Optimum Equalizer and PSK Modulation Technique for MIMO Wireless Communication System

by

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Declaration

This is to certify that the thesis work entitled *"Optimum Equalizer and PSK Modulation Technique for MIMO Wireless Communication System"* has been carried out by *Sabuj Sarkar* in the Department of *Electrical and Electronic Engineering,* Khulna University of Engineering & Technology, Khulna, Bangladesh. The above thesis work or any part of this work has not been submitted anywhere for the award of any degree or diploma.

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This is to certify that the thesis work submitted by Sabuj Sarkar entitled "Optimum Equalizer and PSK Modulation Technique for MIMO Wireless Communication" has been approved by the board of examiners for the partial fulfillment of the requirements for the degree of *Master of Science in Electrical and Electronic Engineering* from the Department of Electrical and Electronic Engineering, Khulna University of Engineering & Technology, Khulna, Bangladesh in February 2013.

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Abstract

Most recently, there have been meaningful and significant advancements in the area of wireless communications, specifically with the introduction of multiple-input multiple-output (MIMO) and the ever-increasing use of equalizers in MIMO wireless communication. Based on phase shift keying (PSK) modulation and from the knowledge of zero-forcing (ZF), maximum likelihood (ML) and minimum mean square error (MMSE) equalizers, this thesis proposes a smart equalizer alternative to existing optimum equalizer termed as Modified MMSE equalizer that reduces bit error rate (BER) via spatial multiplexing (SM), diversity combining and splits up each user's MIMO channel into parallel sub-channels with identical SNRs. ZF equalizer amplifies noise at the time of matrix inversion and the complexity of ML equalizer increases exponentially with increasing number of transmitting antennas and higher order modulation. On the contrary, MMSE and MMSE-SIC cannot exhibit superior performance than ML and ISI is not completely removed, as a result a new optimal equalizer determination is crucial. This thesis sets up a novel technique using MMSE equalizer that eliminates the limitations occurred by the ML, MMSE and ZF equalizers while offering optimal performance. The new equalizer is the upgraded version of MMSE equalizer that incorporates channel equalization and noise addition with successive interference cancellation by means of matrix ordering and modified vector estimation. Channel matrix with noise is first inverted to remove the effects of intersymbol interference (ISI). The channel is then multiplied with the transmitted signal and its interference is cancelled consecutively in accordance to higher signal power forming a proposed matrix order and modified vector. In this way, noise as well as ISI is eliminated in order by subtracting its effect from the subsequent stages and the process continues in an iterative way which optimizes the performance criterion. BER vs. signal-noise ratio (SNR) curve of Modified MMSE equalizer exceeds all of the equalizers considered in this research even than that of the existing optimum ML equalizer i.e. performance characteristics of Modified MMSE is optimal with respect to all of the equalizers. Besides, Modified MMSE equalizer eliminates noise as well as ISI while keeping complexity low as it does not involve any exponential terms akin to ML equalizer. The complexity is reduced from exponential to linear. The new

equalizer technique is compared with other linear and non-linear techniques comprising ZF, ML and MMSE equalizers. MatLab toolbox is exploited for simulation that shows superior performance of the proposed equalizer over the conventional ones. Rayleigh fading channel is used in the simulation as it enables the computation of the expected complexity up to simpler expression in the paradigm. So far, for a realistic perspective, the optimal performance as well as simpler complexity is obtained at the same time and the optimal equalizer can be evaluated efficiently for MIMO wireless communication.

Contents

List of Figures

List of Tables

Abbreviations

3GPP	Third Generation Partnership Project
4G	Fourth Generation
BER	Bit Error Rate
BICM	Bit Interleaved Coded Modulation
CP	Cyclic Prefix
CSI	Channel State Information
EGC	Equal Gain Combining
ISI	Intersymbol Interference
LDC	Linear Dispersion Codes
LTE	Long Term Evolution
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MMSE-SIC	Minimum Mean Square Error with Successive Interference Cancellation
MRC	Maximal Ratio Combining
OFDM	Orthogonal Frequency Division Multiplexing
PDF	Probability Density Function
PSK	Phase Shift Keying
SC	Selection Combining
SDMA	Space Division Multiple Access
SER	Symbol Error Rate
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
STBC	Space Time Block Coding
STC	Space Time Coding

STTC	Space Time Trellis Code
UE	User Equipment
V-BLAST	Vertical Bell Labs Space-Time Architecture
Wi-Fi	Wireless Fidelity
WiMAX	Wireless interoperability for Microwave Access
ZF	Zero Forcing
ZF-SIC	Zero Forcing with Successive Interference Cancellation
ZF-SIC-Sort	Zero Forcing Successive Interference Cancellation with Optimal Ordering

Chapter 1

Introduction

1.1 Introduction

Multiple input multiple output (MIMO) system offers very promising gain in capacity without increasing the use of spectrum, increases reliability and throughput, decreases power consumption and reduces fading, hence leading to an incredible improvement in the data rate of wireless communication systems. The effect of fading and interference always causes an issue for signal recovery in wireless communication. This can be combated with the application of equalizer which compensates intersymbol interference (ISI) created by multipath signal propagation within time dispersive channels. A problem encountered in the design of receivers for MIMO wireless communication systems is the detection of data from noisy and interference environments of the transmitted signals. Designing of optimum equalizer in case of phase shift keying (PSK) modulation is a major obstacle for MIMO wireless communication system. In a real viewpoint, noise and ISI tend to make occasional errors. Thus, developing an equalizer which has this probability of error minimal is encouraging, both from a practical and a theoretical point of view. One way of removing the problem of noise and ISI is by using a novel equalizer that tends to minimize the BER and provides efficient modulation. Unfortunately, such designs tend to result in computationally complex equalizers and for this reason; they are often abandoned in favor of computationally simpler but suboptimal equalizers. However, it is notorious that for many cases the performance gap between the suboptimal and optimal equalizer is significant. This fact makes the research of optimal equalizer further appealing. Moreover, the decreasing cost of computation will result in computationally reasonable optimal designs.

1.2 Background

MIMO is one of the most attractive techniques in wireless communication that uses multiple antennas at both transmitter and receiver and provides improved bit error rate (BER), or data rate compared to conventional communication systems [1-3]. MIMO has distinguished features which offer significant increment in data throughput and link range without additional bandwidth and increase transmit power. Owing to these properties, it has become an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution (LTE) and WiMAX [4-6]. In MIMO wireless communication, multipath fading is a usual phenomenon that causes ISI in the transmitted signal. To remove ISI from the transmitted signal, a strong equalizer is compulsory. The standard linear equalizer methods include the zero-forcing (ZF) technique and the minimum mean square error (MMSE) technique. ZF equalizer applies the inverse of the channel to restore the signal after the channel and to make ISI to zero in a noise free condition. However the inversion step is responsible for noise enhancement. In [7], channel estimation error performance of MIMO ZF receivers in uncorrelated Rayleigh flat fading channels has been investigated. However the detection requires knowledge of the channel state information (CSI) and in practice accurate CSI may not be available. The effect of channel estimation error on the performance of MIMO ZF receivers in uncorrelated Rayleigh flat fading channels has been investigated in [7]. MMSE equalizer is also a linear equalizer that minimizes mean square error (MSE) to maximize the post-detection signal-to-interference plus noise ratio (SINR). As a result, the total noise power and ISI components in the output are minimized. Still, it requires an accurate estimate of the amount of noise present in the system which is hard to obtain in practical systems. MMSE equalizer based receiver for MIMO wireless channel has been suggested in [8] which is a good choice for removing some ISI and minimizes the total noise power. Although it has lower complexity but resulting ISI from the transmitted signal degrades its performance which is yet a major drawback for MIMO wireless channel. Maximum likelihood (ML) is a non-linear equalizer that follows a search process which is performed over all possible symbols and the most likelihood one is chosen. The ML decoder would select the set of symbols that are closest in Euclidean distance to the received signals. In [9] the performance of ML detection has been analyzed over flat fading channels for wireless MIMO system. In this case, a very high data rate can be obtained with little SNR penalty i.e. it has least possible BER but the transceiver structure will be complex exponentially with increasing transmitter size and modulation order. In [10] a soft-decoding ML MIMO demodulation method has been proposed that lowers the complexity of the existing ML method. A MIMO ML demodulator offers optimum performance with significantly lower complexity than the linear equalizer. For 3×3 MIMO communication systems, analysis of full-rate linear and space-time block code under a Rayleigh flat-fading environment has been carried out in [11] by using linear MMSE and ML receivers to minimize the average symbol error rate (SER) for a QPSK transmitted signal. For higher order MIMO the performance of ML equalizer is better than MMSE equalizer at the expense of compound hardware structure. By using spatial multiplexing technique a comparative study of various modulation schemes for MIMO wireless communication has been carried out in [12]. But for higher order PSK modulation, BER tends to increase in a significant manner due to its complex constellation structure. In [13] MIMO bit-interleaved coded modulation (MIMO-BICM) with linear zero-forcing (ZF) receivers has been considered. In particular, a reduction in the performance gap between ZF and ML was observed for increasing number of receiver antennas and higher modulation formats. This is significant, since in these situations, the complexity advantage of the ZF scheme is considerable. Analysis of ZF and MMSE equalizers applied to wireless MIMO systems with larger number receive antennas than transmit antennas has been proposed in [14]. The ZF detector and the MMSE detector have lesser computational calculations as compared to ML-detector as they require only a matrix operation to be carried out. In MIMO system, the performance of V-BLAST with several detectors (ML, ZF, MMSE, STBC, and MRC) in slow fading channels has been analyzed [15, 16]. A drawback of BLAST algorithms is the propagation of decision errors. Also, the interference voiding operation requires that the number of receive antennas be greater than or equal to the number of transmit antennas. Furthermore, due to the interference suppression, early detected symbols benefit from lower receives diversity than later ones. Thus, the algorithm results in unequal diversity advantage for each symbol. Using channel estimation techniques, the BER performance characteristics of MIMO system has been investigated in [17]. However non-linear detectors with

