times u_2} \subseteq \overline{u_1} \times \overline{u_2}$.

Proof: With the help of an example we shall show the proposition.

Let X= {a, b} be a set with fuzzy supra topology $u_1 \times u_2$

$t^* = \{1, 0, \{(a, .5), (b, .5)\}\}$ Again let Y={x, y} and

$s^* = \{1, 0, \{(x, 1), (y, .35)\}\}$ be a supra topology on Y

Let $u_1 \in t^*$ and $u_1 = \{(a, .5), (b, .5)\}$

Also $u_2 \in s^*$, and $u_2 = \{(x,1),(y,.35)\}$

then

$u_1 \times u_2$ ={(a, x, .5), (a, y, .35), (b, x, .5), (b, y, .35)} also closed sets of

$t^* = \{0, 1, \{(a, .5), (b, .5)\}\}$ and closed sets of $s^* = \{0, 1, \{(x, 0), (y, .65)\}\}$

Product fuzzy supra topology = {{(a, x, 0), (a, y, 0), (b, x, 0), (b, y, 0)}, {(a, x, 1), (a, y, 1), (b, x, 1), (b, y, 1)}, {(a, x, .5), (a, y, .65), (b, x, 0), (b, y, .65)}, {(a, x, .5), (a, y, .5), (b, x, 0), (b, y, .5)},{(a, x, .5), (a, y, .5), (b, x, .5), (b, y, .5)}}

Now $u_1 \times u_2$ = {(a, x, .5), (a, y, .5), (b, x, .5), (b, y, .5)}

$\overline{u_1}$ = {(a, .5), (b, .5)}, $\overline{u_2}$ = {(x, 1), (y, 1)}

$\overline{u_1} \times \overline{u_2}$ = {(a, x, .5), (a, y, .5), (b, x, .5), (b, y, .5)}.

Hence $\overline{u_1 \times u_2} \subseteq \overline{u_1} \times \overline{u_2}$ showed.

Theorem 2.4.4.: Let (x_i, t_i^*) where $\forall$ i $\in$J be fuzzy supra topological spaces and X= $\prod_{i \in J} x_i$ where J is the index set and t^* is the product fuzzy supra topology on X.Then (X, t*) is a fuzzy supra topological space.

Proof: Since $\forall$ i $\in$J , (x_i, t_I^*) is fuzzy supra topological spaces . We shall prove that (X, t^*) is supra fuzzy topological spaces. Clearly 0, 1$\in$ t*.As 0, 1$\in$ t_I^* $\forall$ i$\in$ J. Let $\lambda \in t^*$ then $\lambda = \prod_{i \in J} \lambda_{ij}$ where $\lambda_{ij} \in t_i^*$ and $\vee_{i \in J} \lambda_{ij} \in$, t_I^* and each λ_{ij} is fuzzy supra open. So as λ is min λ_{ij}, Hence (X, t*) is fuzzy supra topological space.

2.5. Fuzzy Mapping .

Definition 2.5.1.: A fuzzy mapping f :(X, t) $\rightarrow$(Y, s^*) is called fuzzy s-open if the image of each fuzzy open set in (X, t) is s^* is fuzzy supra open in (Y, s*).

Theorem 2.5.2.: Let (x_1, t_1^*), (x_2, t_2^*) , (y_1, S_1^*),(y_2, S_2^*) be four supra fuzzy topological spaces, Let $f_1 : x_1 \to y_1$ and $f_2 : x_2 \to y_2$ are supra fuzzy continuous then $f_1 \times f_2 : x_1 \times x_2 \to y_1 \times y_2$ are fuzzy continuous function.

Proof: Let λ be a supra open set in $y_1 \times y_2$ then $\lambda = \cup\ \lambda_s \times \mu_r$ where λ_s and μ_r are supra open set in σ_1^* and σ_2^* respectively. Since $f_1 : x_1 \to y_1$ is supra fuzzy continuous $f^{-1}(\lambda_s)$is supra open in x_1 for all supra open set in y_1, Similarly $f_2 : x_2 \to y_2$ is supra fuzzy continuous then $f_2^{-1}(\mu_r)$ is supra open in x_2 . Now

$(f_1 \times f_2)^{-1}(\lambda) = (f_1 \times f_2)^{-1}(\cup\ \lambda_s \times \mu_r) = \cup((f_1 \times f_2)^{-1}(\lambda_s \times \mu_r)$

$= \cup\ f_1^{-1}(\lambda_s) \times f_2^{-1}(\mu_r)$ is fuzzy supra open in $x_1 \times x_2$.

Thus, $f_1 \times f_2$ is FS-continuous.

Lemma 2.5.3.: Let f: $X \to Y$ is FS-continuous and g: $Y \to Y$ is continuous then g of is FS-continuous.

Proof: Let λ be a fuzzy open set in Y. Since g: $Y \to Y$ is fuzzy continuous and obviously $g^{-1}(\lambda)$ is open in Y. Then $g^{-1}(\lambda) \in S$. Where S is the fuzzy topology on Y. Again given that $f : X \to Y$ be FS- Continuous . So that $f^{-1}(g^{-1}(\lambda))$ $\in X \Rightarrow (g \circ f)^{-1}(\lambda) \in t^*$. Hence $g \circ f$ is FS- continuous

For this, we give an example,

Example2.5.4: Let X= {p, q, r}, Y={x, y, z} with two fuzzy topology t_1 on X and t_2 on Y, Where t_1= {1, 0, {(p, .5), (q, .5), (r, 0)}}

t_2= {1, 0, {(x, .5), (y, .25), (z, .5)}}

Let t_1^* ={1,0, {(p, .5),(q, .5),(r, 0)}, {(p, .5),(q, .5),(r, .5)},{(p, .5), (q, .25), (r, .5)}} be a fuzzy supra topology on X, and $t_1 \subset t_1^*$ Again

t_2^*= {1,0, {(x, .5), (y ,.25), (z , .5)}, {(x, .5), (y ,.5), (z , 0)} be a supra topology on Y, and $t_2 \subset t_2^*$ then consider f={(p, x), (q, y), (r, z)} be a mapping from t_1^* to t_2^*

Then f^{-1}{(x, .5),(y ,.25), (z , .5)}= {(p, .5),(q, .25),(r, .5)} $\in t_1^*$ so f is FS-continuous

Let g: $Y \to Y$ be a mapping defined by

g= {(x, x), (y, y), (z, z)} from t_2 to t_2 then clearly g is fuzzy continuous

$(g \circ f)^{-1}$ ({(x, .5),(y,.25),(z, .5)})=$f^{-1}(g^{-1}$

{(p, .5), (q, .25), (r, .5)})= f^{-1}{(p, .5), (q, .25), (r, .5)} = {(p, .5), (q, .25), (r, .5)} $\in t_1^*$
So $g \circ f$ is FS-continuous.

Lemma 2.5.5.: Prove that every fuzzy continuous function is fuzzy supra continuous, but the converse is not necessarily true

Proof: Let (X, t) and(Y, s) be two fuzzy topological space with associated fuzzy supra topology (X, t*) and (Y, s*). Let the function f: (X, t)→(Y ,s) is continuous then $\forall \mu \in s \Rightarrow f^{-1}(\mu) \in t$ .Now since $t \subset t^*$ and $s \subset s^*$ then $\forall\ \mu \in s \Rightarrow \mu \in s^*$ and $f^{-1}(\mu) \in t^*$ .So f is fuzzy supra continuous.

Conversely $\forall \mu \in s^*$ and $f^{-1}(\mu) \in t^*$, $\mu \notin S$ and $f^{-1}(\mu) \notin t$ .So if f is fuzzy supra continuous then f is not fuzzy continuous.

Lemma 2.5.6.: Prove that if f: X→Y and g: Y→Z are FS-continuous then $g \circ f$ is FS-continuous.

Proof: λ be a supra open set in Z. Since g: Y→Z, FS-continuous so $g^{-1}(\lambda) \in Y$,
Again given that f :X→Y is FS-continuous so = $f^{-1}(g^{-1}(\lambda) \in X \Rightarrow (g \circ f)^{-1}(\lambda) \in X$ .Hence, $(g \circ f)$ is FS-continuous.

Lemma 2.5.7.: Prove that every fuzzy open map is fuzzy s-open map and also every fuzzy supra open map is s-open but the converses of these implication is not true .

Proof :- Let (x ,t) and (y, s) below fuzzy topological spaces with the associated fuzzy supra topology (x , t*) and (Y , S*) . Let the function f : (x, t) → (Y , S) is fuzzy open then $\forall$ fuzzy set $\lambda \in t$,f (λ) is fuzzy open in S . Now since $S \subset S^*$ then $\forall\ f(\lambda) \in S \Rightarrow f(\lambda) \in s^* \Rightarrow$ f is fuzzy s–open.

Again let g: $(X, t^*) \rightarrow (Y, s^*)$ is a supra fuzzy open map, then $\forall$ fuzzy supra open set $\mu \in t^*$, g (μ) is fuzzy supra open in s^*. Since every member of t is the member of $t^* \Rightarrow \forall \mu \in t, \mu \in t^* \Rightarrow$ g(μ) is fuzzy supra open in s^*. Therefore, g is s-open.
We shall prove this lemma by using a example.

Example 2.5.8.: Let X be a non empty set and I = [0, 1], and let λ, μ, ω be the fuzzy sets defined respectively by ,

λ (x) =2x when $0 \le x \le \frac{1}{2}$

$= \frac{1}{2}$ when $\frac{1}{2} < x \le 1$

μ (x) $= \frac{1}{2}$ when $0 \le x < \frac{1}{4}$

= 2x when $\frac{1}{4} \le x \le \frac{1}{2}$

=0 when $\frac{1}{2} < x \le 1$

ω (x) =1 when $0 \le x \le \frac{1}{2}$

= 0 when $\frac{1}{2} < x \le 1$. **[53]**

Let $t_1 = \{0, \lambda, 1\}$ be a fuzzy topology on X, and let

$t_1^* = \{0, \lambda, 1, \mu, \omega, \lambda \vee \mu, \lambda \vee \omega\}$, Let f: $(x, t_1) \to (x, t_1)$ be a fuzzy open map on a fuzzy set is defined by $f(x) = x$ if $0 \le x \le \frac{1}{2}$

$= 1 - x$ if $\frac{1}{2} < x \le 1$

Now $[f(\lambda)](x) = \text{Sup}\,[\lambda(x)] = 2 \times \frac{1}{2} = 1$ when $0 \le x < \frac{1}{2}$

= 2x when $\frac{1}{4} \le x \le \frac{1}{2}$

= Sup $[\lambda(x)] = 1 - 1 = 0$, if $\frac{1}{2} < x \le 1$

Hence f (λ) =μ.

Again $[f(1)](x) = \text{Sup}\,[1(x)] = 1$ if $0 \le x \le \frac{1}{2}$ =Sup $[1(x)] = 1 - 1 = 0$ if

$\frac{1}{2} < x \le 1$ since λ and μare fuzzy supra open in t_1^*, hence f is a fuzzy s-open mapping but λ and μ are not fuzzy open in t_1, hence f is not fuzzy open mapping.

Lemma 2.5.9.: Let $f: (X, t^*) \to (Y, s^*)$ be bijective mapping , Then the followings are equivalent

(1) f is a homomorphism.

(2) f is continuous and supra open.

(3) f is continuous and supra closed. **[53]**

Proof: First we will show that (1) $\Rightarrow$ (2)

Since f is homo. Then both f and f^{1} = g (say) is continuous, and since f is bijective then f^{-1} is also bijective. Let $\alpha \in t^*$, as g is continuous. $g^{-1}(\alpha)=f(\alpha)$ is supra open in Y .$\Rightarrow$ f is supra open. Thus (1) $\Rightarrow$ (2)

(2)$\Rightarrow$(1) .Given f is bijective, continuous, and supra open ,Let α be a supra open set in X$\Rightarrow$ f(α)is supra open in Y. As previous g^{-1} = f is supra open in Y. So g= f^{1} is continuous. So f is a homomorphism.

(1) $\Rightarrow$ (3) Let α be supra closed set in X. Then 1-α is fuzzy supra open in Y. Since f is homomorphism. So both f and g= $f^{1}$is continuous, it follows that g^{-1} (1-α) is supra open in Y. But $g^{-1}(1 - \alpha) = 1 - g^{-1}(\alpha) \Rightarrow$

$1 - g^{-1}(\alpha) \in s^*$. Hence $g^{-1}(\alpha) \in (s^*)^c$ thus f is supra closed. Hence (1) $\Rightarrow$ (3)

(3)$\Rightarrow$(1) .

Let $\alpha \in t^* \Rightarrow 1-\alpha \in (t^*)^C$, Since f is supra closed , $f(1-\alpha)= g^{-1}(1-\alpha)=$

$1 - g^{-1}(\alpha) \in (s^*)^c \Rightarrow g^{-1}(\alpha) \in s^*$. $\Rightarrow$ g is continuous and supra closed. So (3) $\Rightarrow$(1)

Lemma 2.5.10.:- Let $f:(X, t^*) \rightarrow (Y, s^*)$ be bijective mapping

Then the followings statements are equivalent.

(1) f is a fuzzy supra open mapping.

(2) f is fuzzy supra closed mapping

(3) f is a fuzzy supra continuous mapping.

Proof: (1) $\Rightarrow$ (2) Firstly we shall show that if f is a fuzzy supra open mapping then f is fuzzy supra closed. Let α be a fuzzy supra closed set in X. Then 1-α is supra open set in X. Since f is fuzzy supra open then $f(1-\alpha) = 1-f(\alpha)$ is supra open in Y. Hence f(α) is supra closed in Y. This implies f is fuzzy supra closed.