SIC are more complex than linear detectors. Kuldeep et. al. [18] proposed a different detection scheme for a 4×4 MIMO system. BER performance characteristics of ZF, MMSE and ML equalizers for MIMO wireless receiver has been investigated in [19]. In wireless communications, MIMO increases data rate and improves performance through multiplexing and diversity combining. But communicating through wireless channel, transmitted signal has to face many obstacles. As a result, ISI, noise addition, time dispersion, attenuation, phase shift etc. phenomena occurs and the receiver cannot retrieve the original data transmitted through the wireless medium. Therefore, the received signal is not the same replica as the transmitted signal. Orthogonal frequency division multiplexing (OFDM) and a cyclic prefix (CP) somehow mitigate the effect of time dispersion. However, ISI and noise is still present in the signal and in general, equalizers are used to surmount this problem. By forcing ISI to zero, ZF equalizer can solve the ISI problem in a noise free environment. But in a noisy condition, it amplifies noise in addition to signal which is a barrier for signal recovery. By applying ZF with successive interference cancellation (ZF-SIC) and successive interference cancellation with optimal ordering (ZF-SIC-Sort), the receiver can decode data packets simultaneously and interferer terms are cancelled in a successive way and even better performance than the former one can be achieved. But the problem of signal distortion still exists because some noises are inherently induced by ZF equalizer. Noise power and ISI components in the output are minimized to some extent by MMSE equalizer. However, the difficulty of the MMSE filter lies in its requirement of accurate SNR estimate which will become much more serious when the ISI effect can no longer be neglected with much higher data rates in next-generation wireless communications. ML equalizer selects the minimum sample from all possible samples. It can exploit all of the available diversity but it is challenging to implement due to its exponential complexity and hard decision performance. Hardware implementation complexity along with noise and ISI is yet a major problem for finding an optimum equalizer. In this thesis the challenge of optimum equalizer selection for MIMO wireless channel in the presence of PSK modulation has been carried out. Design of a novel equalizer that performs even better than existing equalizers is yet a major challenge in achieving efficient modulation for MIMO wireless communications. Hence, in this research, due to the increasing demand of MIMO wireless communication, we are strongly motivated to estimate new optimum equalizer to achieve low BER performance with respect to increasing signal-noise-ratio (SNR).

1.3 Proposal For Optimum Equalizer Selection

The existing problem of signal distortion can be overcome by decreasing BER with respect to increasing SNR. The received signal will be approximately close to the transmitted signal when the BER is lowest. Higher order MIMO and PSK modulation with different constellation points for wireless channel has been proposed in this research to design an optimum equalizer that will aid the receiver to recover the original signal. To develop the optimum equalizer an innovative MMSE equalizer has been proposed that exhibits even superior performance to that of the existing optimum ML equalizer. But as ML is complex in terms of hardware pattern than that of novel MMSE. This novel MMSE is termed as Modified MMSE i.e. proposed optimum equalizer in this research. By using MatLab, simulation has been carried on for ZF, ZF-SIC, ZF-SIC-Sort, MMSE, MMSE-SIC, Modified MMSE and ML equalizer respectively, for higher order MIMO wireless channel and different PSK modulation. Especially, 2×2, 2×3 and 2×3 MIMO combinations and B-PSK, Q-PSK, 8-PSK, 16-PSK and 32-PSK has been employed for developing the proposed optimum equalizer. Using fixed number of transmitting antennas, receiving antennas are verified by 2, 3 and 4 respectively. First, BPSK modulation has been implemented for all these equalizers. Then step by step simulation has been done for QPSK, 8-PSK, 16-PSK and 32-PSK modulation. While the processes that are followed by each individual equalizer are stated below:

ZF equalizer proceeds by canceling the interference terms using the following weight matrix:

$$W_{ZF} = \left[H^H H\right]^{-1} H^H$$

$$\hat{x} = W_{ZF} x$$

The inverse of the channel are applied to the received signal to restore the signal before the channel and it forces the ISI to zero. When the off diagonal elements in the matrix $H^H H$ are not zero, ZF equalizer cuts short the interfering terms at the time of equalization process. But in that process an amplification of noise occurs. ZF with successive interference cancellation, the receiver can decode data packets simultaneously and BER can be improved further and ZF with successive interference cancellation and optimal ordering the receiver decode the signal first which comes with higher power. It ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. Yet noise is present in the received signal and hence the ZF equalizer is not the best possible equalizer in presence of noise. ML equalizer selects the set of symbols which are closest in Euclidean distance to the received all signals. It compares the received signals with all possible transmitted signal vector that is modified by channel matrix H and estimates transmit symbol vector x according to the maximum likelihood principle.

$$x = arg_{x_k} \in \{x_1 x_2 \cdots x_R\}\, min \left\| r - Hx_k \right\|^2$$

Where, x is the estimated symbol vector. At the receiver, the most likelihood transmitted signal is determined, as the one for which $\|r - Hx_k\|^2$ is minimal i.e. which is closest to the received vector is said to be the most likelihood signal to be transmitted. MMSE equalizer estimates a random vector x on the basis of observations y to choose a function x(y) which minimizes the MSE. MMSE detector finds a coefficient W which minimizes

$$W = \left[H^H H + N_0 I\right]^{-1} H^H$$

MMSE equalizer minimizes the total noise power and ISI components in the output. The linear MMSE equalizer achieves minimum MSE among all estimators of the form AY + b. Where, Y is a random vector, A is a matrix and b is a vector. While it simplifies computation process but a matter of concern is that it cannot eliminate the ISI entirely. As a result its performance degrades with high SNR and cannot provide satisfactory results. MMSE-SIC proceeds by successively voiding out the interference terms. As a result its performance is better than linear MMSE equalizer. As ML is an optimal method among the existing equalizing techniques but its complexity increases with increasing transmitting antennas and modulation order. So an alternate method that outperforms all the existing equalizers even that of ML equalizer while keeping complexity lower than that of the existing methods termed as Modified MMSE is implemented in this thesis. Proposed Modified MMSE, successive interference is cancelled based on prescribed modified matrix order, which is termed as Modified MMSE. The Modified MMSE performs even better than that of ML equalizer. It is the proposed optimal equalizer that provides the solution of the existing problem. This is carried out similar to that MMSE technique but additionally interference is cancelled in an updated ordered matrix approach. While transmitted signal from one antenna is considered the interference from other antenna is deducted on the basis of optimum power of the transmitted symbol. This process continues until all the interference terms are cancelled. Finally it gives optimized performance in terms of decreased BER with respect to SNR. Modified MMSE equalizer along with ZF, ZF-SIC, ZF-SIC-Sort, ML, MMSE, MMSE-SIC are simulated and compared. Throughout this research, simulating with MatLab program and comparing all of these equalizers with point to point basis in terms of performance as well as complexity concern has been carried out to decide about the proposed optimum equalizer. The clear point that is imposed in this process is that to develop the proposed optimum equalizer that performs best considering all of the terms and conditions that may present in the wireless channel and can degrade the performance of MIMO wireless channel. The case is imposed here is that to fix the transmitting antennas and varying the receiving antennas with consecutive increase of PSK modulation. From all the studies that have been conducted throughout the research, estimates the novel equalizer which is the optimum equalizer for MIMO wireless communication system considering PSK modulation irrespective of all conditions considered in this research.

1.4 Thesis Outline

This thesis is organized as follows:

Chapter 1 is an introductory chapter which focuses the purpose of this thesis.

In Chapter 2, MIMO wireless communication is described. The concept of MIMO wireless transceiver system, data transmission and channel matrix are described in the first part of the chapter. Later classification, operating principle and formation of MIMO are explained and finally Rayleigh, Rician fading channel as well as Okumura and Hata model are analyzed.

Chapter 3 describes PSK modulation technique. Basics of PSK modulation, requirements of PSK in MIMO wireless communication, its classification as well as constellation and implementation with mathematical model are explained.

Different types of equalizers are introduced in chapter 4. Throughout this chapter, different equalizers with existing mathematical models are discussed.

Chapter 5 explains the proposed Modified MMSE equalizer. Problems of specific equalizer are mentioned in order to design proposed equalizer. Overview and the structure of the proposed equalizer is also explained. Flowchart diagrams of proposed Modified MMSE equalizer along with existing equalizers are provided. Later, comparison of proposed Modified MMSE with MMSE and MMSE-SIC are also analyzed. Finally, proposed optimum equalizer judgment for PSK modulation and different MIMO combination are analyzed from simulation and graphical point of view and their performances are also evaluated.

Chapter 6 concludes the thesis, with a summary and some suggestions on further research.

Chapter 2

MIMO Wireless Communication

2.1 Introduction

Wireless communications and its recent explosion in multiple-input multiple-output (MIMO) systems have opened a new era for researchers. To be able to get an exact analysis of MIMO systems and to make statements about performance criterions, we need an adequate explanation of the basic channel and its properties in terms of fading, time variance, linearity, intersymbol interference (ISI), etc. A complementary description of a MIMO channel is a research area of itself, and a lot of investigations are carried out for the classification and description of MIMO transmission phenomena and their impact on performance parameters. In this thesis, we are interested in identifying the key parameters of MIMO systems to find an optimal performance for the MIMO wireless channel in different scenarios. To avoid difficulty, we will choose a very basic MIMO transmission model which is strong enough to provide basic insights into MIMO communications while being sufficiently simple in its analytical representation. This chapter explains basic MIMO wireless system, data propagation model, channel matrix for a real communication environment, and the necessary assumptions to verify the choice of this representation. Furthermore, we develop operating principle, formation and classification of MIMO in terms of hierarchical arrangement and transmitter state knowledge. Finally, channel characteristics such as diversity, multiplexing and fading are described in order to get further information about wireless MIMO.

2.2 MIMO Wireless Transceiver System

In wireless communication, the use of multiple antennas at both the transmitter and receiver is known as MIMO. It is a spatial multiplexing technique and one of the several forms of smart antenna technology. MIMO technique has attracted unlimited attention in wireless communications that offers multiple parallel sub channels at the same frequency, therefore giving higher capacities over the same bandwidth. It achieves this target by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency or to achieve a diversity gain that improves the link reliability. For this reason, MIMO is one of the most up-to-date topics of international wireless research. It takes advantage of a radio-wave phenomenon called multipath where transmitted information bounces off walls, ceilings, and other objects, reaching the receiving antenna multiple times via different angles and at slightly different times. This initiates multipath behavior by using multiple smart transmitters and receivers with an added spatial dimension to dramatically increase performance and range. Smart antennas use spatial diversity, which puts additional antennas to good use. If there are more antennas than spatial streams, as in a 2×3 antenna configuration, then the third antenna can add receiver diversity and increase range. In order to implement MIMO, either the user (mobile device) or the base stations (BS) need to support MIMO. Optimal performance and range can only be obtained when both the station support MIMO. Conventional wireless devices cant take advantage of multipath because they use a single-input single-output (SISO) technique. Systems that use SISO can only send or receive a single spatial stream at one time. MIMO is a significant innovation and technique which has been adapted to work for a few more wireless standards beside the 802.11, such as 4G standards.

A typical MIMO transceiver system containing C transmitting-antennas and $D \geq C$ receiving-antennas are depicted in figure 2.1. The signal from each of the C transmitters can reach each of the D channel receivers on different paths. MIMO works best if these paths are distinct spatial, resulting in reception of signals that are uncorrelated. Multi-paths aids channels de-correlation, and thus increase the efficiency of spatial multiplexing.

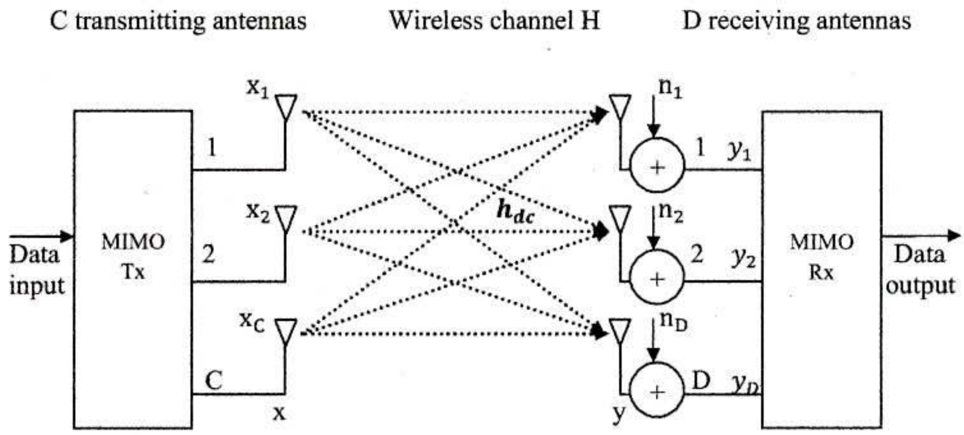

FIGURE 2.1: MIMO Tx-Rx model.

The received signal in the D^{th} antenna is given by

$$y_D = \sum_{C=1}^{C} h_{CD} x_C + n_D \tag{2.1}$$

In this case the wireless channel is narrowband, time-invariant, and can be represented by the $C \times D$ matrix H. The $C \times 1$ dimensional transmitter vector x consists of the independent input symbols $x_1, x_2, \ldots, x_C$ and y is the received symbol vector consisting of output symbols $y_1, y_2, \ldots, y_D$ and c is the fading corresponding to the path from transmitting antenna C to receiving antenna D, where, $D = 1,2,3,\cdots$,D. n_D is the noise corresponding to receiving antenna D. So, MIMO transmission can be expressed by the general form of,

$$y = Hx + n \tag{2.2}$$

2.3 Data Propagation Through MIMO

The figure 2.2 describes data propagation in a MIMO system. We consider 6-bit data stream which is splitting up into C equal rate data streams, where C indicates the number of transmitting antennas, which is three for the present case. Each of the lower bit rate sub streams are transmitted from one of the antennas. All are transmitted at the same time and at the same frequency, therefore they mix together in the channel. Since all sub streams are being transmitted at the same frequency, it is very spectrally efficient.

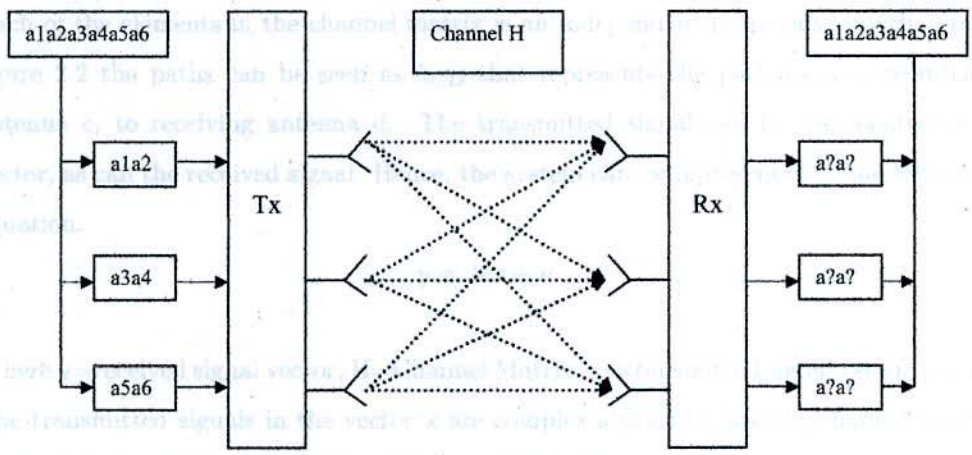

FIGURE 2.2: Block diagram of data propagation through MIMO.

Each of the receiving antennas picks up all of the transmitted signals superimposed upon one another. If the channel H is a sufficiently rich scattering environment, each of the superimposed signals will have propagated over slightly different paths and hence will have differing spatial signatures. The spatial signatures exist due to the spatial diversity at both ends of the link and create independent propagation channels. Each transmitting-receiving antenna pair can be treated as parallel sub channels, this will become obvious when channel H is analyzed. Since the data is being transmitted over parallel channels, one channel for each antenna pair, the channel capacity increases in proportion to the number of transmitting-receiving pairs.

2.4 MIMO Channel Matrix, H

Since each of the receiving antennas detects all of the transmitted signals, there are $C \times D$ independent propagation paths, where there are C transmitting and D receiving antennas. This allows the channel to be represented as $C \times D$ matrix. By using $C \times D$ MIMO system, the channel matrix H is as obtained as follows:

$$H = \begin{bmatrix} h_{11} & h_{12} & .. & h_{1c} \\ h_{21} & h_{22} & .. & h_{2c} \\ .. & .. & .. & .. \\ h_{d1} & h_{d2} & .. & h_{dc} \end{bmatrix}$$