(2) $\Rightarrow$(3) Let α be a fuzzy supra closed set in X. We have since f is fuzzy supra closed, f (α) is fuzzy supra closed in Y, $f^{1}(f(\alpha)) = \alpha \in t^*$ as f is bijective. Hence f is continuous.

(3) $\Rightarrow$ (1), since f is fuzzy supra continuous. Then $f^{-1}(\alpha) \in t^*$, for each supra open set $\alpha \in s^*$, Hence it is clear that f is fuzzy supra open.

Implication: With example, we prove that if f is FS-continuous, but f is not fuzzy continuous.

Example 2.5.11.: Let X= {p, q, r}, Y={x, y, z} with two fuzzy t_1 on X and t_2 on Y, Where t_1= {1, 0, {(p, .5), (q, .5), (r, 0)}}

t_2 = {1, 0, {(x, .5), (y, .25), (z, .5)}}

Let t_1^*= {1, 0, {(p, .5), (q, .5), (r, 0)}, {1, 0, {(p, .5), (q, .5), (r, .5)}}, {1, 0, {(p, .5), (q, .25), (r, .5)}} is a fuzzy supra topology on X, and $t_1 \subset t_1^*$ Again t_2^*= {1, 0, {(x, .5), (y ,.25), (z , .5)}, {(x, .5), (y, .5),(z, 0)}be a Fuzzy supra topology on Y. $t_2 \subset t_2^*$ then consider f={(p, x),(q, y),(r, z)} be a mapping from t_1^* to t_2^*. Then f^{-1}{(x, .5),(y ,.25),(z , .5)}= {(p, .5),(q, .25),(r, .5)} $\in t_1^*$ so f is FS-continuous, but f is not fuzzy continuous. Because under the composition of mapping

f^{-1}{(x, .5),(y ,.25),(z , .5)} $\notin t_1$

Lemma 2.5.12.: Show the property if f: $X \rightarrow Y$ is FS-continuous and g: $Y \rightarrow Y$ is continuous then g o f is FS-continuous

Proof: Let λ be a supra open set in Z. Since g: $Y \rightarrow Y$, FS-continuous so $g^{-1}(\lambda) \in Y$, Again given that f : $X \rightarrow Y$ is FS-continuous so $f^{-1}(g^{-1}(\lambda) \in X \Rightarrow$ $(g \circ f)^{-1}(\lambda) \in X$ .Hence, $(g \circ f)$ is FS-continuous.

Example: For this, we give an example, Let X= {p, q, r}, Y={x, y, z} with two fuzzy topology t_1 on X and t_2 on Y, Where

t_1= {1, 0, {(p, .5), (q, .5), (r, 0)}}

t_2= {1, 0, {(x, .5), (y, .25), (z , .5)}} Let t_1^* ={1,0, {(p, .5),(q, .5),(r, 0)}, {1,0, {(p, .5), (q, .5), (r, .5)}}, {1, 0, {(p, .5),(q, .25),(r, .5)}} be a fuzzy supra topology on X, and $t_1 \subset t_1^*$ Again t_2={1, 0, {(x, .5), (y ,.25), (z , .5)}, {(x, .5), (y ,.5),(z , 0)}be a supra topology on Y, $t_2 \subset t_2^*$ then consider f={(p, x), (q, y), (r, z)} be a mapping from t_1^* to t_2^* . Then f^{-1}{(x, .5), (y, .25), (z, .5) = {(p, .5), (q, .25), (r, .5)} $\in t_1^*$ so f is FS-continuous. Let g: $Y \rightarrow Y$ is a mapping defined by g= {(x, x), (y, y), (z, z)} from t_2 to t_2^*then clearly g is fuzzy continuous

$(g \circ f)^{-1}$({(x, .5), (y, .25), (z, .5)})

$=f^{-1}(g^{-1}\{(p, .5), (q, .25), (r, .5)\})$

$= f^{-1}\{(p, .5), (q, .25),(r, .5)\}$

$=\{(p, .5), (q, .25), (r, .5)\} \in t_1^*$

So $g \circ f$ is FS- continuous.

CHAPTER- THREE

R_0 and R_1 Fuzzy Supra Topological Spaces

3.1: Introduction:- In this chapter, we introduce and study some R_0 and R_1 properties in fuzzy supra topological space and obtain their several features. In short we write FSR_0 and FSR_1 for fuzzy supra R_0 and R_1 respectively.

3.2. R_0 Fuzzy Supra Topological Space.

3.2.1. Definition (fuzzy Supra R_0 space):-

Let (X, t*) be fuzzy supra topological space. In view of the multiplicity of the possible definitions, we currently name as FSR_0 (i), FSR_0 (ii), FSR_0 (iii) and FSR_0 (iv).

(a) (X, t*) is an FSR_0 (i) space iff $\forall$ x, y$\in$X, x$\neq$ y, whenever there exist $\lambda \varepsilon t^*$ with $\lambda(x) =1$ and $\lambda(y) =0$ then there also exist $\mu \varepsilon t^*$ with $\mu(x) =0$, and $\mu(y) =1$.

(b) (X, t*) is an FSR_0(ii) space if and only if $\forall$ x, y$\in$X, x$\neq$ y, whenever there exist $\lambda \varepsilon t^*$with $\lambda(x)>0$ and$\lambda(y) =0$ then there also exist $\mu \varepsilon t^*$ with $\mu(x) = 0$, and $\mu(y)>0$.

(c) (X, t*) is an FSR_0 (iii) space if and only if $\forall$ x, y$\in$X, x$\neq$y, whenever there exist $\lambda \varepsilon t^*$ with $0\leq \lambda(x) \leq \alpha<\lambda(y) \leq 1$ then there also exist ε t* with

$0\leq \mu(y) \leq \alpha<\mu(x) \leq 1$, where $\alpha\in I_1$.

(d) (X, t*) is an FSR_0 (iv) space if and only if $\forall$ x, y$\in$X, x$\neq$y, whenever there exist $\lambda \varepsilon t^*$ with $\lambda(x) <\lambda(y)$ there also exist $\mu \varepsilon t^*$ with $\mu(x)> \mu(y)$.

3.2.2 Now we give example to show the non implications among FSR_0 (i), FSR_0 (ii), FSR_0(iii), FSR_0 (iv).

Proof: For this, we give some example,

Example (1) Let X={x, y, z} and

$t^* = \{1, 0, \lambda= \{(x, 1), (y, \frac{1}{2}), (z, 0)\}, \{(x, 1), (y, \frac{1}{2}), (z, \frac{1}{2})\},$

$\{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}\}$ on X.

Let $\lambda= \{(x, 1), (y, \frac{1}{2}), (z, 0)\}$.Here $\lambda(x) =1$, $\lambda(z) =0$, but there does not exist any

$\mu \in t^*$ such that $\mu(z)=1$ and $\mu(x)=0$ but $\lambda(x)>0$and $\lambda(z)=0$, also $\mu(x)=0$ and $\mu(z)>0$, Hence clearly (X, t*) is FSR_0(ii) but not FSR_0(i),

$$\text{So } FSR_0(ii) \not\Rightarrow FSR_0(i)$$

Example (2) Let X={x, y, z} and

$t^*=\{1,0, \lambda=\{(x, 1),(y, \frac{1}{3}), (z, 0)\}, \{(x, 1), (y, \frac{1}{2}), (z, \frac{1}{2})\}, \mu=\{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}\}$ on X. Here $\lambda(x)=1$, $\lambda(y)=\frac{1}{3}$, $\mu(x)=0$, $\mu(y)=\frac{1}{2}$ taking $\alpha=.4$

also $0\leq\mu(x)\leq.4<\mu(y)\leq1$ with $0\leq\lambda(y)\leq.4<\lambda(x)\leq1$

So (X, t*) is an FSR_0(iii) but (X, t*) is not FSR_0 (i)

Example (3) Let X={x, y, z} and

t*={1,0, u={(x, 1), (y, .4), (z, 0)},

w = {(x, .9), (y, .5), (z, 0)}, v= {(x, 0), (y, 0), (z, 1)}} on X.

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, w, 1} observe that (X, t^*) is an FSR_0(i) but (X, t*) is not FSR_0(iv) for $\alpha=.6$, $w(x)>w(y)$ there does not exist $u(x)<u(y)$, So $FSR_0(i) \not\Rightarrow FSR_0(iv)$

Example (4) Let X={x, y, z} and u, v εI^X where

t* = {1, 0, u ={(x, .8), (y, .7), (z, .3)},

v = {(x, .3), (y, .8), (z, .2)},

w= {(x, .8), (y, .8), (z, .3)}}

Consider the fuzzy supra topology t* on X is generated by {0, u, v, w, 1} for $\alpha=.75$, $0\leq v(x)\leq.75<v(y)\leq1$ and $0\leq u(y)\leq.75<u(x)\leq1$ so (X, t*) is an FSR_0(iii) but (X, t*) is not FSR_0(iv), since $u(y)>u(z)$, but no v(y) with

$v(y)<v(z)$.

Theorem 3.2.3: Prove that (X, t*) is FSR_0(ii) if and only if (X, t*) is FSR_0(iii)

Prove: Let (X, t*) is FSR_0(ii) space, we shall prove that (X, t*) is FSR_0(iii) Let x, y $\in X$ with $x\neq y$ and $v\in t^*$such that $0\leq v(x)\leq0<v(y)\leq1$

.i, e $v(x)=0$ and $v(y)>0$ since (X, t*) is FSR_0 (ii) space then $\exists$ $u\varepsilon t^*$ such that $u(x)>0$, $u(y)=0$ then it is clear that $0\leq u(y)\leq0<u(x)\leq1$ Hence (X, t*) is FSR_0(iii) where $\alpha=0$

Conversely suppose that (X, t*) is FSR_0(iii), we shall prove that (X, t*) is FSR_0(ii). Let x, y εX with $x \neq y$ and vε t* such that v(x) =0 and v(y)>0, it can be written as $0 \le v(x) \le 0 < v(y) \le 1$. Since (X, t*) is FSR_0(iii) space then ∃ $u \in t^*$ such that $0 \le u(y) \le 0 < u(x) \le 1$ that is u(y)=0 and u(x)>0, Hence (X, t^*) is FSR_0(ii) this completes the proof.

Theorem 3.2.4: Let (X, t*) be a fuzzy supra topological space and $I(t^*) = \{u^{-1}(0, 1]; u\varepsilon t^*\}$ then

(a) (X, t*) is FSR_0(iii) if and only if (X, I(t*)) is supra R_0.

(b) (X, t*) is FSR_0(i) <≠> (X, I (t^*)) is supra R_0.

(c) (X, t*) is FSR_0(ii) <≠> (X, I (t*)) is supra. R_0.

(d) (X, t^*) is FSR_0 (iv) <≠> (X, I (t^*)) is supra R_0.

Proof :- (a) Let (X, t*) is FSR_0(iii), we shall prove that (X, I(t*)) is supra R_0. Let x, y εX with x≠y and Mε I(t*)with xεM,

y∉M, or x∉M, yεM suppose $x \in M$, y∉M, we can write $M= u^{-1}(0, 1]$, where uε t*. that is

$0 \le u(y) \le 0 < u(x) \le 1$ Since (X, t*) is FSR_0(iii), Then ∃ vε t*, such that $0 \le v(x) \le 0 < v(y) \le 1$, that is v(x)= 0, and v(y)>0, it follows that $x \notin v^{-1}(0,1]$,

$y \varepsilon v^{-1}(0, 1]$, and also $v^{-1}(0, 1] \varepsilon I(t^*)$.

Hence it is clear that (X, I(t*)) is supra R_0.

Conversely suppose that (X, I(t*)) is supra R_0 we shall prove that (X, t*) is FSR_0(iii) .Let x, y $\in$X with $x \neq y$ and $u \in t^*$.with $0 \le u(x) \le 0 < u(y) \le 1$,

that is u(x)=0,and u(y)>0 ,it follows that $x \notin u^{-1}(0,1]$,$y \varepsilon u^{-1}(0,1]$, and $u^{-1}(0,1] \varepsilon I(t^*)$. Since(X, I(t*)) is supra R_0 , then ∃ Mε I(t*) ,such that xεM, y ∉ M , we can write $M= v^{-1}(0, 1]$, where vεt*.It follows that v(x)>0,v(y)=0, i.e $0 \le v(y) \le 0 < v(x) \le 1$. Hence it is clear that (X, t*) is FSR_0(iii). This completes the proof (a). To prove (b), (c), (d) we shall give some example.

Example: 1. Let X={x, y, z} and u, v, w εI^X where

t* ={1, 0, u={(x, 1), (y, 0), (z, .3)},

v = {(x, 0), (y, 1), (z, .2)},

w = {(x, 1), (y, 1), (z, .3)}}

Consider the fuzzy supra topology t*on X is generated by {0, u, v, w 1 } , now u(x)=1,u(y)=0 and v(x)=0, v(y)=1 . Hence it is clear that (X, t*) is FSR_0(i) .

Again I(t*)={X, {x, z}, {y, z}} It is clear that I(t*) is a supra topology on X .and (X, I(t*)) is not supra R_0 , since y, zεX ,$y \neq z$ and $\{x, z\} \in I(t^*)$ With

zε{x, z}, $y \notin \{x, z\}$ but there is no Uε I(t*) with $z \notin U$, yεU Thus (X, t*) is

FSR_0(i) <≠> (X,I(t*)) is supra R_0.

Example: 2. Let X={x, y, z,} and u, v, w εI^X where

t*={1, 0, u={(x, 1), (y, 0), (z, .3)},

v= {(x, 0), (y, 1), (z, .2)},

w = {(x, 1), (y, 1), (z, .3)}}

Consider the fuzzy supra topology t*on X is generated by {0, u, v, w, 1 } ,Now u(x)=1, u(y)=0 and v(x)=0, v(y)=1 .Hence it is clear that (X, t*) is FSR_0(ii) and also FSR_0(iv) .

Again I(t*)={X, {x, z}, {y, z}} It is clear that I(t*) is a supra topology on X , and (X, I(t^*)) is not supra R_0, since y, zεX, $y \neq z$ and {x, z}εI(t*) with

zε{x, z} , $y \notin \{x, z\}$ but there is no Uε I(t*) with $z \notin U$, yεU .Thus (X, t^*) is

FSR_0 (iv) <≠> (X,I(t^*)) is supra R_0.

Example:3. Let X={x, y} and u, v, wεI^X where

t* ={1, 0, u={(x, 1), (y, 0)},

v = {(x, 0), (y .2)}, w={(x, 1), (y, .2)}}

Consider the fuzzy supra topology t* on X is generated by {0, u, v, w, 1}, Now u(x) =1, u(y)=0 and v(x)=0, v(y)=.2, Hence it clear that (X, t*)is not FSR_0 (i) .

Again I(t*)={X, {x},{y}} It is clear that I(t*) is a supra topology on X and (X, I(t*)) is supra R_0,since x, yεX ,$x \neq y$ and {x}ε I(t*) with xε{x} , $y \notin \{x\}$but

there is {y}ε I(t^*) with $x \notin \{y\}$, $y \in \{y\}$.

Thus (X, t*) is FSR_0(i) < ≠ > (X, I(t*)) is supra R_0 .

Example: 4 . Let X={x, y} and u, v, $w \in I^X$ where

t*={1, 0, u={(x, 1), (y, 0)},v ={(x, .2), (y, 0)}}

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, 1 } , now u(x)=1, u(y)=0 and v(x)=.2, v(y)=0, Hence it clear that (X, t*)is not FSR_0(iv). Again I(t*) ={X} It is clear that I(t*) is a supra topology on X, and (X, I(t*)) is supra R_0 .

Definition 3.2.5.: Let f be a real valued function on a fuzzy supra topological space, If {x: f(x)> α} is supra open and for every real α where $\alpha \in I_I$ then f is called lower semi continuous function.

Definition 3.2.6: Let X be non empty set and T* be a supra topology on X, and let t*=ω(T*) be the set of all lower semi continuous functions from (X, T*) to I with usual topology. Thus t* = ω(T*) = {$u \in I^X$: u^{-1} (α, 1]$\in$ T* } for each $\alpha \in I_I$, t*=ω(T*) turns out to be a fuzzy supra topology on X (cf.,e.g., Lowen **[47]).**

Let P be the property of a Supra topological space(X, T*) and FSP be its

fuzzy supra topological analogue. Then FSP is called good extension of P "if and only if the statement (X, T*) has P if and only if (X, ω(T*)) has FSP" holds good for every supra topological space (X, T*)

Theorem 3.2.7: Let (X, T*) be supra topological space. Then (X, T*) is R_0 if and only if (X, ω(T*)).is R_0(P) where P= (i), (ii), (iii), (iv) .

Proof: Let (X, ω(T*)).be FSR_0(i) we shall prove that (X, T*) is R_0 . Let

x, y εX with $x \neq y$ and UεT* with $x \in U$, and $y \notin U$, by the definition of lsc $I_U \varepsilon$ ω(T*)and with $I_U(x)=1$, $I_U(y)=0$, Since (X, ω(T*)).is FSR_0(i) ,then $\exists$ $v \in \omega(T^*)$ such that v(x)=0, v(y)=1 then $x \notin v^{-1}$ (α, 1] and yε v^{-1}(α, 1] as v(x)=0, v(y)=1 . Hence it is clear that (X, T*) is R_0 spaces.

Conversely suppose that (X, T*) is R_0 spaces. We shall prove that (X, ω(T*)) is FSR_0(i) .Let x, y $\in$X with $x \neq y$ and there exist $u \in$ ω(T*) such that u(x) =1, u(y) =0 then u^{-1} (0,1] ε T*; .hence xε u^{-1} (0, 1], $y \notin u^{-1}$ (0, 1] as u(x) =1, u(y)=0 , Since(X, T*) is R_0 space then, $\exists$ Vε T* such that $x \notin V$ and yεV , but $1_v \varepsilon$ ω(T*) and $1_v(x)$ =0, $1_v(y)$=1. Hence it is clear that (X, ω(T*))is R_0(i). Hence (X, T*) is R_0if and only if (X, ω(T*)) is R_0(i).

In the same way we prove that

(X, T*) is R_0 if and only if (X, ω(T*)) is R_0 (ii).

(X, T*) is R_0 if and only if (X, ω(T*)) is R_0(iii).

(X, T*) is R_0 if and only if (X, ω(T*)) is R_0(iv).

Thus it is seen that $R_0(P)$ is good extension of its supra topological counter part. P= (i), (ii), (iii), (iv). This completes the proof.

3.3 R_I Fuzzy Supra Topological Space

In this section, we study several property of fuzzy supra R_I topological space.

Definition: 3.3.1:- Let (X, t*) be a fuzzy supra topological space then

(a)FSR_I(i) space if and only if $\forall x, y \in X, x \neq y$, whenever there exist $w \varepsilon t^*$ with $w(x) \neq w(y)$,Then $\exists$ u, $v \varepsilon t^*$ such that u(x)=v(y)=1 and $u \wedge v=0$.

(b) FSR_I(ii) space if and only if $\forall x, y \in X, x \neq y$, whenever there exist $w \in t^*$ with $w(x) \neq w(y)$ Then $\exists$ u, $v \varepsilon t^*$ such that u(x)>0,v(y)>0 and $u \wedge v=0$

(c) FSR_I(iii) space if and only if $\forall x, y \in X, x \neq y$, whenever there exist $w \in t^*$ with $w(x) \neq w(y)$ then for $\alpha \varepsilon I_1$ $\exists$ u, $v \varepsilon t^*$ such that $u(x)>\alpha$, $v(y)>\alpha$ and $u \wedge v \leq \alpha$.

Lemma 3.3.2.: Show the following implications are true:

(X, t*) is FSR_I(i) $\Rightarrow$ (X, t*) is FSR_I(iii)

$\Uparrow$ (X, t*) is FSR_I(ii).

Proof: First, suppose that (X t*,) is FSR_I(i) we shall prove that(X, t*) is FSR_I(iii) . Let x, $y \varepsilon X$ with $x \neq y$ and $w \varepsilon t^*$such that $w(x) \neq w(y)$, Since (X, t*) is FSR_I(i) then $\exists$ u, $v \varepsilon t^*$, u(x)=v(y)=1 and $u \wedge v=0$. Now it is clear that when $w \in t^*$ with $w(x) \neq w(y)$, for $\alpha \varepsilon I_1$, $\exists$ u, $v \varepsilon t^*$ $u(x)>\alpha$, $v(y)>\alpha$ and $u \wedge v \leq \alpha$.

Hence it is clear that(X, t*) is FSR_I(iii)

.Next, suppose that (X, t*) is FSR_I (ii) we shall prove that(X, t*) is FSR_I(iii).Let x, $y \in X$ with $x \neq y$ and $w \in t^*$ such that $w(x) \neq w(y)$ .Since (X, t*) is F SR_I(ii) then $\exists$ u, $v \varepsilon t^*$ such that u(x)>0,v(y)>0 and $u \wedge v=0$ .Hence for $\alpha \in I_1$ $\exists$ u, $v \varepsilon t^*$, $u(x)>\alpha$, $v(y)>\alpha$ and $u \wedge v \leq \alpha$. So it is clear that (X, t*) is FSR_I(iii).

Now, we give some examples to show the non implications among FSR_I(i), FSR_I(ii) and FSR_I(iii) .

Example 3.3.3: Let X={x, y} and u, v, $w \in I^X$ where

$t^* = \{1, 0, u= \{(x, 1), (y, 0)\},$

$v = \{(x, 0), (y, .2)\}, w= \{(x, 1), (y, .2)\}\}$

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, w, 1}. Here $w \varepsilon t^*$ with $w(x) \neq w(y)$, since $w(x)=1$, $w(y) =.2$, now $u(x)=1$, $u(y)=0$ and $v(x)=0$, $v(y)=.2$, so u, $v \varepsilon t^*$ with $u(x)>0$, $v(y)>0$ and $u \wedge v=0$ hence (X, t*) is FSR_I(ii) but (X, t*) is not FSR_I(i) .

Example 3.3.4:- Let X={x, y} and u, v, $w \in I^X$ where

$t^* = \{1, 0, u=\{(x, .8), (y, .2)\}, v =\{(x, .1), (y, .7)\}, w=\{(x, .8), (y, .7)\}\}$

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, w, 1 },

Here $w \varepsilon t^*$ with $w(x) \neq w(y)$, since $w(x)=.8$, $w(y)=.7$, now $u(x)=.8$, $u(y)=.2$ and $v(x)=.1$, $v(y)=.7$, so u, $v \varepsilon t^*$ with $0.2<\alpha<.8$ .We have $u(x)> \alpha$, $v(y)> \alpha$ and $u \wedge v \leq \alpha$.So (X, t*) is FSR_I(iii) but $u(x)>0$, $v(y)>0$ and $u \wedge v \neq 0$,hence

(X, t*) is not FSR_I(ii) also (X, t*) is not FSR_I(i) . This completes the proof.

Theorem 3.3.5.: Let (X, t*) be a fuzzy supra topological space and

$I(t^*) = \{u^{-1}(0, 1]; u \varepsilon t^*\}$ then

(a)(X, t*) is FSR_I(i) $\Rightarrow$ (X, I(t*)) is supra R_1

(b) (X, t*) is FSR_I (ii) $\Rightarrow$ (X, I(t*)) is supra R_I

(c) (X, t*) is FSR_I(iii) $\Rightarrow$ (X, I(t*)) is supra R_I.

Proof: (a) Let (X, t*) is FSR_I (i), we shall prove that (X, I (t*)) is supra R_I . Let x, $y \in X$ with $x \neq y$ and $M \in I(t^*)$ with $x \varepsilon M$, $y \notin M$, we can write $M= w^{-1} (0, 1]$ where $w \varepsilon t^*$. Then we see that $w(x)>0$, $w(y) \leq 0$, therefore $w(x) \neq w(y)$, Since (X, t*) is FSR_I(i) , Then $\exists$ u, $v \in t^*$, such that $u(x)=1$, $v(y)=1$ and $u \wedge v=0$.It follows that $\exists$ $u^{-1} (0, 1]$, $v^{-1} (0, 1]$, and

$x \varepsilon u^{-1} (0, 1]$, $y \varepsilon v^{-1} (0, 1]$, and $u^{-1} (0, 1] \cap v^{-1} (0, 1]= \varphi$, as $u \wedge v=0$. Hence it is clear that (X, I (t*)) is supra R_I.

(b) Let (X, t*) is FSR_I(ii), we shall prove that (X, I(t*)) is supra R_I. Let x, y εX with $x \neq y$ and $M \varepsilon I(t^*)$ with $x \varepsilon M$, $y \notin M$, So we can write $M= w^{-1} (0, 1]$, where $w \varepsilon t^*$,Now we have $w(x)>0$, and $w(y) \leq 0$ with $w(x) \neq w(y)$. Since (X, t*) is FSR_I(ii) then for $\alpha \in I_1$ $\exists$ u, vεt* u(x)>0, v(y)>0 , and $u \wedge v=0$. It follows that $\exists$ u^{-1} (0, 1], v^{-1} (0, 1] ε I(t*) with and xε u^{-1} (0, 1], yε v^{-1}(0, 1], and

u^{-1} $(0, 1] \cap v^{-1} (0, 1]=\varphi$,as $u \wedge v=0$. So it is clear that (X, t*)) is supra R_I.

(c) Since (X, t*) is FSR_I(iii) , we shall prove that (X, I(t*)) is supra R_I. Let x, y εX with $x \neq y$ and Mε I(t*) with xεM, y $\notin$ M , So we can write M= w^{-1} (α, 1] , where wε t*,Now we have w(x)> α, and w(y)≤ α with w (x)≠ w(y).Since (X, t*) is FS R_I(ii) then for $\alpha \in I_1$ $\exists$ u, v∈ t* with u(x)> α,v (y)> α , and $u \wedge v \leq .\alpha$ It follows that $\exists$ u^{-1} (α, 1], v^{-1} (α, 1] ∈ I(t*) with x∈ u^{-1} (α, 1], y∈ v^{-1} (α, 1], as u(x)> α, v(y)> α and u^{-1} $(\alpha, 1] \cap v^{-1}$ (α, 1] =φ ,as $u \wedge v \leq .\alpha$. So it is clear that (X, I(t*)) is supra R_I .

Example:3.3.6. Let X={x, y} and u, v, w$\in I^X$ where

t* ={1, u={(x, .8), (y, .2)},

v = {(x, .1), (y, .7)}, w= {(x, .8), (y, .7)}} is the fuzzy supra topology on X, here wεt^* with w(x)≠w(y), since w(x)=.8, w(y)=.7, Now u(x)=.8, u(y)=.2 and v(x)=.1, v(y)=.7,so u, vεt* but u(x)>0, v(y)>0 and u∧v ≠0 . Hence (X, t*) is not FSR_I(ii). Also (X, t*) is not FSR_I(i), since u(x) ≠1 and v(y) ≠1 and u∧v ≠0.Also I(t*) ={X, {x},{y}}, Then clearly I(t*) is a topology on X and

(X, t*)) is a supra R_I space.

Example:3.3.7. Let X={x, y} and u, v$\in I^x$ where

t* = {1, 0, u = {(x, .8), (y, .2)}, v = {(x, .1), (y, .7)}}

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, 1},

Now u(x) =.8, u(y) =.2 and v(x) =.1, v(y) =.7, here u, vε t* for α=.9 we have (X, t*) is not FSR_I(iii) space .Also I (t*) ={X, ϕ}, Then clearly (X, I(t*)) is supra R_1 space.