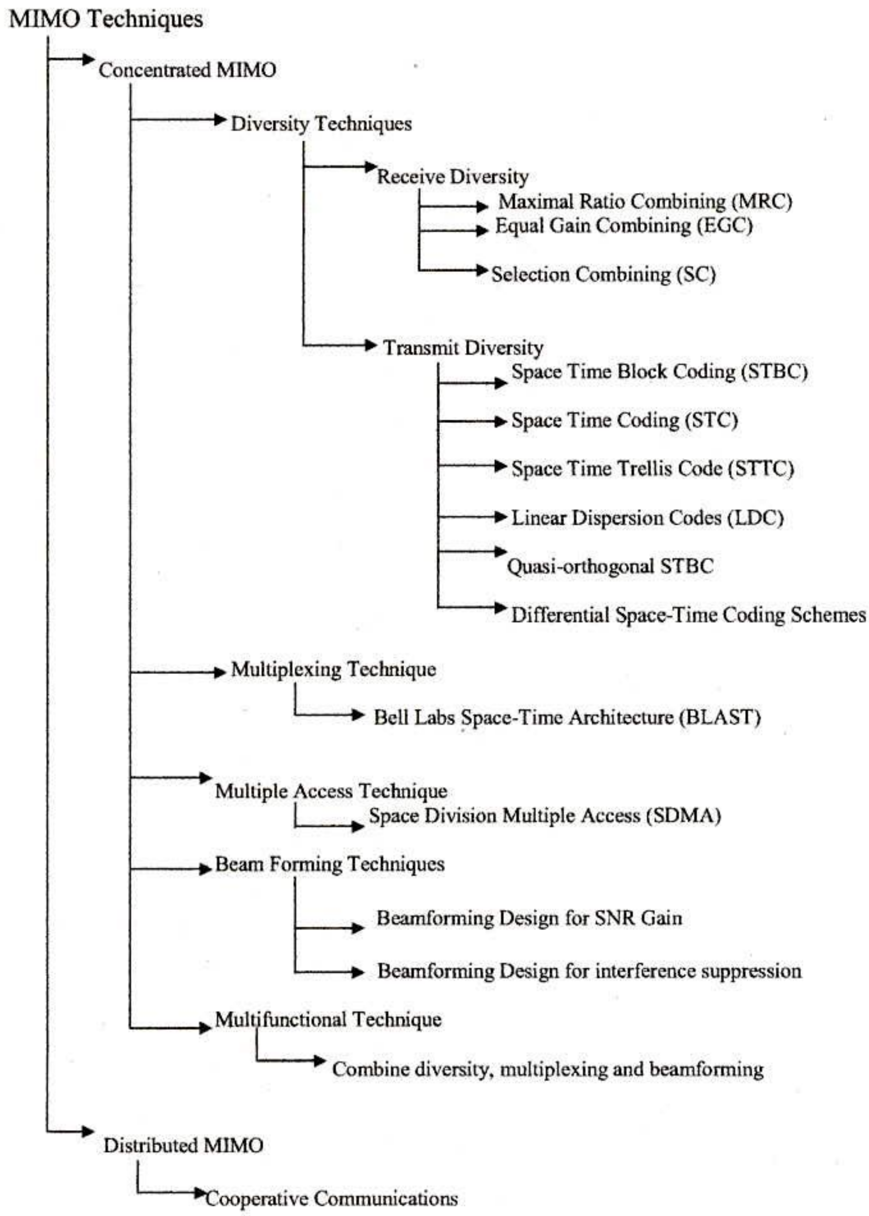

FIGURE 2.3: Classification of MIMO techniques.

Diversity Techniques

Wireless communication in the presence of channel fading has been regarded as one of the immense research challenges in recent times. In a fading channel, the associated severe attenuation often results in decoding errors. A natural way of overcoming this problem is to allow the receiver to have several replicas of the same transmitted signal, while assuming that at least some of them are not severely attenuated. This technique is referred to as diversity, where it is possible to attain diversity gains by creating independently fading signal replicas in the time, frequency or spatial domain. In case of narrowband frequency-flat fading, the optimum combining strategy in terms of maximizing the SNR at the combined output is maximal ratio combining (MRC). MRC is often used in large phased-array systems. In selection combining, the strongest signal is selected from all of the received signals while all the received signals are summed coherently in equal-gain combining. All the three combining techniques are said to achieve full diversity order, which is equal to the number of receiving antennas. On the other hand, the idea of transmitter diversity corresponds to the transmission of the same signal over multiple transmitting antennas. Transmitter diversity includes, space time block coding (STBC), space time coding (STC), space time trellis code (STTC), linear dispersion codes (LDC), quasi-orthogonal STBC, differential space-time coding schemes.

Multiplexing Techniques

STBC and STTC are capable of providing diversity gains in aid of improving the achievable system performance. However, this BER performance improvement is often achieved at the expense of a rate loss since the STBC and STTC may result in a throughput loss compared to single-antenna-assisted systems. As a design option, a specific class of MIMO systems is designed for improving the achievable spectral efficiency of the system by transmitting the signals independently from each of the transmitting antennas, hence resulting in a multiplexing gain.

Beamforming

Multiple antennas can be used on behalf of attaining either spatial diversity or spatial multiplexing gains. However, multiple antennas can also be used in order to improve the SNR at the receiver or the signal-to-interference-plus-noise ratio (SINR) in a multi-user scenario. This can be achieved by utilizing beamforming techniques. Beamforming constitutes an effective process of reducing the multiple access interference, where the antenna gain is increased in the direction of the desired user, whilst reducing the gain towards the interfering users.

Multi Functional MIMO

The MIMO schemes presented in the previous sections are uni-functional i.e. they can achieve either diversity gain or multiplexing gain or beamforming gain. A multi-functional MIMO scheme can attain a combination of the three gains. V-BLAST is capable of achieving full multiplexing gain, while STBC can achieve full antenna diversity gain. Therefore, by combining V-BLAST and STBC, an improved transmitter diversity gain can be achieved as compared to pure V-BLAST, while ensuring that the overall bandwidth efficiency is higher than that of pure STBC due to the independence of the signals transmitted by different STBC layers.

2.5.2 Distributed MIMO

Wireless channels suffer from multipath propagation of the signals that result in channel fading. Exploiting multiple transmitting antennas is an effective method that can be used for counteracting the effects of the channel fading by providing diversity gains. BER performance is significantly improved by transmitter diversity, when the different transmitter antennas are spatially located so that the paths arriving from each transmitting antenna to the destination experience independent fading that can be achieved by having a distance between the different antennas, which is significantly higher than the carriers wavelength. However, considering a handheld mobile phone, it is not a feasible option to position the transmitter antennas far enough in order to achieve independent fading. Conversely, the spatial fading correlation caused by insufficiently high antenna spacing at the transmitter or receiver of a MIMO system results in a degradation of both the achievable capacity and the BER performance of MIMO systems. The problem of correlation of the transmitter signals can be circumvented by introducing a new class of MIMOs also referred to as distributed MIMOs or cooperative communications.

2.6 Classifications Based on Channel Knowledge

MIMO techniques can be further classified regarding the quality of channel knowledge at the transmitter. Such as:

i. When Exact Transmitter Channel Knowledge is Known

ii. When Partial Transmitter Channel Knowledge is Known

iii. When No Transmitter Channel Knowledge is Known

When Exact Transmitter Channel Knowledge is Known

In order to maximize the diversity gain, we consider a $C \times D$ MIMO system. Usually, this can be done through transmitting the same signal from all transmitter antennas after weighting by $C \times 1$ vector w_c. At the receiving array, the antenna outputs are combined into a scalar signal z through a weighted addition according to a $D \times 1$ vector w_d .Consequently, the transmission is described by

$$y = \sqrt{E_s} H w_c k + n \tag{2.3}$$

$$d = w_d^H y = \sqrt{E_s} w_d^H H w_c k + w_d^H n \tag{2.4}$$

Maximization of the received SNR comes in terms of maximizing $\left\|w_d^H H w_c k\right\|_F^2 / \|w_d\|_F^2$. To solve this problem, we need to use the singular value decomposition (SVD) of H as

$$H = U_H \sum V_H^H \tag{2.5}$$

Where,U_H and V_H are $D \times d(H)$ and $C \times d(H)$ unitary matrices, d (H) being the rank of H and

$$\sum H = diag\left\{\sigma_1, \sigma_2, \ldots, \sigma_{d(H)}\right\} \tag{2.6}$$

is the diagonal matrix containing the singular values of H. Using this particular decomposition of the channel matrix, it is easily shown in [21] that the receive SNR is maximized when w_c and w_d are the transmitter and receiver singular vectors corresponding to the maximum singular value of H,

$$\sigma_{max} = \left\{\sigma_1, \sigma_2, \ldots, \sigma_{d(H)}\right\} \tag{2.7}$$

This technique is known as the dominant Eigen mode transmission, and Equation (2.4) may be rewritten as:

$$z = \sqrt{E_s}\sigma_{max}k + \hat{n} \tag{2.8}$$

Where, $\hat{n} = w_d^H n$ has a variance equal to σ_n^2.

From Equation (2.8), it is easily observed that the array gain is equal to $\epsilon\left\{\sigma_{max}^2\right\} = \epsilon\left\{\lambda_{max}\right\}$, Where, λ_{max} is the largest Eigen value of HH^H. The array gain for i.i.d. Rayleigh channels is thus bounded as follows:

$$max\left\{n_c, n_d\right\} \leqslant g_a \leq n_c n_d \tag{2.9}$$

In the i.i.d. Rayleigh case, the asymptotic array gain of a dominant Eigen mode transmission (i.e., for large n_c, n_d) is given by:

$$g_a = \left(\sqrt{n_c} + \sqrt{n_d}\right)^2 \tag{2.10}$$

Finally, the diversity gain is obtained by upper and lower-bounding the error rate at high SNR (assuming that the Chernoff bound is a good approximation of the SER at high SNR)

$$\widehat{N}_s\left(\frac{kd_{min}^2}{4min\left\{n_c, n_d\right\}}\right)^{-n_c n_d} \geq \widehat{P} \geq \widehat{N}_s\left(\frac{kd_{min}^2}{4}\right)^{-n_c n_d} \tag{2.11}$$

The above equation implies that the error rate maintains a slope of as $n_c n_d$ a function of the SNR: the dominant Eigen mode transmission extracts a full diversity gain of $n_c n_d$.