Theorem 3.3.8: Let (X, t*) be a fuzzy supra topological space .Consider the following statements:

(1) (X, T*) be a supra R_I space.

(2) (X, ω(T*)) be an FSR_I (i) space.

(3) (X, ω(T*)) be an FSR_I(ii) space.

(4) $(X, \omega(T^*))$ be an FSR_I(iii) space.

Proof: First suppose that (X, T^*) be a supra R_I space. We shall prove $(X, \omega(T^*))$ be an FSR_I(i)space. Let $x, y \varepsilon X$ with $x \neq y$ and $m \varepsilon \omega(T^*)$ such that $m(x) \neq m(y)$ ie either $m(x)<m(y)$ or $m(x)>m(y)$. Suppose $m(x) <r<m(y)$. Then it is clear that $m^{-1}(r, 1] \varepsilon T^*$ as $m \varepsilon \omega(T^*)$ and $x \notin m^{-1}(r, 1]$, $y \varepsilon m^{-1}(r, 1]$.Since (X, T^*) be a supra R_I space. $\exists U, V \varepsilon T^*$ such that $x \varepsilon T^*$, $y \varepsilon T^*$ such that $U \cap V = \varphi$ .The definition of lower semi continuous $I_U, I_v \varepsilon \omega(T^*)$ and with $I_U(x)=1$, $I_v(y)=1$, and $I_U \wedge I_v=0$. So$(X, \omega(T^*))$ is FS R_I (i) and $(X, \omega(T^*))$ is FS R_I (ii) space.

Further it can be easily shown that (2) $\Rightarrow$ (4) and (3) $\Rightarrow$(4).

We therefore prove that (4) $\Rightarrow$(1)

Suppose that $(X, \omega(T^*))$ be an FSR_I(iii) space. We shall prove (X, T^*) be a supra R_I space. Let $x, y \varepsilon X$ with $x \neq y$ and $M \in T^*$ such that $x \in M$ and $y \notin M$ or $x \notin M$ and $y \in M$. Suppose $x \in M$ and $y \notin M$. But 1_M is lower semi continuous from (X, T^*) into I., So $1_M \in \omega(T^*)$ and $1_M(x)=1$, $1_M(y)=0$ that is $1_M(x) \neq 1_M(y)$.

Since $(X, \omega(T^*))$ is FSR_I(iii)space for $\alpha \in I_1$ then $\exists$ $u, v \in \omega(T^*)$ such that $u(x)>\alpha$, $u(y)>\alpha$ and $u \wedge v \leq \alpha$ .Now we observed that from definition of lower semi continuous function $u^{-1}(\alpha, 1)$, $v^{-1}(\alpha, 1) \in T^*$ such that $x \in u^{-1}(\alpha, 1)$, $y \in v^{-1}(\alpha, 1)$ and $u^{-1}(\alpha, 1) \cap v^{-1}(\alpha, 1)=\phi$, Thus (X, T^*) be a supra R_I space.

This completes the proof..

CHAPTER-FOUR

T_0 and T_1 Fuzzy Supra Topological spaces

4.1. Introduction: We discuss here the separation axiom T_0, T_1, fuzzy supra topological space and obtain their several properties, we write FST_0, in short for fuzzy supra T_0. The letter T is used here from the German word "Trennung'' Also Alexandroff and Hopf introduced T_i in mathematics.

4.2.:T_0 Fuzzy Supra Topological Space

Definition 4.2.1.: FST_0-Space: Let (X, t^*) be a fuzzy supra topological space, and then (X, t^*) is

(a) FST_0 (i) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \varepsilon t^*$ such that $\lambda(x)=1$, $\lambda(y)=0$,or $\exists$ $\mu \in t^*$ $\mu(x)=0$, $\mu(y)=1$.

(b) FST_0(ii) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \varepsilon t^*$ such that $\lambda(x)>0$, $\lambda(y)=0$, or $\exists \mu \in t^*$ such that $\mu(x)=0$, $\mu(y)>0$.

(c) FST_0(iii) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \varepsilon t^*$ such that $0 \leq \lambda(x) \leq 0 < \lambda(y) \leq 1$ or $\exists \mu \in t^*$ such that $0 \leq \mu(y) \leq 0 < \mu(x) \leq 1$.

(d) FST_0 (iv) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \varepsilon t*$ such that $\lambda(x) < \lambda(y)$ or $\lambda(y) < \lambda(x)$

Lemma 4.2.2.: The following are true:

(a)$\Rightarrow$(b) and (a) $\Rightarrow$ (c), also(b) $\Rightarrow$ (c) $\Rightarrow$ (d)

(a) **Proof:** (a) $\Rightarrow$ (b)

From (a), since $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \in t^*$ such that $\lambda(x)=1$, $\lambda(y)=0$, or $\mu(x)=0$, $\mu(y)=1$ then clearly $\lambda(x)>0$, $\lambda(y)=0$, or $\mu(x)=0$, $\mu(y)>0$ which is (b).

(a) $\Rightarrow$ (c)

From (a), since $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda \in t^*$ such that $\lambda(x)=1$, $\lambda(y)=0$; it follows that $0 \leq \lambda(y) \leq 0 \leq \lambda(x) \leq 1$ .Similarly we can show $0 \leq \mu(x) \leq 0 \leq \mu(y) \leq 1$

Hence it is clear that (a) $\Rightarrow$ (c).

(b) $\Rightarrow$ (c)

From (b), since $\forall$ x, y $\in$X, x $\neq$ y, $\exists$ $\lambda \in t^*$ such that λ(x)>0, λ(y)=0, or μ(x)=0, μ(y)> 0, when λ(x)>0, λ(y) = 0 then clearly that $0 \leq \lambda(y) \leq 0 \leq \lambda(x) \leq 1$ and when μ(x)=0, μ(y)>0,then $0 \leq \mu(x) \leq 0 \leq \mu(y) \leq 1$.

Hence it is clear that (b) $\Rightarrow$ (c)

(c) $\Rightarrow$ (d)

From (c), $\forall$ x, y $\in$X, with x $\neq$ y, $\exists$ $\lambda \in t^*$ such that $0 \leq \lambda(x) \leq 0 < \lambda(y) \leq 1$ or $\exists$ $\mu \in t^*$ Such that $0 \leq \mu(y) \leq 0 < \mu(x) \leq 1$. Now from $0 \leq \lambda(x) \leq 0 < \lambda(y) \leq 1$, we observe that $\lambda(x) \neq \lambda(y)$,Again from $0 \leq \mu(y) \leq 0 < \mu(x) \leq 1$ we have $\mu(x) \neq \mu(y)$. Hence (X, t^*) is FST_0(iv) -space.

Lemma 4.2.3.: Let (X, t^*) and (X, $t^*{}_1$) be two fuzzy supra topological spaces where t^* and $t^*{}_1$ be two fuzzy supra topology on X. Let t^* is finer than $t^*{}_1$, Then we show if $t^*{}_1$, is a fuzzy supra T_0 space then t^* is also fuzzy supra T_0 space.

Proof: Let x, y be two distinct point in X. Since $t^*{}_1$, is fuzzy supra T_0 space then $\exists$ $\lambda \in t^*{}_1$, such λ(x)=1, λ(y)=0 or λ(x)=0, λ(y)=1. Also since $t^*{}_1 \subset t^*$ so for t^*, λ(x) =1, λ(y) =0 or λ(x) =0, λ(y) =1 is hold. Hence t^* is also a fuzzy supra T_0 space.

Theorem 4.2.4: Let (X, t^*) and (Y, s^*) be two fuzzy supra topological spaces and f: X$\rightarrow$Y be one-one, onto and supra open map then

(1) (X, t^*) is T_0(i) $\Rightarrow$ (Y, s^*) is T_0(i) .

(2) (X, t^*) is T_0(ii) $\Rightarrow$(Y, s^*) is T_0(ii).

(3) (X, t^*) is T_0(iii) $\Rightarrow$(Y, s^*) is T_0(iii).

Proof: Suppose (X, t^*) is T_0 (i), we shall prove that (Y, s^*) is T_0 (i). Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, $\exists$ $x_1, x_2 \in X$ with $f(x_1) = y_1$ and $f(x_2) = y_2$; and $x_1 \neq x_2$ as f is one-one. Again (X, t^*) is T_0(i) , then $\exists$ u$\in t^*$ such that u(x)=1, u(y)=0.

Now $f(u)(y_1) = \{\text{Sup } u(x_1): f(x_1) = y_1 \}$

$= 1$

$f(u)(y_2) = \{\text{Sup } u(x_2): f(x_2) = y_2\} = 0$

Since f is supra open map then $f(u) \in s^*$ as $u \in t^*$. Hence we observe that $\exists$

$f(u) \in s^*$ such that $f(u)(y_1) = 1$, and $f(u)(y_2) = 0$. Hence it is clear that (Y, s^*) is T_0(i).

Similarly (b), (c), (d) can be proved.

Now we give example to show the non implications among FST_0(i), FST_0(ii), FST_0(iii), FST_0(iv) .

Example: 4.2.5 Let X={x, y, z} and

$t^* = \{1, 0, \lambda = \{(x, \frac{3}{4}), (y, \frac{1}{2}), (z, 0)\}, \{(x, \frac{3}{4}), (y, \frac{1}{2}), (z, \frac{1}{2})\}, \{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}\}$ on X.

Let $\lambda = \{(x, 1), (y, \frac{1}{2}), (z, 0)\}$ .Here $\lambda(x) = \frac{3}{4}$, $\lambda(z)=0$, $\lambda(x)>0$ and $\lambda(z)=0$, also $\mu(x)=0$ and $\mu(z)>0$ So clearly it is FST_0 (ii) but there does not exist any $\mu \in t^*$ such that $\mu(y)=1$ and $\mu(x)=0$ So, $FST_0(ii) \neq> FST_0(i)$

Example: 4.2.6 Let X={x, y, z} and

$t^* = \{1, 0, \lambda = \{(x, \frac{5}{8}), (y, \frac{1}{2}), (z, \frac{3}{4})\}, \{(x, \frac{5}{8}), (y, \frac{1}{2}), (z, \frac{3}{4})\},$

$\mu = \{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}$ on X

Here $\lambda(x) = \frac{5}{8}$, $\lambda(y) = 0$, $\mu(x) = 0$, $\mu(y) = \frac{1}{2}$ we have $0 \leq \mu(x) \leq 0 < \mu(y) \leq 1$

So (X, t*) is an FST_0(iii) but (X, t*) but there does not exist any $\mu \in t*$ such that $\mu(y)=1$ and $\mu(x) = 0$, is not FST_0(i). Hence $FST_0(iii) \neq> FST_0(i)$.

4.3. α-T_0 Fuzzy Supra Topological Space .

Definition 4.3.1.: Let (X, t^*) be a fuzzy supra topological space, and then (X, t^*) is said to be

(a) $FS\alpha$-T_0(i) –Space if and only if $\forall$ x, y $\in$X, x $\neq$y , $\exists$ $\lambda \in t^*$ such that $\lambda(x)=1$, $\lambda(y) \leq \alpha$ or $\lambda(x) \leq \alpha$, $\lambda(y)=1$.

(b) $FS\alpha$-T_0 (ii) –Spaces if and only if $\forall$ x, y $\in$X, x $\neq$y, $\exists$ $\lambda \in t^*$ such that $\lambda(x)>\alpha$, $\lambda(y)=0$, or $\lambda(x)=0$, $\lambda(y)> \alpha$.

Lemma 4.3.2 .: Let (X, t^*) be a fuzzy supra topological space, and if $0 \leq \alpha \leq \beta \leq 1$ .Then (1) (X, t^*) is α - T_0 (i) $\Rightarrow$ (X, t^*) is β - T_0 (i)

(2) (X, t^*) is β - T_0(ii) $\Rightarrow$ (X, t^*) is α - T_0(ii)

Proof : Suppose that (X, t^*) be a fuzzy supra topological space, and(X, t^*) is α-T_0(i) , we shall prove that (X, t^*) is β - T_0 (i) .Let x, y $\in$X, with x $\neq$y , Since (X, t^*) is α - T_0(i) then for $\alpha \in I_1$, $\exists$ $\lambda \in t^*$ such that λ (x)=1, $\lambda(y) \leq \alpha$ or $\lambda(x) \leq \alpha$, $\lambda(y)=1$.This implies that $\lambda(x)=1$, $\lambda(y) \leq \beta$. Since $0 \leq \alpha \leq \beta \leq 1$, hence it is clear that (X, t^*) is β - T_0 (i) .Similarly if X, $t^*)$ is β - T_0 (ii). Let x, y $\in$X, with x $\neq$y for $\beta \in I_1$, $\exists$ $\lambda \in t^*$ such that $\lambda(x)> \beta$,$\lambda(y)=0$, or $\lambda(x)=0$, $\lambda(y)> \beta$.Hence $\lambda(x)>\alpha$, $\lambda(y)=0$, $\lambda(x) = 0$, $\lambda(y)>\alpha$; since $\alpha \leq \beta$. So $\Rightarrow$ (X, t^*) is α - T_0 (ii).