When Partial Transmitter Channel Knowledge is Known

The exploitation of the array gain may also be possible if the transmitter has only a partial channel knowledge. Perfect channel knowledge at the transmitter has been covered in previous section, but requires a high rate feedback link between the receiver and the transmitter to keep the latter continuously informed about the channel state. By contrast, exploiting only the channel statistics or a quantized version of the channel at the transmitter requires a much lower rate feedback link. Precoding techniques generally consist in combining a multi-mode beamformer spreading the code words in orthogonal directions related to the channel distribution with a constellation shaper, or more simply, a power allocation scheme. There are naturally many similarities with the various Eigen mode transmissions of previous section, the difference being that the Eigen beams are now based on the statistics of H rather than on the instantaneous value of H [25]. Similarly, antenna selection techniques may rely only on partial channel knowledge, choosing transmitter or receiver antennas based on the first and second-order statistics of H. Intuitively, this comes to choose the antenna pairs with the lowest correlation. Naturally, such a technique does not minimize the instantaneous error performance, but only the average error rate. As a result, it leads mostly to a coding gain and small diversity advantage. A generalization of antenna selection consists of exploiting a limited amount of feedback at the transmitter through quantized precoding. This technique relies on a codebook of precoding matrices, i.e. a finite set of precoders, designed off-line and known to both the transmitter and receiver. The receiver estimates the best precoder as a function of the current channel and then feeds back the index of the best precoder in the codebook.

When No Transmitter Channel Knowledge is Known

When the transmitter has no channel knowledge, the presence of multiple antennas at both sides may allow extracting diversity and increasing the capacity. This is achieved through the use of so-called STC, which expand symbols over the antennas i.e. over space and over time. STBCs are the simplest types of spatial temporal codes that exploit the diversity offered in systems with several transmitter antennas. Alamouti first designed a simple transmission diversity technique for systems having two transmitter antennas [22]. This method provides full diversity and requires simple linear operations at both transmission and reception side. The encoding and decoding processes are performed with blocks of transmission symbols. Alamoutis simple transmitter diversity scheme is extended in [23] and [24] due to the theory of orthogonal designs for larger numbers of transmitting antennas.

2.7 Operating Principle of MIMO

MIMO can be operated under the three following stages, such as a) Precoding, b) Spatial multiplexing (SM) and c) Diversity coding

2.7.1 Precoding

Precoding means a multi-stream beamforming. In a wider-sense, it is considered to be all spatial processing that occurs at the transmitter. In single-layer beamforming, the same signal is emitted from each of the transmitter antennas with appropriate phase weighting such that the signal power is maximized at the receiver output. The benefits of beamforming are to increase the signal gain from constructive combining and to reduce the multipath fading effect. In the absence of scattering, beamforming results in a well defined directional pattern. When the receiver has multiple antennas, the transmitter beamforming cannot simultaneously maximize the signal level at all of the receiving antennas and precoding is used. But precoding requires knowledge of the channel state information (CSI) at the transmitter.

2.7.2 Spatial multiplexing

Spatial multiplexing (SM) requires MIMO antenna configuration. In SM, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmitter antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial features, the receiver can separate these streams by creating parallel channels. At higher signal-noise ratio (SNR), SM is a very powerful technique for increasing channel capacity. The maximum number of spatial streams is limited by the lesser number of antennas at the transmitter or receiver. It can be used with or without transmit channel knowledge and used for simultaneous transmission to multiple receivers, known as space-division multiple accesses (SDMA). The scheduling of receivers with different spatial characteristics allows good separability. SM can also be combined with precoding when the channel is known at the transmitter or combined with diversity coding when decoding reliability is in tradeoff.

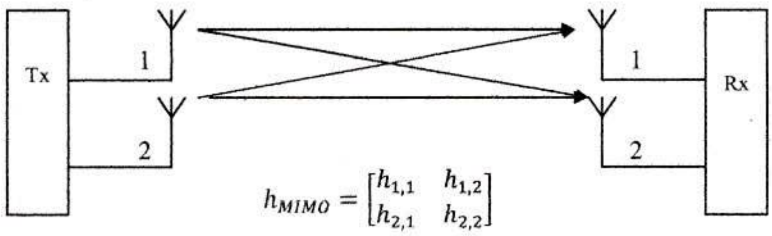

FIGURE 2.4: Spatial multiplexing phenomena for 2×2 MIMO configurations.

If transmission matrix H is known, the cross components can be calculated on the receiver. In the open-loop method, the transmission includes special segments that are also known to the receiver. The receiver can perform channel estimation. In the closed-loop method, the receiver reports the channel status to the transmitter via a special feedback channel. This makes it possible to respond to varying conditions.

2.7.3 Diversity coding

Diversity coding techniques are used when there is no channel knowledge at the transmitter. In diversity methods a single stream is transmitted, but the signal is coded using technique called space-time coding (STC). The signal is emitted from each of the transmitter antennas using certain principles of full or near orthogonal coding. Diversity exploits the independent fading in the multiple antenna links to enhance signal diversity. If the channel state information is unknown then no beamforming or array gain is achieved from diversity coding. Diversity may be considered in two ways, such as transmitter diversity and receiver diversity.

Transmitter Diversity

Transmitter diversity occurs when there are more transmitting antennas than receiving antennas. The simplest scenario uses two transmitting antennas and one receiving antenna (MISO, 2×1). Figure 2.5 describes the transmit diversity in case of MISO.

In this case, the same data is transmitted repeatedly over two antennas. This method has the advantage that the multiple antennas and repeated coding is moved from the mobile UE to the base station, where these processes are simpler and cheaper to implement. To generate an additional signal, STC is used which is first developed by Alamouti for two

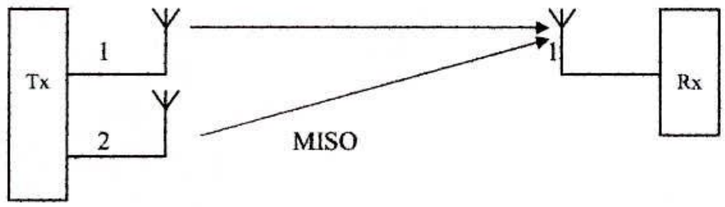

FIGURE 2.5: Transmitter diversity.

antennas [20]. STC additionally improve the performance and make spatial diversity usable. The signal copy is transmitted not only from a different antenna but also at a different time. This delayed transmission is called delayed diversity. STC combine spatial and temporal signal copies as illustrated in figure 2.6. The signals $s_1$and$s_2$are multiplexed in two data chains. After that, a signal replication is added to create the Alamouti space-time block code (STBC).

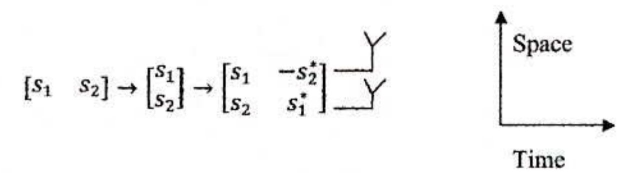

FIGURE 2.6: Alamouti coding.

Receiver Diversity

Receiver diversity uses more antennas on the receiver side than on the transmitter side. The simplest scenario consists of two receiving antennas and one transmitting antenna (SIMO, 1×2). Figure 2.7 describes the receive diversity in case of SIMO. Since special

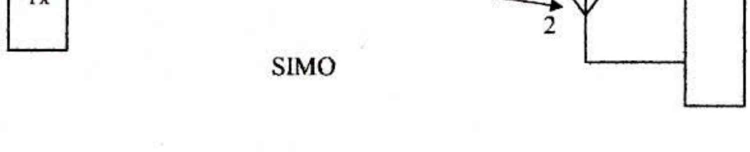

FIGURE 2.7: Receiver diversity.

coding methods are not needed, this scenario is very easy to implement. Only two RF paths are needed for the receiver.

2.8 Formation of MIMO

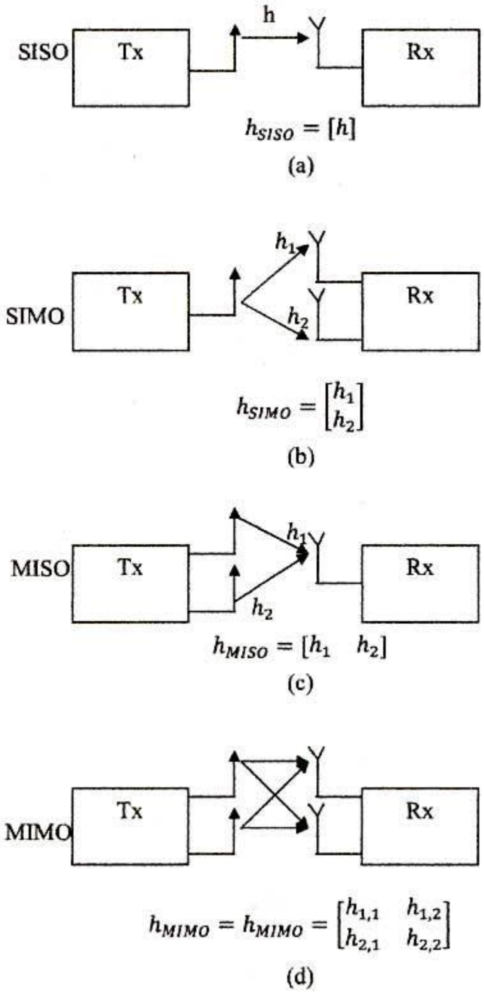

FIGURE 2.8: Formation of different transmitting-receiving antenna (a) SISO, (b) SIMO, (c) MISO, (d) MIMO.