Example:4.3.3. Let X={x, y} and consider the fuzzy supra topology

$t^* = \{1, 0, \lambda=\{(x, 1), (y, \frac{2}{5})\}, \{(x, 1), (y, \frac{1}{2}),\}, \mu = \{(x, 0), (y, \frac{1}{2})\}\}$ on X. Taking $\alpha=\frac{1}{5}$, and $\beta=\frac{3}{5}$ then (X, t^*) is β - T_0(i) but (X, t^*) is not α - T_0 (i)

Example:4.3.4. Let X={x, y} and consider the fuzzy supra topology

$t^*=\{1,0, \lambda=\{(x, 1),(y, \frac{2}{5}) \}, \{(x, 1), (y,\frac{1}{2}),\}, \mu = \{(x, 0), (y, \frac{1}{2})\}\}$ on X. Taking $\alpha=\frac{1}{5}$ and $\beta=\frac{3}{5}$ then from (X, t^*) we have $\mu(x)$ =0 and $\mu(y)= \frac{1}{2} > \frac{1}{5}$, So (X, t^*) is α- T_0(ii) $\neq>$ (X, t^*) is not β - T_0 (ii), since $\mu(y) = \frac{1}{2} < \frac{3}{5}$

Lemma4.3..5: Every discrete fuzzy supra topological space is a FST_0 space.

Proof: Let (X, t^*) is an discrete fuzzy supra topological space and let x, y be distinct points of X. Since the space is discrete then for $\alpha \in I_1$, there exist $\lambda \in t^*$ such that

$\lambda(x)=1$, $\lambda(y)\leq \alpha$ or $\lambda(x)\leq\alpha$, $\lambda(y)=1$ .Then from definition of T_0 space (X , t^*) is a FST_0 space.

Lemma 4.3.6.: Every indiscrete fuzzy supra topological space is not a FST_0 Space.

Proof: Let (X, t^*) is an indiscrete fuzzy supra topological space and let x, y be distinct points of X. Since the spaces is indiscrete then for $\alpha \in I_1$, there does not exist $\lambda \in t^*$ such that. $\lambda(x)=1$, $\lambda(y) \leq \alpha$ or $\lambda(x) \leq \alpha$,$\lambda(y)=1$. Hence (X, t^*) is not FST_0.

4.4: T_1 Fuzzy Supra Topological Space.

Definition 4.4.1.: FST_1-Space: Let (X, t^*) be a fuzzy supra topological space, and Then (X, t^*) is said to be fuzzy supra T_1- Spaces

(a)FST_1(i) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda, \mu \in t^*$ such that $\lambda(x)=1$, $\lambda(y)=0$ and $\mu(x)=0$, $\mu(y)=1$.

(b) FST_1(ii) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda, \mu \in t^*$ such that $\lambda(x)>0$, $\lambda(y)=0$, and $\mu(x)= 0$, $\mu(y)>0$.

(c) FS T_1 (iii) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists \lambda, \mu \in t^*$ such that

$0\leq \lambda(x) \leq 0<\lambda(y) \leq 1$ and $0\leq \mu(y) \leq 0< \mu(x) \leq 1$

(d) FST_1 (iv) –space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists \lambda, \mu \in t^*$ such that $\lambda(x)< \lambda(y)$ and $\mu(y)< \mu(x)$.

Implication 4.4.2.: From the above definition, we see that

(a)$\Rightarrow$(c) and(b) $\Rightarrow$ (c) $\Rightarrow$ (d)

Proof: Let (X, t^*) be a T_1 (i) fuzzy supra topological space, we shall prove that (a) (X, t^*) is T_1(iii). Since (X, t^*) be a T_1(i) then $\forall$ x, y $\in X$, $x \neq y$, $\exists$ $\lambda, \mu \in t^*$ s. t $\lambda(x)=1$, $\mu(y)=0$ and $\mu(x)=1$, $\lambda(y)=0$.So it is clear that $0\leq \lambda(y)\leq0<\lambda(x) \leq 1$ and $0\leq \mu(x) \leq0<\mu(y) \leq 1$, Hence (a)$\Rightarrow$(c).

Next suppose that (X, t^*) is T_1(ii) ,we shall prove that (X, t^*) is T_1(iii)-Space. Since (X, t^*) is T_1(ii), then if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$ λ, $\mu \varepsilon t^*$ such that $\lambda(x)>0$,

$\lambda(y)=0$,and $\mu(x)=0$, $\mu(y)>0$.Which shows that $0\leq\lambda(y)\leq 0<\lambda(x)\leq 1$ and $0\leq\mu(x)\leq 0<\mu(y)\leq 1$ .Hence (X, t^*) is T_I(iii).

Finally suppose that (X, t^*) is T_I (iii), we shall prove that (X, t^*) is T_I(iv) .Since (X, t^*) is T_I (iii), so $\forall$ $x, y \in X, x \neq y, \exists\, \lambda, \mu \in t^*$ such that
$0\leq \lambda(x)\leq 0<\lambda(y)\leq 1$and $0\leq\mu(y)\leq 0<\mu(x)\leq 1$, this implies that
$\lambda(x)<\lambda(y)$ and $\mu(y)<\mu(x)$.

Now we show by examples the non implications among FST_I (i), FST_I (ii), FST_I (iii), FST_I (iv).

Example:4.4.3. Let $X=\{x, y, z\}$ and $t^* = \{1, 0, \lambda= \{(x, \frac{3}{4}), (y, \frac{1}{2}), (z, 0)\}, \{(x, \frac{3}{4}), (y, \frac{1}{2}), (z, 1)\}, \mu=\{(x, 0), (y, \frac{1}{2}), (z, 1)\}\}$on X. Here $\lambda(x)>0$, $\lambda(z)=0$, and also $\mu(x)=0$, $\mu(z)>0$ So clearly it is FST_I (ii) but there does not exist $\exists\ \lambda, \mu \in t^*$such that $\lambda(x)=1$, $\lambda(y)=0$ and $\mu(x)=0$, $\mu(y)=0$.So it is clear that
FST_I (ii) $\neq>$ FST_I(i)

Example: 4.4.4 .Let $X=\{x, y, z\}$ and
$t^*=\{1, 0, \lambda=\{(x, \frac{5}{8}), (y, 0), (z, \frac{3}{4})\}, \{(x, \frac{5}{8}), (y, \frac{1}{2}), (z, \frac{3}{4})\},$
$\mu= \{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}\}$ on X

Here $\lambda(x)=\frac{5}{8}$, $\lambda(y)=0$; $\mu(x)=0$, $\mu(y)= \frac{1}{2}$ we have $0\leq\mu(x)\leq 0<\mu(y)\leq 1$ and $0\leq\lambda(y)\leq 0<\lambda(x)\leq 1$ so (X, t^*) is an FST_I(iii) but there does not exist any $\lambda, \mu \in t^*$ such that $\lambda(x)=1$, $\lambda(y)=0$ and $\mu(x)=0$, $\mu(y)=0$.So it is clear that
$F\,ST_I$ (iii) $\neq>$ $F\,ST_I$ (i).

Example: 4.4.5. Let $X=\{x, y, z\}$ and
$t^*=\{1, 0, \lambda=\{(x, \frac{5}{8}), (y, 0), (z, \frac{3}{4})\}, \{(x, \frac{5}{8}), (y, \frac{1}{2}), (z, \frac{3}{4})\}, \mu=\{(x, 0), (y, \frac{1}{2}), (z, \frac{1}{2})\}\}$on X.

Here $\lambda(x)=\frac{5}{8}$, $\lambda(y)=0$, $\mu(x)=0$, $\mu(y)=\frac{1}{2}$ we have $0\le\mu(x)\le 0<\mu(y)\le 1$ and $0\le\lambda(y)\le 0<\lambda(x)\le 1$ so (X, t^*) is an FST_1(iii) but there does not exist any $\lambda,\mu\in t^*$ $\lambda(x)<\lambda(y)$, $\mu(y)<\mu(x)$ .such that So it is clear that

$$FST_1\text{ (iii)} \neq> F\,ST_1\text{ (iv)}$$

Theorem 4.4.6: Let (X, T^*) be a supra topological space .Consider the following statements.

(1) (X, T^*) be a Supra-T_1 space.

(2) $(X, \omega(T^*))$ be an FST_1(i) space.

(3) $(X, \omega(T^*))$ be an FST_1(ii) space.

(4) $(X, \omega(T^*))$ be an FST_1(iii) space.

(5) $(X, \omega(T^*))$ be an FST_1(iv)space.

Then the following implications are true .

$(1)\Rightarrow(2)\Rightarrow(4)\Rightarrow(5)\Rightarrow(1)$

$\Downarrow$ $\Uparrow$

(3) (3)

Proof:- Let $(X\ T^*,)$ is a supra- T_1-space. We shall prove that$(X, \omega(T^*))$ be an FST_1(i) space. Since (X, T^*) is Supra T_1-space, then for two distinct points x and y in X with, $x\neq y$ $\exists$ supra open set U and $V\in T^*$ such that $x\in U$ and $y\notin U$, and $x\in V$, $y\notin V$. But from definition of lower semi continuous function 1_U,

$1_V\in\omega(T^*)$ and $1_U(x)=1$, $1_U(y)=0$ also $1_V(x)=0$, $1_V(y)=1$. Hence

$(X, \omega(T^*))$ is FST_1(i) and also $(X, \omega(T^*))$ is FST_1(ii). Further it is easily prove that $(2)\Rightarrow(4)$, $(3)\Rightarrow(4)$ and $(4)\Rightarrow(5)$

So we prove only $(5)\Rightarrow(1)$

Suppose $(X, \omega(T^*))$is FST_1(iv) space we shall prove $(X, T^*)$is Supra-T_1(i).Let x, $y\in X$ with $x\neq y$ .Since $(X, \omega(T^*))$is FST_1 (iv)space if $\exists$ u, $v\in\omega(T*)$ such that $u(x)<u(y)$ and $v(x)>v(y)$. Let r, s $\in I$ be such that $u(x)<r<u(y)$ and $v(x)>s>v(y)$. Then we have $u^{-1}(r, 1]$, $v^{-1}(s, 1]$,and $x\notin u^{-1}(r, 1]$, $y\in v^{-1}(r, 1]$and $x\in u^{-1}(r, 1]$,$y\notin v^{-1}(s, 1]$.Hence (X, T^*) is Supra- T_1 space. This completes the proof.

Thus it is seen that T_1(p) is a good extension of its supra topological counter part.

Theorem 4.4.7: Let (X, t^*) and (Y, s^*) be two fuzzy supra topological spaces and $f:X \rightarrow Y$ be continuous and one-one, supra open map then

(1) (Y, s^*) is T_1(i) $\Rightarrow$.(X, t^*) is T_1(i)

(2) (Y, s^*) is T_1(ii) $\Rightarrow$ (X, t^*) is T_1(ii)

(3) (Y, s^*) is T_1(iii) $\Rightarrow$ (X, t^*) is T_1(iii)

Proof: Suppose(Y, s^*) is T_1 (i) we shall prove (X, t^*) is T_1(i) , Let $x_1, x_2 \in X$ with $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ in Y as f is one- one . Now since (Y, s^*) is T_1 (i) $\exists$ u, v $\in t^*$, such that $u(x_1) = 1$, $u(x_2) = 0$,and $v(x_1)=0$, $v(x_2)=1$ this implies that f^{-1} $(u(x_1))=1$,

f^{-1} $(u(x_2))=0$ and $f^1 v(x_1)=0$, $f^1(v(x_2))= 1$, since f is continuous and u, v$\in$ s^* implies that f^{-1} (u) ,f^{-1} (v) $\in t^*$ with $f^1(u(x_1))=1$,

$f^1(u(x_2))=0$ and $f^1(v(x_1))=0$, $f^1(v(x_2))= 1$, Hence (X, t^*) is T_1(i)

Similarly (2), (3) can be proved.

Theorem 4.4.8.: Let (X, t^*) be a fuzzy supra topological space and

$I(t*) = \{u^{-1}(0, 1]; u \in t^*\}$ then

(a) (X, t^*) is FST_1(i) $\Rightarrow$ $(X, I(t^*))$ is supra T_1

(b) (X, t^*) is FS T_1 (ii) $\Rightarrow$ $(X, I(t^*))$ is supra T_1

(c) (X, t^*) is FST_1(iii) $\Rightarrow$ $(X, I(t^*))$ is supra T_1

Proof: - (a) Let (X, t^*) is FS T_1 (i) we shall prove that $(X, I(t^*))$ is T_1 . Let x, y $\in$X with $x \neq y$ .Since (X, t^*) is FST_1(i) , Then $\exists$ u, v$\in t^*$, such that u(x)=1, u(y)=0 and v(x)=0,v(y)=1.It follows that $\exists$ $u^{-1}(0, 1]$, v^{-1} (0, 1] $\in I(t^*)$ and $x \in u^{-1}(0, 1]$ and $y \notin u^{-1}$ (0, 1] and $x \notin v^{-1}$ (0, 1], $y \in v^{-1}(0, 1]$. Hence it is clear that $(X, I(t^*))$ is supra T_1.

(b) Let (X, t^*) is FST_1(ii) we shall prove that $(X, I(t^*))$ is supra T_1. Let x, y $\in$X with $x \neq y$ .Since (X, t^*) is FST_1(ii) then $\exists$u, v$\in$ $\mathbf{t}^*$ u(x)= 0, u(y)>0 and v(x)>0, v(y)=0.Since $u^{-1}(0, 1]$, $v^{-1}(0, 1] \in I(t*)$ and it is clear that $x \in u^{-1}(0,1]$, $y \notin u^{-1}(0, 1]$, $x \notin v^{-1}(0, 1]$,$y \in v^{-1}(0, 1]$. So it is clear that (X, I(t*)) is supra T_1.

(c) Since (X, t^*) is FST_1(iii) , we shall prove that $(X, I(t^*))$ is supra T_1. Let

x, y $\in$X with $x \neq y$ .Since (X, t^*) is FS T_1(iii) then $\exists$ u, v$\in t^*$with $0 \leq u(x) \leq 0 < u(y) \leq 1$ and $0 \leq v(y) \leq 0 < v(x) \leq 1$. It follows that $\exists$ $u^{-1}(0,1]$,$v^{-1}(0,1] \in I(t^*)$ with $x \in u^{-1}(0, 1]$, $y \notin u^{-1}(0, 1]$, $x \notin v^{-1}(0, 1]$, $y \in v^{-1}(\alpha, 1]$. So it is clear that $(X, I(t^*))$ is supra T_1.

Example: 4.4.9. Let X={x, y} and u, v, w$\in I^X$ where

t^* = {1, 0, u={(x, .8), (y, .2)}, v ={(x, .1), (y, .7)}, w={(x, .8), (y, .7)}}

Consider the fuzzy supra topology t^* on X is generated by {0, u, v, w, 1},

Here $w \in t^*$with $w(x) \neq w(y)$, since w(x)=.8, w(y)=.7, Now u(x)=.8, u(y)=.2 and v(x)=.1, v(y)=.7, so u, v $\in t^*$. Hence (X, t^*) is not FST_1(ii) also (X, t^*) is not FST_1(i), Since $u(x) \neq 1$and $v(y) \neq 1$ .Also $I(t^*)$ ={X, {x}, {y}}, Then clearly $(X,I(t^*))$ is supra T_1.

Example: 4.4.10. Let X={x, y} and u, v$\in I^X$ where

t^* ={1,0, u={(x, .8), (y, .2)}, v ={(x, .1), (y, .7)}} Consider the fuzzy supra topology t^* on X is generated by {0,u, v, 1 } , Now u(x)=.8, u(y)=.2 and v(x)=.1, v(y)=.7, Here u, v$\in t^*$, we have $(X t^*,)$ is not FS T_1 (iii) space. Also $I(t^*)$={X, {x}, {y}}, Then clearly $(X, I(t^*))$ is supra T_1.

CHAPTER-FIVE

T_2- Fuzzy Supra Topological space

5.1. Introduction:

In this chapter, we introduce and study on various concept of Fuzzy Supra Hausdorffness that is T_2- Fuzzy Supra Topological space. In short we write FST_2 for T_2- Fuzzy Supra Topological space.

5.2. T_2 Fuzzy Supra Topological space.

5.2.1 Definition:- Let (X, t*) be a fuzzy supra topological space, and then (X, t*) is said to be

(a) FST_2(i) Space if and only if $\forall$ x, y $\in$X, x $\neq$y , $\exists\lambda, \mu \in t^*$ such that $\lambda(x)=1$, $\mu(y)=1$ and $\lambda\wedge\mu=0$.

(b)FST_2(ii) Space if and only if $\forall$ x, y $\in$X, x $\neq$y , $\exists$ $\lambda, \mu\in t^*$ such that $\lambda(x)>0$, $\mu(y)>0$ and $\lambda\wedge\mu=0$.

(c)FST_2(iii) Space if and only if $\forall$ x, y $\in$X, x $\neq$y , $\exists\lambda, \mu\in t^*$ $\lambda(x)>\alpha$, $\mu(y)>\beta$ where $\alpha,\beta\in I_1$ and $\lambda\wedge\mu=0$.

Lemma 5.2.2.: Show the following implication are true,

(X, t*) is FST_2(i) $\Rightarrow$ (X, t*) is FST_2(iii) $\Leftarrow$(X, t^*) is FST_2(ii)

Proof:- Let (X, t^*) be a fuzzy supra topological space, and (X, t*) is FST_2(i), we shall prove that (X, t*) is FST_2(iii) . Let x, y $\in$X with x $\neq$y, since (X, t^*) is FST_2(i), $\exists$ $\lambda, \mu\in t^*$ such that $\lambda(x)=1$, $\mu(y)=1$ and $\lambda\wedge\mu=0$ .So for $\alpha,\beta\in I_0$ it is clear that $\lambda(x)>\alpha$, $\mu(y)>\beta$. Hence it is clear that (X, t*) is FST_2(iii).

Next suppose that (X, t*) is FST_2(ii), We shall prove that (X, t^*) is FST_2(iii) .

Let x, y $\in$X with x $\neq$y, since (X, t^*) is FST_2(ii), $\exists$ $\lambda, \mu\in t^*$ such that $\lambda(x)>0$, $\mu(y)>0$, and $\lambda\wedge\mu=0$. So for $\alpha,\beta\in I_0$ it is clear that $\lambda(x)>\alpha$, $\mu(y)>\beta$. Hence it is clear that (X, t^*) is FST_2(iii).

Now we give some example to show the non implication among Supra- T_2(i), FST_2(ii), FST_2(iii) .

Example5.2.3: Let X={x, y} and$\lambda,\mu,\omega \in I^X$, whereλ, μ and ω are defined by $\lambda(x)= 0.7$, $\lambda(y)=0$; $\mu(x)=0$ and $\mu(y) =0.8$; $\omega(x)= 0.7$, $\omega(y)=0.8$, Consider the fuzzy supra topology t^* on X generated by $\{1, 0, \lambda, \mu, \omega\}$. Now for $\alpha=0.6$, $\beta=0.7$, it is clear that (X, t^*) is FST_2(iii) but not FST_2(i).

Example 5.2.4: Let X = {x, y} and$\lambda, \mu, \omega \in I^X$, whereλ, μ and ω are defined by $\lambda(x)= 0.7$, $\lambda(y) =0$; $\mu(x) =0$ and $\mu(y) =0.8$; $\omega(x)= 0.7$, $\omega(y)=0.8$, Consider the fuzzy supra topology t* on X generated by $\{1, 0, \lambda, \mu, \omega\}$. It is clear that (X, t^*) is FST_2 (ii) but not FST_2(i).

Theorem 5.2.5.: Let (X, T^*) be a supra topological space .Consider the following statements:

(1) (X, T*) be a Supra- T_2 space.

(2) $(X, \omega(T^*))$ be a FST_2(i) space.

(3) $(X, \omega(T^*))$ be a FST_2(ii) space.

(4) $(X, \omega(T^*))$ be a FST_2(iii) space.

Then the following implications are true.

$$
\begin{array}{ccc}
(1)\Rightarrow(2) & \Rightarrow(4) & \Rightarrow(1) \\
\Downarrow & \Uparrow & \\
(3) & (3) &
\end{array}
$$

Proof: Let (X, T*) is a Supra- T_2-space. We shall prove that $(X, \omega(T^*))$ be an FST_2(i) space. Since (X, T^*) is a Supra - T_2 -space, then for all two distinct points x and y in X With $x \neq y$ $\exists$ two disjoint supra open set U and V such that $x \in U$ and $y \in V$. and $U \cap V=\varphi$ Again from definition of lower semi continuous function 1_U, $1_V \in \omega(T^*)$

and $1_U(x)=1$, $1_V(y)=1$ and $1_U \wedge 1_V=0$. If $1_U \wedge 1_V \neq 0$, then $\exists$ $z \in X$ such that $(1_U \wedge 1_V)(z) \neq 0 \Rightarrow 1_U(z) \neq 0$ and $1_V(z) \neq 0$, $\Rightarrow z \in U$, $z \in V$, $\Rightarrow z \in U \cup V$, $\Rightarrow U \cup V=\varphi$ a contradiction ,so that $1_U \wedge 1_V=0$,and consequently $(X, \omega(T^*))$ is FST_2(i) and also $(X, \omega(T^*))$ is FST_2(ii) .

Obviously (2) $\Rightarrow$(4) and (3) $\Rightarrow$(4),

Now we show only (4) $\Rightarrow$(1)

Suppose that (X, ω (T^*)) is FST_2(iii) space . We shall prove that (X, T^*) is a Supra-T_2space. Let x, y $\in$X with x $\neq$y, Since (X, $\omega(T^*)$) is FST_2 (iii) then from definition $\forall$ x, y $\in$X, x $\neq$y , $\exists\lambda$, $\mu \in \omega(T^*)$ $\lambda(x) > \alpha$, $\mu(y) > \beta$ and $\lambda \wedge \mu$ =0.So $\lambda^{-1}(\alpha, 1]$, $\mu^{-1}(\beta, 1] \in T^*$ where $\alpha, \beta \in I_1$ and $x \in \lambda^{-1}(\alpha, 1], y \in \mu^{-1}(\beta, 1]$. Moreover

$\lambda^{-1}(\alpha, 1] \cap \mu^{-1}(\beta, 1] = \phi$ and if

$z \in \lambda^{-1}(\alpha, 1] \cap \mu^{-1}(\beta, 1]$ then

$z \in \lambda^{-1}(\alpha, 1]$ and $z \in \mu^{-1}(\beta, 1] \Rightarrow$ $(\lambda\wedge\mu)(z)>0$, a contradicts that $(\lambda\wedge\mu)(z)=0$. Hence (X, T*) is a Supra-T_2 space. This completes the proof.

Thus it is seen that is a good extension of supra topological counter part.

Theorem 5.2.6.: Let (X, t^*) be a fuzzy supra topological space and I (t*) = {u^{-1} (0, 1]; u $\in$ t*} then

(a) (X, t*) is FST_2(i) $\Rightarrow$ (X, I(t^*)) is supraT_2

(b) (X, t*) is FST_2(ii) $\Rightarrow$ (X, I (t*)) is supra T_2

(c) (X, t*) is FST_2 (iii) $\Rightarrow$ (X, I (t*)) is supra T_2

Proof: - (a) Let (X, t*) is FST_2(i) we shall prove that (X, I(t^*)) is T_2 . Let x, y $\in$X with x$\neq$y .Since (X, t^*) is FST_2(i) , Then $\exists$ u, v$\in$ t*, such that u(x)=1, ,v(y)=1and $u\wedge v$=0 ,.It follows that $\exists$

$u^{-1}(0, 1]$, v^{-1} (0, 1] $\in$ I(t^*) and x$\in$ u^{-1} (0, 1] y$\in$ v^{-1} (0, 1] and. u^{-1} (0, 1] $\wedge v^{-1}$ (0, 1] =0 as $u\wedge v$=0 Hence it is clear that (X, I(t*)) is supra T_2.

(b) Let (X, t*) is FST_2(ii) we shall prove that (X, I(t*)) is supra T_2 .Let x, y $\in$X with x$\neq$y .Since (X, t^*) is FST_2(ii) then $\exists$u, v$\in t^*$ u(x) > 0, v(y)>0and $u\wedge v$=0.Since $u^{-1}(0, 1]$, $v^{-1}(0, 1]$ $\in$ I(t*) so that x$\in$ u^{-1} (0, 1], y$\in$ v^{-1} (0, 1] .and $u^{-1}(0, 1] \wedge v^{-1}(0, 1]$ =0 as $u\wedge v$=0.So it is clear that (X, I(t^*)) is supra T_2 .

(c) Since (X, t^*) is FS T_2 (iii) , we shall prove that (X, I(t^*)) is supra T_2. Let x, y $\in$X with x$\neq$y .Since (X, t^*) is FST_2(iii) then $\exists$ u, v$\in$ $\mathbf{t}^*$with u(x)>α and v(y).>β and $u\wedge v$=0 It follows that $\exists$ u^{-1} (0,1], v^{-1} (0,1] $\in$ I(t^*) with x$\in$ u^{-1}(0, 1], y$\in$ v^{-1} (0, 1].

So it is clear that (X, I (t^*)) is supra T_2 .

Definition 5.2.7: Let (X, t^*) be a fuzzy supra topological space, and then (X, t^*) is said to be α- fuzzy supra -T_2 Space

(a)(X, t^*) is $FST_2(i)$ space if and only if $\forall$ x, y $\in X$, $x \neq y$, $\exists$,λ, $\mu \in t^*$ such that $\lambda(x)=1=\mu(y)$and $\lambda \wedge \mu \leq \alpha$.

(b)(X, t^*) is $FST_2(ii)$ Space if and only if, $\forall$ x, y $\in X$, $x \neq y$, $\exists$ λ, $\mu \in t^*$ such that $\lambda(x) > \alpha$, $\mu(y) > \alpha$ and $\lambda \wedge \mu = 0$.

(c) (X, t^*) is $FST_2(iii)$ Space if and only if, $\forall$ x, y $\in X$, $x \neq y$, $\exists$λ, $\mu \in t^*$ such that $\lambda(x) > \alpha$, $\mu(y) > \alpha$ and $\lambda \wedge \mu \leq \alpha$.

Lemma 5.2.8.: The following implications are true

(X, t^*) is α - $T_2(i)$ $\Rightarrow$ (X, t^*) is α - $T_2(iii)$

$\Uparrow$ (X, t^*) is α - $T_2(i)$

Proof:- Let(X, t^*) be a fuzzy supra topological spaces, and (X, t^*) is α - $T_2(i)$, we shall prove that (X, t^*) isα - $T_2(iii)$, .Let x, y $\in X$, with $x \neq y$, Since (X, t^*) is α - $T_2(i)$ then for$\alpha \in I_1$, $\exists$λ, $\mu \in t^*$ such that $\lambda(x)=1=\mu(y)$ and $\lambda \wedge \mu \leq \alpha$ .This implies that $\lambda(x) > \alpha$ and$\mu(y) > \alpha$ .. Hence it is clear that (X, t^*) is α - $T_2(iii)$

Next suppose that (X, t^*) is α - T_2 (i),we shall prove that (X, t^*) isα - T_2 (iii), Since (X, t^*) is α - T_2 (i) then for$\alpha \in I_1$, $\exists$λ, $\mu \in t^*$ such that $\lambda(x) > \alpha$ and $\mu(y) > \alpha$ .and $\lambda \wedge \mu = 0$.Now it is clear that $\lambda(x) > \alpha$ and $\mu(y) > \alpha$ .and $\lambda \wedge \mu \leq \alpha$ .Hence . (X, t^*) is α - T_2 (iii).

Now we give some example to show the non implication among α - T_2(i), α - T_2(ii), α - T_2(iii).