MIMO can be splitted up by SISO, SIMO, and MISO. Transceiver having single antenna both in input and output is called single-input single-output (SISO) system. When the transceiver has single antenna in its input and multiple antennas in its output is known as single-input and multiple-output (SIMO). While the input contains multiple antennas and output has single antenna, it is called multiple-input and single-output (MISO) antenna. All of these formations as well as traditional MIMO are depicted by the following figure.

MIMO can be further divided into two forms, such as: a) single user MIMO, b) multi user MIMO.

2.8.1 Single User MIMO (SU-MIMO)

When the data rate is to be increased for a single UE, this is called single user MIMO (SU-MIMO). Principal single-user MIMO techniques are Bell Laboratories Layered Space-Time (BLAST), Per Antenna Rate Control (PARC), Selective Per Antenna Rate Control (SPARC).

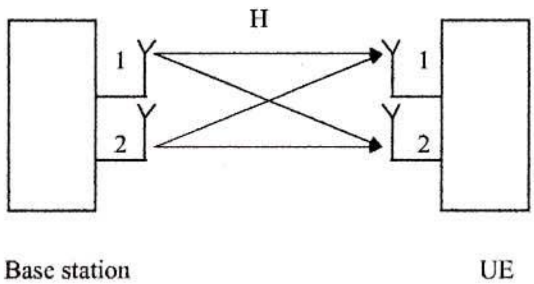

FIGURE 2.9: SU-MIMO.

2.8.2 Multi User MIMO (MU-MIMO)

When the particular streams are allotted to numerous users, this is called multi user MIMO (MU-MIMO). Recently, the research on multi-user MIMO technology has been emerging. Especially this mode is useful in the uplink because the complexity on the UE side can be kept at a minimum by using only one transmitting antenna. This is also called 'two-way MIMO'. While full MU-MIMO can have higher potentials, research in this area is more active. In recent 3GPP and WiMAX standards, MU-MIMO is being treated as one of the candidate technologies adoptable in the specification by Samsung, Intel, Qualcomm, Ericsson, TI, Huawei, Philips, Alcatel-Lucent, Freescale, et al. since MU-MIMO is more feasible to low complexity mobiles with small number of reception antennas than SU-MIMO with the high system throughput capability. Enhanced multiuser MIMO employ: 1) advanced decoding techniques; 2) advanced precoding techniques.

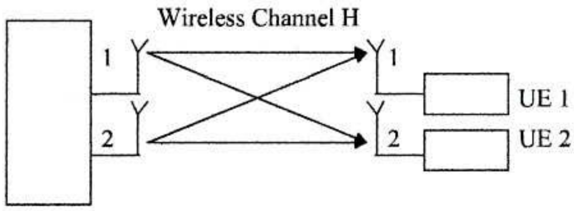

FIGURE 2.10: MU-MIMO.

2.9 Rayleigh Fading Model

Rayleigh fading is a numerical model for the effect of a propagation environment on a wireless signal when there is no line of sight signal. The magnitude of a signal that passes through such a transmission medium varies randomly according to a Rayleigh distribution i.e. the radial component of the sum of two uncorrelated Gaussian random variables. It is regarded as an acceptable model for tropospheric and ionospheric signal propagation as well as heavily built-up urban environments on wireless signals. It is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver. In case of a dominant line of sight, Rician fading may be more applicable. When there are many objects in the environment that scatter the radio signal then it is a suitable model before it arrives at the receiver. If there is no dominant component of the scatterer, then such a process will have zero mean and phase evenly distributed between 0 and 2 radians. So, Rayleigh distributions are defined for fading of a channel when all the received signals are reflected signals and there is no dominant component. The envelope of the channel response will therefore be Rayleigh distributed. In the case of random variable D, the probability density function will be in the form of:

$$p_D(d) = \begin{cases} \frac{2d}{\Omega} e^{-d^2/\Omega} & , d \geq 0 \\ 0 & , d < 0 \end{cases} \tag{2.12}$$

Where, $\Omega = E\left(D\right)^2$. The gain and phase elements of a channel's distortion are conveniently represented as a complex number. In this case, Rayleigh fading is exhibited by the assumption that the real and imaginary parts of the response are modelled by independent and identically distributed zero-mean Gaussian processes so that the amplitude of the response is the sum of two such processes.

2.10 Rician Fading Model

Rician fading is a stochastic model for wireless propagation irregularity caused by partial cancellation of a wireless signal by itself the signal arrives at the receiver by several different paths hence exhibiting multipath interference, and at least one of the paths is changing. Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others. In Rician fading, the amplitude gain is characterized by a Rician distribution. A Rician fading channel can be described by two parameters: Kand . K is the ratio between the power in the direct path and the power in the other, scattered, paths. is the total power from both paths ($\Omega = \nu^2 + 2\sigma^2$), and acts as a scaling factor to the distribution. The received signal amplitude R is then Rice distributed with parameters $\nu^2 = \frac{K}{1+K}\Omega$ and V . The resulting PDF then is:

$$f(x) = \frac{2(K+1)x}{\Omega} exp\left(-K - \frac{(K+1)x^2}{\Omega}\right) I_0\left(2\sqrt{\frac{(K+1)}{K}}x\right) \tag{2.13}$$

Where, $I_0(.)$ is the 0^{th} order modified Bessel function of the first kind.

2.11 Okumura Model

The Okumura model for urban areas is a wireless propagation model that was built using the data collected in the city of Tokyo, Japan. The model is ideal for using in cities with many urban structures but not many tall blocking structures. The model served as a base for the Hata model. Okumura model was built into three modes. The ones for urban, suburban and open areas. The model for urban areas was built first and used as the base for others. Okumura's model is one of the most widely used models for signal prediction in urban areas. This model is applicable for frequencies in the range 150 MHz to 1920 MHz and distances of 1 km to 100 km. It can be used for base station antenna heights ranging from 30 m to 1000 m and mobile station antenna heights ranging from 1 m to 10 m.

The Okumura model is formally expressed as:

$$L = L_{FSL} + A_{MU} - H_{MG} - H_{BG} - \sum K_{correction} \tag{2.14}$$

Where,

L = The median path loss. Unit: Decibel (dB)

L_{FSL} = The Free space Loss. Unit: Decibel (dB)

A_{MU}= Median attenuation. Unit: Decibel (dB)

H_{MG}= Mobile station antenna height gain factor.

H_{BG}= Base station antenna height gain factor.

$K_{correction}$= Correction factor gain (such as type of environment, water surfaces, isolated obstacle etc.)

2.12 Hata Model

Hata Model refers to a wireless propagation model that is based on the Okumura model. It in turn has developed separate models for varying environments:

a. Hata model for urban areas

b. Hata model for suburban areas

c. Hata model for open areas

Hata Model for Urban Areas

In wireless communication, the Hata model for urban areas, also known as the Okumura-Hata model for being a developed version of the Okumura model, is the most widely used radio frequency propagation model for predicting the behaviour of cellular transmissions in built up areas. This model incorporates the graphical information from Okumura model and develops it further to realize the effects of diffraction, reflection and scattering caused by city structures. This model also has two more varieties for transmission in suburban areas and open areas. Hata model predicts the total path loss along a link of terrestrial microwave or other type of cellular communications. This particular version of the Hata model is applicable to the wireless propagation within urban areas. This model is suited for both point-to-point and broadcast transmissions and it is based on extensive empirical measurements taken. This model is applicable for frequencies in the range 150 MHz to 1500 MHz and distances of 1 km to 20 km. It can be used for base station antenna heights ranging from 30 m to 200 m and mobile station antenna heights ranging from 1 m to 10 m.

The Hata model for urban areas is formulated as following:

$$L_U = 69.55 + 26.16log_{10}f - 13.82log_{10}f - C_H + [44.9 - 6.55log_{10}h_B]\, log_{10}d \qquad (2.15)$$

For small or medium sized city,

$$C_H = 0.8 + (1.1log_{10}f - 0.7)h_M - 1.56log_{10}f \qquad (2.16)$$

and for large cities,

$$C_H = \begin{cases} 8.29\,(log_{10}\,(1.54h_M))^2 - 1.1 & \text{,if } 150 \le f \le 200 \\ 3.2\,(log_{10}\,(11.75h_M))^2 - 4.971 & \text{,if } 150 \le f \le 200 \end{cases} \qquad (2.17)$$

Where,

L_U= Path loss in urban areas. Unit: decibel (dB)

h_B= Height of base station antenna. Unit: meter (m)

h_M= Height of mobile station antenna. Unit: meter (m)

f = Frequency of transmission. Unit: Megahertz (MHz).

C_H= Antenna height correction factor

d = Distance between the base and mobile stations. Unit: kilometer (km).

Though based on the Okumura model, the Hata model does not provide coverage to the whole range of frequencies covered by Okumura model. Hata model does not go beyond 1500 MHz while Okumura provides support for up to 1920 MHz.