Example:5.2.9. Let X={x, y} and consider the fuzzy supra topology,

$t^*=\{1, 0, \lambda=\{(x, \frac{7}{10}),(y, 0)\}, \mu=\{(x, 0),(y, \frac{7}{10})\}\}$ on X, where λ, $\mu \in I^X$ For α =.5 (X, t^*) is α - T_2(ii) but (X, t^*) is not α - T_2(i)

Example:5.2.10. Let X={x, y} and consider the fuzzy supra topology

$t^* = \{1, 0, \lambda=\{(x, 1), (y, \frac{2}{5})\}, \mu=\{(x, \frac{2}{5}),(y, 1)\}\}$ on X, where λ, $\mu \in I^X$ .For α =.5 we see that(X, t^*) is α - T_2 (i) but (X, t^*) is not α - T_2 (ii)

Example:5.2.11. Let X={x, y} and consider the fuzzy supra topology and, $\lambda, \mu \in I^X$

$t^* =\{1, 0, \lambda=\{(x, \frac{9}{10}), (y, \frac{3}{10})\}$, $\mu=\{(x, \frac{2}{5}),(y, \frac{9}{10}), (x, \frac{9}{10}), (y, \frac{9}{10})\}$ on X, For

$\alpha =.5$

(X, t^*) is α - T_2(iii) but (X, t^*) is not α -T_2 (ii)

Lemma 5.2.12.: Let (X, t^*) be a fuzzy supra topological spaces, and if $0 \le \alpha \le \beta \le 1$

Then (1) (X, t^*) is α - T_2(i) $\Rightarrow$ (X, t^*) is β - T_2(i)

(2) (X, t^*) is β - T_2(ii) $\Rightarrow$ (X, t^*) is α - T_2(ii)

(3) (X, t^*) is 0 - T_2(ii) $\Rightarrow$ (X, t^*) is 0- T_2(iii)

Proof: Suppose that (X, t^*) be a fuzzy supra topological spaces, and(X, t^*) is α - T_2(i),

we shall prove that (X, t^*) is β - T_2(i) .Let $x, y \in X$, with $x \neq y$, Since (X, t^*) is α - T_2 (i) then for $\alpha \in I_1$, $\exists \lambda, \mu \in t^*$ such that λ (x)=1=μ(y) and $\lambda \wedge \mu \le \alpha$ .This implies that λ(x)=1=μ(y) and $\lambda \wedge \mu \le \beta$, Since $0 \le \alpha \le \beta \le 1$. Hence it is clear that (X, t^*) is β - T_2(i)

Example:5.2.13. Let X={x, y} and consider the fuzzy supra topology

$t^* = \{1, 0, \lambda= \{(x, 1), (y, \frac{2}{5})\}, \mu=\{(x, 0), (y, 1)\}\}$ on X, taking $\alpha = \frac{1}{5}$ and $\beta = \frac{3}{5}$ then

(X, t^*) is β - T_2(i) but (X, t^*) is not α - T_2(ii)

Similarly if X, t^*) is β - T_2 (ii). Let $x, y \in X$, with $x \neq y$ for $\beta \in I_1$, $\exists \lambda, \mu \in t^*$ such that $\lambda(x) > \beta$,$\mu(y) > \beta$ and $\lambda \wedge \mu$=0.Henceλ (x)>α, , μ(y)> α as $\alpha \le \beta$. So $\Rightarrow (X, t^*)$ is α - T_2(ii).

Example:5.2.14. Let X={x, y} and consider the fuzzy supra topology

$t^* =\{1, 0, \lambda=\{(x, \frac{2}{5}), (y, \frac{2}{5})\}, \{(x, \frac{2}{5}), (y, \frac{1}{2})\}, \mu=\{(x, 0), (y, \frac{1}{2})\}\}$, on X taking $\alpha = \frac{1}{5}$ and $\beta = \frac{3}{5}$ then from (X, t^*) we have $\lambda(x)= \frac{2}{5} > \frac{1}{5}$, λ (y)= $\frac{2}{5}$, and μ (x) =0 μ (y)= $\frac{1}{2} > \frac{1}{5}$ so$(X, t^*)$is α- T_2 (ii) but (X, t^*) is not β - T_2 (ii) since λ (x)= $\frac{2}{5} < \frac{3}{5}$ and μ (y)= $\frac{1}{2} < \frac{3}{5}$.

Finally Suppose that (X, t^*) is 0- T_2 (ii) , we shall prove that (X, t^*) is 0 - T_2 (iii) . Let x, y $\in X$, with $x \neq y$, Since (X, t^*) is 0- T_2 (ii) then for $\alpha \in I_1$, $\exists$ $\lambda, \mu \in t^*$ s.t $\lambda(x)>0$, $\mu(y)>0$ and $\lambda\wedge\mu=0$. Hence it is clear that (X, t^*) is 0- T_2(iii).

Conversely Suppose that (X, t^*) is 0- T_2 (ii),we shall prove that (X, t^*) is 0 - T_2 (iii). Let x, y $\in X$, with $x \neq y$, Since (X, t^*) is 0- T_2(iii) then for $\alpha \in I_1$, $\exists \lambda, \mu \in t^*$ such that $\lambda(x)>0$, $\mu(y)>0$ and $\lambda \wedge \mu \leq 0$.This implies that $\lambda(x)>0$, $\mu(y)>0$ and $\lambda \wedge \mu=0$ Hence it is clear that (X, t^*) is 0- T_2 (ii).

This completes the proof.

Theorem 5.2.15: Let (X, t^*) and (Y, s^*) be two fuzzy supra topological spaces and f: $X \to Y$ be one-one ,onto and supra open map then

(1). (X, t^*) is α - T_2(i) $\Rightarrow$ (Y, s^*) is α - T_2(i)

(2) (X, t^*) is α - T_2(ii) $\Rightarrow$ (Y, s^*) is α - T_2(ii)

(3) (X, t^*) is α - T_2(iii) $\Rightarrow$ (Y, s^*) is α - T_2(iii)

Proof:- Suppose(X, t^*) is α - T_2(i) we shall prove (Y, s^*) is α - T_2 (i), Let $y_1, y_2 \in Y$, with $y_1 \neq y_2$, Since f is onto then $\exists$ $x_1, x_2 \in X$ with $f(x_1)= y_1$, $f(x_2)= y_2$ and $x_1 \neq x_2$ as f is one-one . Again (X, t^*) is α - T_2(i), for $\alpha \in I_1$ u, v $\in t^*$ such that $u(x_1)=1=v(x_2)$a and $u \wedge v \leq \alpha$,

Now $f(u)(y_1)=\{\text{Sup } u(x_1) : f(x_1)= y_1\}$

$=1$

$f(v)(y_2)=\{\text{Sup } v(x_2) : f(x_2)= y_2\}$

and $f(u\wedge v)(y_1) =\{\text{Sup } ((u\wedge v))(x_1) : f(x_1)= y_1\}$;

$f(u\wedge v)(y_2) =\{\text{Sup } ((u\wedge v))(x_2) : f(x_2)= y_2\}$

Hence $f(u\wedge v) \leq \alpha \Rightarrow f(u) \wedge f(v) \leq \alpha$.

Since f is supra open then f(u) ,f(v) $\in s^*$ such that $f(u)(y_1)=1$ and $f(v)(y_2)=1$and $f(u\wedge v) \leq \alpha$ .Hence (Y, s^*) is α - T_2 (i).

Similarly (b) and (c) can be proved.

Theorem 5.2.16: Let (X, t^*) and (Y, s^*) be two fuzzy supra topological spaces and $f: X \rightarrow Y$ be continuous and one-one supra open map then

(1) (Y, s^*) is α - T_2(i) $\Rightarrow$.(X, t^*) is α - T_2(i)

(2) (Y, s^*) is α - T_2 (ii) $\Rightarrow$ (X, t^*) is α - T_2(ii)

(3) (Y, s^*) is α - T_2 (iii) $\Rightarrow$ (X, t^*) is α - T_2(iii)

Proof: Suppose(Y, s^*) is α - T_2 (i) we shall prove (X, t^*) is α - T_2 (i) , Let $x_1, x_2 \in X$ with $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ in Y as f is one- one . Now since (Y, s^*) is α - T_2 (i) $\exists$ u, v $\in s^*$, such that $u(f(x_1))=1= v(f(x_2))=0$and $u \wedge v \leq \alpha$ for $\alpha \in I_1$ this implies that

$f^{-1}(u(x_1))=1, f^{-1}(v(x_2))=1$ and $f^{-1}(u \wedge v) \leq \alpha$ i.e $f^{-1}(u) \wedge f^{-1}(v) \leq \alpha$

since u,v $\in s^*$, , since f is continuous and u, v$\in s^*$implies that f^1 (u) ,f-^{1} (v) $\in t^*$ with $f^{-1}(u(x_1))=1$, $f^{-1}(v(x_2))=1$and $f^{-1}(u) \wedge f^{-1}(v) \leq \alpha$ Hence (X, t^*) is T_1(i) Similarly (2), (3) can be proved.

Lemma 5.2.17.: Show that every FST_2 space is FST_1 spaces.

Proof: Let (X, t^*) be a fuzzy supra topological spaces, Since (X, t^*) is FS-T_2 (ii) – Spaces if and only if $\forall$ x, y $\in X$, $x \neq y, \exists$ $\lambda, \mu \in t^*$ such that $\lambda(x)>0$, $\mu(y)>0$, and $\lambda \wedge \mu=0$. $\Rightarrow\lambda(x)>0$, $\mu(y)=0$ and $\mu(y)>0$ but$\lambda(x)=0$. Hence (X, t^*) is FST_1 spaces.

Lemma 5.2.18.: Give an example to show every FST_1 spaces need not FST_2 space .

Proof: Let X={x, y, z} and

$t^*=\{1, 0, \lambda=\{(x, \frac{3}{4}),(y,\frac{1}{2}), (z, 0),\{(x, \frac{3}{4}),(y, \frac{1}{2}), (z, 1)\}, \mu=\{(x, 0), (y, \frac{1}{2}),$

(z, 1)}on X. Here $\lambda(x)>0$, $\mu(z)>0$ So clearly it is FST_1(ii) but $\lambda \wedge \mu \neq 0$,

Hence (X, t^*) is not FST_2space.

Lemma: 5.2.19. Prove that every fuzzy - T_2 space is fuzzy supra - T_2 space.

Proof: From the definition of fuzzy topology and fuzzy supra topology it is clear that if t is a fuzzy topology on a set X and $t^*$is a fuzzy supra topology on X, then $t \subseteq t^*$, and fuzzy - T_2 space and fuzzy supra - T_2 space are applied same conditions on fuzzy topology as well as fuzzy supra topology respectively. So it is clear that every fuzzy - T_2 space is fuzzy supra - T_2 space.

CHAPTER-SIX

Compactness and Connectedness in FSTS

6.1. Introduction:- In this chapter we introduce and study some compactness property of fuzzy supra topological spaces.

6.2. Compactness Property of FSTS

Definition 6.2.1: Let (X, t*) be a fuzzy supra topological space and $\alpha\varepsilon[0, 1]$ .A collection $\Phi \subset I^X$ is said to be a fuzzy supra open α - shading if and only if for every point $x\varepsilon X$, there exists $\lambda\varepsilon$ Φsuch that $\lambda(x)>\alpha$, where $\alpha\varepsilon I$, and each member of Φ is a fuzzy supra open set.

Definition 6.2.2.: A fuzzy Supra topological space (X, t*) is supra compact, if every supra open cover of X by members of t^* contains a finite sub cover. that is if $\mu_i \varepsilon t^*$ for all $i\varepsilon J$ and $\vee_{i\in J} \mu_i =1$ where $i\varepsilon J$, then there are finitely many indices $I_1, I_2, ...I_n \varepsilon I$, Such that $\vee_{j=1}^{n} \mu_{ij} =1$.

Definition 6.2.3.: Let (X, t^*) be a fuzzy supra topological space and $\alpha\varepsilon I$ then
(X, t*) is said to be α- supra compact if every fuzzy supra open α- shading of the space has a finite α- sub shading. **[47]**

Theorem 6.2.4.: Let (X, t*) be a fuzzy supra topological space. Then the following are equivalent.

(1) μ_i such that $i \in J$ is a cover of X.

(2) $\vee_{i\in J} \mu_i =1$ where $i\varepsilon J$ for all $x \varepsilon X$.

(3) $\wedge_{i\in j} \mu_i =0$ where $i\varepsilon J$ for all $x \varepsilon X$

Proof: (1) $\Rightarrow$(2) is clear from the definition of cover. Since μ_i such that $i \in J$ is a cover of X means $\vee_{i\in J} \mu_i =1$ where $i\varepsilon J$ for all $x \in X$.

(2)□ (3) Since $\wedge_{i\in j} \mu_i =\inf\{\mu_i\}$ where $i\varepsilon J$ for all $x \in X$.

$= 1\text{-}\sup \{\mu_i\}$ where $i \in J$ for all $x \varepsilon X$.

$=1-1=0$

$(3) \Rightarrow (1)$, from (3) as above it can be shown that $\vee_{i\in J} \mu_i = 1$. Which implies that μ_i is a cover of X.

Theorem 6.2.5.: Let (X, t*) and (Y, S*) be a fuzzy supra topological space, with (X, t*) supra compact, and let f: $X \rightarrow Y$ be a supra fuzzy continuous surjection mapping. Then (Y, S*) is also supra compact.

Proof: Let $u_i \varepsilon S^*$ for each iεJ and assume that $\vee_{i\in J} u_i=1$. Now for each x εX,

$\vee_{i\in J} f^1(u_i)(x) =1$, So the t^*- open fuzzy sets $f^1(u_i)$, iεJ cover of X . Thus for finitely many indices $I_1, I_2, ...I_n$ εI such that $\vee_{i\in J} f^1(u_{ij}) =1$, If u is a fuzzy set in Y the condition f is surjection mapping into Y implies that , for any y εY,

$f(f^{-1}(u))(y)= \sup \{ f^1(u_i)(z): z\varepsilon f^{-1}(y)\}$

$= \sup \{u(f(z)):f(z)=y\} =u(y)$

so $f(f^1(u))=u$ .Thus, any fuzzy sets in Y.

$1=f(1)=f(\vee_{i\in J} f^1 (u_{ij})) = \vee_{i\in J} f(f^1(u_{ij})) = \vee_{i\in J} u_{ij}$ .Therefore (Y, S*) is supra compact .

Theorem 6.2.6: Let (X, t^*) and (Y, S^*) be a fuzzy supra topological spaces, and let f: $X \rightarrow Y$ be a supra fuzzy continuous surjection mapping. Let A is an fuzzy supra compact set in (X, t*) Then f(A) is also supra compact in(Y, S*) .

Proof: Let B= $\{G_i : i \in J \}$, where G_i be a fuzzy supra open cover of f(A). Then by definition of supra continuity A=

$\{f^1(G_i): \ i \in J \}$ is the fuzzy supra open cover of A. Since A is fuzzy supra compact, there exists a finite sub cover of A, that is G_{ik}, k=1, 2, 3……n, such that $A \subseteq \bigcup_{k=1}^{n} f^1 (G_{ik})$. Hence

$f(A) \subseteq f(\bigcup_{k=1}^{n} f^1(G_{ik})) = \bigcup_{k=1}^{n} f(f^1(G_{ik})) \subseteq \bigcup_{k=1}^{n} G_i$.

Therefore f(A) is fuzzy supra compact

Corollary 6.2.7. Let (X, t*) be a fuzzy supra topological space. Then(X, t*) be α- supra compact iff for all α-centered collection of fuzzy supra closed set F in X, there exists $x \in X$ and for all supra closure of $\mu(x) \geq 1-\alpha$ $\forall$ $\mu \in F$. **[47]**

Proof:-Firstly suppose (X, t^*) is α-supra compact then from definition for all supra open set $\mu \in F$ with $\mu(x) \geq \alpha$ with $\alpha \in [0, 1]$. Hence the complement of μ is $(1-\mu)(x)= 1-\mu(x)$. Hence there exist $x \in X$. $\mu(x) \geq 1-\alpha$. So supra closure of $\mu(x)$ $\geq 1-\alpha$.

Conversely suppose for all $\mu \in F$ supra closure of $\mu(x) \geq 1-\alpha$.So F is the collection of supra closed set with $\mu(x) \leq \alpha$. Hence by the theorem A supra fuzzy topological space (X, t*) is α-supra compact, iff $\forall$ collection F of supra closed set λ in X with $\lambda(x) \leq \alpha$, there is $x \in X$ such that $\mu(x) \geq 1-\alpha$ where $\mu \in F$,So (X, t^*) is α-supra compact .

Theorem 6.2.8: Let (X, t^*) and (Y, s^*) be two fuzzy supra topological spaces .Then the product $(X \times Y, \delta^*)$ is fuzzy supra compact if and only if (X, t^*) and (Y, s^*) are fuzzy supra compact.

Proof: First suppose that $(X \times Y, \delta^*)$ is fuzzy supra compact, then we can define a fuzzy continuous surjection mapping κ_l and π_2 from $(X \times Y, \delta^*)$ to (X, t*) and (Y, s^*) respectively .Then by a theorem (X, t*) and (Y, s*) are fuzzy supra compact.

Conversely let (X, t*) and (Y, s*) are fuzzy supra compact. Since $\delta^* = \{G_i \times H_i : G_i \in t^* \text{and } H_i \in s^* \text{for } i \in J\}$ where G_i and H_i are fuzzy supra open set. We claim that if $\{G_i : i \in J\}$ is a cover of X. and $(H_i : i \in J)$ is a cover of Y. That is if $\bigvee_{i \in J} G_i(x)=1$ for all $x \in X$, and if $\bigvee_{i \in J} H_i(y)=1$ where $i \in J$ for all $y \in Y$. Then $\bigvee_{i \in J} \{(G_i \times H_i)(x,y)\}= \text{Sup } \{\min\{ G_i(x), H_i(y)\}\}$. Hence we have finite subset J' of for which $\bigvee_{i \in J} G_i(x)=1$ or $\bigvee_{i \in J} H_i(y)=1$. Whence we have

$\delta^* = \{G_i \times H_i : G_i \varepsilon t^* \text{and } H_i \ \varepsilon\ s^* \text{for } i \ \varepsilon\ J'\}$ is a finite sub cover of $(X \times Y, \delta^*)$ .Hence$(X \times Y, \delta^*)$ is fuzzy supra compact.

Theorem:6.2.9. Every fuzzy supra compact subset of a fuzzy supra T_2-space is closed.

Proof: Let (X, t^*) is a fuzzy supra T_2-space and λ is fuzzy supra compact subset of X. We shall prove that λ is closed. If $\lambda^c = 0$, then λ is closed. Since 0 is an open set. If $\lambda^c \neq 0$ Now since (X, t^*) is a fuzzy supra T_2-space then $\forall$ x, y εX, x $\neq$ y , $\exists$ λ, με t* such that λ(x)>0, μ(y)>0

and λ∧μ=0. Now μ(y) is an open cover of λ Since X is supra compact .then $\exists$ finite number of points $y_1, y_2, y_3, \ldots\ldots y_n$ such that $\lambda \subset \bigcup_{i=1}^{n} y_i$ if $x \in \lambda^C$ then $x \notin \bigcup_{i=1}^{n} y_i$ Since λ∧μ=0. Which shows that λ^C is open. Hence λ is closed.

6.3: Connectedness Property of FSTS

In the following section we have investigated certain fuzzy supra connected concepts and we will use the abbreviation FSC for the terms 'fuzzy supra connected'.

Definition 6.3.1.: Let (X, t*) be a FSTS. Two fuzzy set λ and μ in X are said to be Q-separated if there exist (t^*-supra closed) or t^*-supra open fuzzy sets u and v in X such that $\lambda \leq u$, $\mu \leq v$ and λ∧v=0,or μ∧u=0 $\Rightarrow \bar{\lambda} \wedge \mu = 0 = \lambda \wedge \bar{\mu}$ .If λ and μ are crisp then they are Q-separated iff they are separated. **:[49]**

Definition 6.3.2: Let (X, t^*) be a FSTS. Letλ andμ are two supra fuzzy closed subsets in X. Then λ and μ are said to be disjoint if λ∧μ=0.

Theorem 6.3.3: Let (X, t*) be a FSTS. λ and μ are two supra fuzzy closed subsets in X. We shall prove that λ∧μ=0 if and only if $\bar{\lambda} \wedge \mu = 0 = \lambda \wedge \bar{\mu}$ in other word two fuzzy supra closed subsets of a FSTS (X, t*) is separated if and only if they are disjoint.

Proof: Let (X, t*) be a FSTS and λ and μ are two fuzzy supra closed subsets in X, Then $\bar{\lambda} = \lambda$ and $\bar{\mu}$ =.μ Let λ and μ are separated then by definition $\bar{\lambda} \wedge \mu = 0$ and $\lambda \wedge \bar{\mu} = 0$, Since $\bar{\lambda} = \lambda$and $\bar{\mu} = \mu$ so λ∧μ=0 and λ∧μ=0 $\Rightarrow$ λ and μ are disjoint.

Conversely let λ and μ are two disjoint supra fuzzy closed subsets of (X, t*) then λ∧μ=0. We shall prove that they are separated, i.e

$\overline{\lambda} \wedge \mu=0$ and $\lambda \wedge \overline{\mu}=0$, now $\lambda\wedge\mu=0$

$\Rightarrow \overline{\lambda} \wedge \mu=0$ and $\lambda \wedge \overline{\mu}=0$. Since $\overline{\lambda}=\lambda$ and $\overline{\mu}=\mu$. Hence λ and μ are separated.

Theorem 6.3.4: Let (X, t^*) be a FSTS. λ And μ are two separated fuzzy supra subset in X. We shall prove that if $\lambda\vee\mu$ is fuzzy supra closed then $\lambda$and μ are individually fuzzy supra closed.

Proof: Let λ and μ are two fuzzy separated supra subsets in X such that $\lambda\vee\mu$ is fuzzy supra closed. We shall prove that λ and μ are individually fuzzy supra closed.

Since λ and μ are two fuzzy separated supra subsets in X, so we have $\lambda\vee\overline{\mu}=0$ and $\overline{\lambda}\wedge\mu=0$. Also $\lambda\vee\mu$ is fuzzy supra closed $\Rightarrow \overline{\lambda\vee\mu}=\overline{\lambda}\vee\overline{\mu}=\lambda\vee\mu$

$\therefore \overline{\lambda}=\overline{\lambda}\wedge(\overline{\lambda}\vee\overline{\mu})=\overline{\lambda}\wedge(\overline{\lambda}\vee\mu)=(\overline{\lambda}\wedge\lambda)\vee(\overline{\lambda}\wedge\mu)=\lambda\vee 0=\lambda$

Similarly $\overline{\mu}=\mu$.

Hence λ and μ are fuzzy supra closed.

Theorem 6.3.5: Let (X, t^*) be a FSTS. λ and μ are two separated fuzzy supra subsets in X. We shall prove that if $\lambda\vee\mu$ is fuzzy supra open then λ and μ are individually fuzzy supra open.

Proof:- Let λ and μ are two fuzzy separated supra subsets in X such that $\lambda\vee\mu$ is fuzzy supra open . We shall prove that λ and μ are individually fuzzy supra open . Since λ and μ are two fuzzy separated supra subsets in X, So we have $\lambda\vee\overline{\mu}=0$ and $\overline{\lambda}\wedge\mu=0$. Also $\lambda\vee\mu$ is fuzzy supra open. $\overline{\mu}$ being closed . So $(\overline{\mu})^c$ is closed. Hence $(\lambda\vee\mu)\wedge(\overline{\mu})^c$ is fuzzy supra open. So

$(\lambda\vee\mu)\wedge(\overline{\mu})^c=[\lambda\wedge(\overline{\mu})^c]\vee[\mu\wedge(\overline{\mu})^c]=\lambda\wedge 0=\lambda$

[Since$\lambda\vee\overline{\mu}=0\Rightarrow\lambda\leq(\overline{\mu})^c$]

This shows that λ is open, similarly it can be prove that μ is open.

Definition 6.3.6: Let (X, t^*) be a FSTS. We call (X, t^*) is

(1)FSC (I) iff there does not exist non-zero Q-separated supra fuzzy set u and v in t^* with $1=u\vee v$.

(2)FSC (II) iff there does not exist non-zero separated supra fuzzy sets u and v in X with $1= u \vee v$.

(3) FSC (III) iff there do not exist $u, v \in t^* - \{0, 1\}$ with $u \vee v>0$ and $u \wedge v= 0$.

Theorem 6.3.7: The following statements are equivalent for an FSTS (X, t^*) ;

(a) (X, t^*) is FSC (I).

(b)There does not exist non-zero disjoint t^*-supra closed crisp sets whose supremum is 1.

(c)There does not exist non-zero disjoint t^*-supra open crisp sets whose supremum is 1.

(d) (X, t^*) is FSC (II).

Proof: (a) $\Rightarrow$(b)

If there exist Crisp subsets 1_A and $1_B \in t^{*C}$, $1_A \neq 0 \neq 1_B$, such that $1_A \vee 1_B =1$ and $1_A \wedge 1_B =0$. Then clearly 1_A and 1_B are t^* separated.

So (X, t^*) is not FSC (I).Which contradicts (a).

(b) $\Rightarrow$(c)

If there exist Crisp subsets 1_A and $1_B \in t^*$, $1_A \neq 0 \neq 1_B$, such that $1_A \vee 1_B =1$ and $1_A \wedge 1_B =0$. Then clearly 1_A and 1_B are t^*- closed. Which contradicts (b) .

(c) $\Rightarrow$(d) If (X, t^*) is not (d) then $u, v \in I^X - \{0\}$ such that u and v are separated and $1=u \vee v$.

Now there exist λ and $\mu \in t^*$ such that $u \leq \lambda$ and $v \leq \mu$ and $\lambda \wedge v=0=\mu \wedge u$, but then λ and μ are crisp also $\lambda \wedge \mu =0$ and $\lambda \vee \mu=1$ contradicting (c)

Theorem 6.3.8 A supra topological space (X, T^*) is connected iff $(X, \omega(T^*))$.is FSC(i)

Proof:- First ,let (X, T^*) is connected . If $(X, \omega(T^*))$.is not FSC(i) then there exist two non-zero fuzzy supra open sets u, v in $\omega(T^*)$ such that $1 = u \vee v$. and $u \wedge v= 0$. Then we see that $u^{-1}(\alpha,1]$, $v^{-1}(\alpha,1] \in T^*, \alpha \in I_1$ and $u^{-1}(\alpha,1] \cup v^{-1}(\alpha,1]=X$ and $u^{-1}(\alpha,1] \cap v^{-1}(\alpha,1]=\phi \Rightarrow (X, T^*)$ is not connected, a contradiction.

Conversely if $(X, \omega(T^*))$.is FSC(i) but $(X, T^*.)$ is not connected then there exist

U and $V \in T^{*}$ with $U \neq \phi \neq V$ such that $U \cap V = \phi$ and $U \cup V = X$, Now I_U, 1_v being l.s.c belong to (ω (T^{*})) and $I_U \neq 0 \neq 1_v$ .Moreover $I_U \vee 1_v = 1$ and $I_U \wedge 1_v = 0$, Showing that (X, ω (T^{*})).is not FSC(i) ,which is also a contradiction.

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