Hata Model for Suburban Areas

The Hata model for suburban areas, also known as the Okumura-Hata model for being a developed version of the Okumura model, is the most widely used model in radio frequency propagation for predicting the behavior of cellular transmissions in city outskirts and other rural areas. This model incorporates the graphical information from Okumura model and develops it further to better suit the need. This model also has two more varieties for transmission in urban areas and open areas. Hata model predicts the total path loss along a link of terrestrial microwave or other type of cellular communications and is a function of transmission frequency and the average path loss in urban areas. This model is based on Hata model for urban areas and uses the median path loss from urban areas and is applicable for frequencies in the range 150 MHz to 1.500 GHz.

Hata model for suburban areas is formulated as,

$$L_{SU} = L_U - 2\left(log\frac{f}{28}\right)^2 - 5.4 \tag{2.18}$$

Where,

L_{SU}= Path loss in suburban areas. Unit: decibel (dB)

L_U= Average path loss in urban areas. Unit: decibel (dB)

f = Frequency of transmission. Unit: megahertz (MHz).

Hata Model for Open Areas

The Hata model for open areas, also known as the Okumura-Hata model for being a developed version of the Okumura model, is the most widely used model in wireless propagation for predicting the behavior of cellular transmissions in open areas. This model incorporates the graphical information from Okumura model and develops it further to better suit the need. This model also has two more varieties for transmission in urban areas and suburban areas. Hata model for open areas predicts the total path loss along a link of terrestrial microwave or other type of cellular communications and is a function of transmission frequency and the median path loss in urban areas. This particular version of Hata model is applicable to the transmissions in open areas where no obstructions block the transmission link. This model is suited for both point-to-point and broadcast transmissions and is applicable for frequencies in the range 150 MHz to 1.500 GHz.

The Hata model for open areas is formulated as:

$$L_0 = L_U - 4.78\,(logf)^2 + 18.33logf - 40.94 \tag{2.19}$$

Where,

L_O= Path loss in open area. Unit: decibel (dB)

L_U= Path loss in urban area. Unit: decibel (dB)

f= Frequency of transmission. Unit: Megahertz (MHz).

This model is dependent on the Hata model for urban areas.

Chapter 3

Phase Shift Keying Modulation

3.1 Introduction

In electronics and telecommunications, modulation is the process of varying one or more properties of a high-frequency periodic waveform called the carrier signal with a modulating signal that contains information to be transmitted. The three key parameters of a modulating signal are its amplitude, phase and frequency. Any of these properties can be modified in accordance with a low frequency signal to obtain the modulated signal. In general a high-frequency sinusoid waveform is used as carrier signal. Digital data transmission can be classified by three major categories such as: amplitude-shift keying (ASK), frequency-shift keying (FSK) and phase-shift keying (PSK). But in this chapter, we are concentrated ourselves mainly on the discussion of PSK. This chapter describes the basics of PSK modulation, requirements of PSK in MIMO wireless communication, its classification, constellation diagram as well as the mathematical analysis of bit error rate (BER) for all of the higher order PSK modulation. Finally applications of PSK are represented in the last part of this chapter.

3.2 Basics of PSK Modulation

PSK is a modulation process where the input signal shifts the phase of the output waveform to one of a fixed number of states. This can be achieved simply by defining a relative phase shift from the carrier, usually equi-distant for each required state. Hence a two level phase modulated system has two relative phase shifts from the carrier, + or - 90^0. In general, this method will lead to an improved BER performance. However, the resulting signal will probably not be constant amplitude and not be very spectrally efficient due to the rapid phase discontinuities. Some additional filtering will be required to limit the spectral occupancy. PSK modulation requires coherent generation and as such if an IQ modulation technique is employed this filtering can be performed at baseband. The signal can be expressed as

$$V_0(t) = \sqrt{2S}\left[\omega_0 t + \frac{2\pi(i-1)}{M}\right], \text{i=1,2,...,M} \tag{3.1}$$

$$-T_S/2 \leq t \leq T_S/2$$

Where,

S = the average signal power over the signaling interval, T_S

M= 2^N number of acceptable phase states

N = the number of bits needed to quantize M

There are two principal ways of using the phase of a signal in this way: By viewing the phase itself as carrying the information, in that case the demodulator must have a reference signal to compare the received signal's phase; or by viewing the change in the phase as carrying informationdifferential schemes, some of which do not need a reference carrier. A convenient way to represent PSK schemes is on a constellation diagram. This shows the points in the complex plane where the real and imaginary axes are termed as the in-phase and quadrature axes respectively due to their 90^0 separation. Such a representation on perpendicular axes lends itself to clear-cut implementation. The amplitude of each point along the in-phase axis is used to modulate a cosine (or sine) wave and the amplitude along the quadrature axis is used to modulate a sine (or cosine) wave. In PSK, the constellation points chosen are usually positioned with uniform angular spacing around a circle. This gives maximum phase-separation between adjacent points and thus the best immunity to corruption. They are positioned on a circle so that they can all be transmitted with the same energy. In this way, the moduli of the complex numbers they represent will be the same and so that will the amplitudes needed for the cosine and sine waves.

3.3 Requirements of PSK in MIMO Wireless Communication

There are different modulation schemes that can be used to modulate the data: - namely BPSK, QPSK, m-QAM, FSK etc. In space communication power is severely limited. FSK is not generally used as it would require very high bandwidth to modulate our data resulting in very low bandwidth efficiency. Higher constellation QAM also cant be used in our case as that would require very high C/N ratio. The choice is between QPSK and BPSK. The advantage of BPSK is that it requires the lowest C/N ratio. The drawback is that the data rate achieved using BPSK is very low. QPSK is basically two BPSK links operating on the same radio channel with their carriers in phase quadrature. Therefore the BER of a QPSK remains the same as BPSK. At the same time the data rate is doubled. The only penalty we pay is in terms of C/N ratio. QPSK requires 3 dB more C/N ratio than BPSK. Our research demands high data rate without losing much on bandwidth and power. Because of these tradeoffs we decided on using QPSK modulation scheme for the raw data received from the rover. PSK is less susceptible to errors than ASK while it requires/occupies the same bandwidth as ASK, high power efficiency, system is the simplest. In higher order PSK, data can be transmitted at a faster rate, relative to the number of phase changes per unit time, than is the case in BPSK. The main advantage of PSK is its excellent signal-to-noise ratio (S/N or SNR), which allows communication under adverse conditions such as severe fading, noise, or interference where other communications modes fail.

3.4 Classification of PSK Modulation

According to the phase shifting principle of a signal the PSK modulation may be classified by the following categories:

a. BPSK,

b. QPSK,

c. 8-PSK,

d. 16-PSK,

e. 32-PSK,

f. 64-PSK

g. 128-PSK

h. 256-PSK

However, in the present thesis, BPSK, QPSK, 8-PSK, 16-PSK and 32-PSK are used for the implementation of optimum equalizer. Hence, we will discuss the classification of the above mentioned PSK one by one.

Binary Phase Shift Keying (BPSK)

One of the simplest forms of PSK is binary-phase shift keying (BPSK). It is sometimes called phase reversal keying (PRK) or 2PSK. It uses two phases which are separated by 180^0 and so can also be termed as 2-PSK. Fig 3.1 shows BPSK constellation diagram. On I and Q diagram, I state has two different values. There are two possible locations in the state diagram, so a binary one or zero can be sent. This modulation is the most robust of all the PSKs since it takes the highest level of noise or distortion to make the demodulator to reach an incorrect decision. It is, however, only able to modulate at 1 bit/symbol and so is unsuitable for high data-rate applications. In the presence of an arbitrary phase-shift introduced by the communications channel, the demodulator is unable to tell which constellation point is which. As a result, the data is often differentially encoded prior to modulation. BPSK is functionally equivalent to 2-QAM modulation.

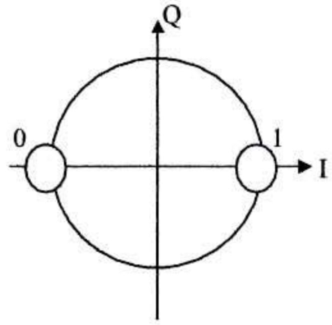

FIGURE 3.1: BPSK constellation diagram.

Implementation

The general form of BPSK can be expressed by the following equation:

$$s_n(t) = \sqrt{\frac{2E_b}{T_b}\cos\left(2\pi f_c t + \pi\left(1-n\right)\right)}, \text{n=0,1} \tag{3.2}$$

This yields two phases, 0 and π. In a more specific way, binary data is often carried by the following signals:

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}\cos\left(2\pi f_c t + \pi\right)} = -\sqrt{\frac{2E_b}{T_b}\cos 2\pi f_c t}, \text{for binary 0} \tag{3.3}$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}\cos\left(2\pi f_c t\right)}, \text{for binary 1} \tag{3.4}$$

Where, f_c is the carrier frequency of the wave. Hence, the signal-space can be represented by the single basis function

$$\phi(t) = \sqrt{\frac{2}{T_b}\cos\left(2\pi f_c t\right)} \tag{3.5}$$

Where, 1 is represented by $\sqrt{E_b}\phi(t)$ and 0 is represented by $-\sqrt{E_b}\phi(t)$

TABLE 3.1: Phase separation of BPSK modulation.

Symbol	Bit	Phase
S_1	0	0^0
S_2	1	180^0

Bit Error Rate

BER of BPSK in AWGN can be calculated as:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}erfc\left(\frac{E_b}{N_0}\right) \tag{3.6}$$

Quadrature Phase Shift Keying (QPSK)

Another more common type of phase modulation that is widely used in wireless communication is quadrature phase shift keying (QPSK). QPSK uses four points on the constellation diagram, equispaced around a circle. Sometimes it is known as quaternary PSK, quadriphase PSK, 4-PSK, or 4-QAM. With four phases, QPSK can encode two bits

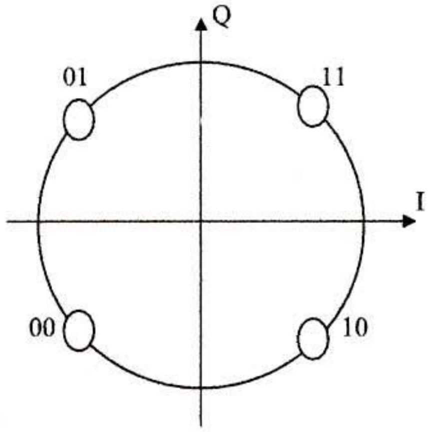

FIGURE 3.2: QPSK constellation diagram.

per symbol, shown in the Fig. 3.2 with gray coding to minimize the BER. By quadrature means the signal shifts between phase states which are separated by 90 degrees. The signal shifts in increments of 90 degrees from 45 to 135, 45, or 135 degrees. These points are chosen as they can be easily implemented using an I/Q modulator. Only two I values and two Q values are needed and this gives two bits per symbol. There are four states because $2^2 = 4$. As it has four states so it is a more bandwidth-efficient type of modulation than BPSK, possibly twice as efficient. From the mathematical analysis of QPSK it is seen that it can be used either to double the data rate compared with a BPSK system while maintaining the same bandwidth of the signal, or to maintain the data-rate of BPSK but halving the bandwidth needed. The advantage of QPSK over BPSK is evident i.e. QPSK transmits twice the data rate in a given bandwidth compared to BPSK at the same BER. The drawback is that QPSK transceivers are more complicated than BPSK. However, with modern electronics technology, the penalty in cost is very moderate. As with BPSK, there are phase uncertainty problems at the receiving end, and differentially encoded QPSK is often used in practice.

TABLE 3.2: Phase separation of QPSK modulation.

Symbol	Bits	Phase
S_1	00	45^0
S_2	01	135^0
S_3	11	225^0
S_4	10	315^0

Implementation

QPSK implementation is more universal than BPSK and also indicates the implementation of higher-order PSK. The symbols in the constellation diagram in terms of the sine and cosine waves can be expressed by:

$$s_n(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + (2n-1)\pi/4\right), \text{n=1,2,3,4.} \tag{3.7}$$

This yields the four phases $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ as needed. This results in a two-dimensional signal space with unit basis functions.

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos\left(2\pi f_c t\right) \tag{3.8}$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos\left(2\pi f_c t\right) \tag{3.9}$$

The first basis function is used as the in-phase component of the signal and the second as the quadrature component of the signal. Hence, the signal constellation consists of the signal-space of 4-points

$$\pm\sqrt{\frac{E_s}{2}}, \pm\sqrt{\frac{E_s}{2}}$$

The factors of 1/2 indicate that the total power is divided equally between the two carriers. Comparing these basis functions with that for BPSK show clearly how QPSK can be viewed as two independent BPSK signals. But the signal-space points for BPSK do not need to split the symbol (bit) energy over the two carriers in the scheme shown in the BPSK constellation diagram. QPSK systems can be implemented in a number of ways. QPSK transmitter and receiver structure is shown below.

The binary data stream is split into the in-phase and quadrature-phase components. These are then separately modulated onto two orthogonal basis functions. In this implementation, two sinusoids are used. Afterwards, the two signals are superimposed, and the resulting signal is the QPSK signal. The polar non-return-to-zero encoding is

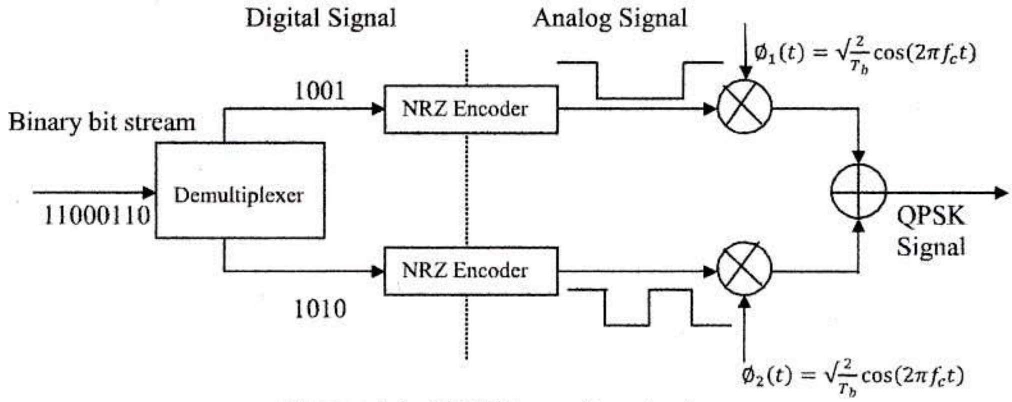

FIGURE 3.3: QPSK transmitter structure.

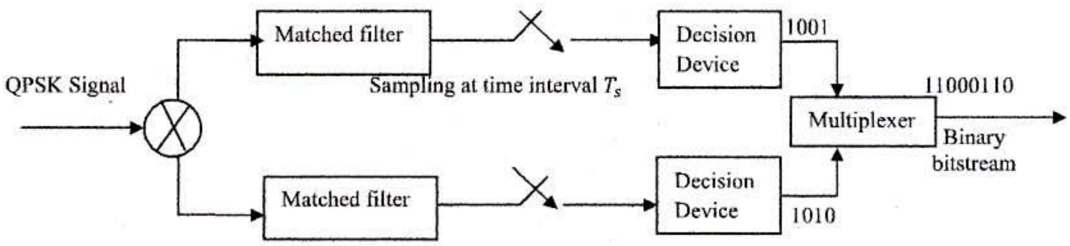

FIGURE 3.4: Receiver structure for QPSK.

used here. These encoders can be placed before for binary data source, but have been placed after to illustrate the conceptual difference between digital and analog signals involved with digital modulation. The matched filters can be replaced with correlators. Each detection device uses a reference threshold value to determine whether a 1 or 0 is detected.

Bit Error Rate

QPSK can be viewed as two independently modulated quadrature carriers. With this interpretation, the even (or odd) bits are used to modulate the in-phase component of the carrier, while the odd (or even) bits are used to modulate the quadrature-phase component of the carrier. BPSK is used on both carriers and they can be independently demodulated. As a result, the probability of bit-error for QPSK is the same as for BPSK:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{3.10}$$

8-Phase Shift Keying(8-PSK)

8-PSK is usually the higher order modulation of PSK constellation deployed. It has eight constellation points. Each set of bits is just one bit different from its neighbor. So while phase is shifted in the neighboring position then there is a probability of error is only one bit.

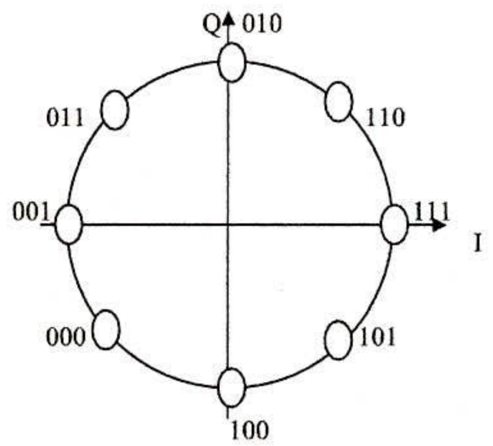

FIGURE 3.5: Constellation diagram for 8-PSK with gray coding.

The eight positions are created with x and y distances or by phases of sines and cosines in communications. We have two basis functions, a sine and a cosine and each configuration has a different phase to indicate a specific bit pattern. We use eight different phase values $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$ and 2π . Each of these phase shifts is 45 degrees apart. Each of these is applied to the sine and the cosine to give us a total of eight values. 8-PSK transmitted signal shows smaller phase transitions than QPSK which is an important factor but since the signals are also less distinctly different from each other, makes 8-PSK prone to higher bit errors. Because, we can pack more bits per symbol, with each symbol transmitted, we can convey three bits. The throughput of 8-PSK is 50% better than QPSK which can transmit just 2-bits per symbol as compared to 3-bits for 8-PSK. 8-PSK is the first of the bandwidth-efficient modulations.

TABLE 3.3: Phase separation of 8-PSK modulation.

Symbol	Bits	Phase
S_1	000	0^0
S_2	001	45^0
S_3	011	90^0
S_4	010	135^0
S_5	110	180^0
S_6	111	225^0
S_7	101	270^0
S_8	100	315^0

8-PSK Implementation

The symbols in the constellation diagram in terms of the sine and cosine waves can be expressed by:

$$s_n(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + (2n-1)\pi/4\right), \text{n=1,2,3,4,5,6,7,8.} \tag{3.11}$$

his generates eight phases $\pi/4$, $\pi/4$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $7\pi/4$ and 2π as needed.

Bit Error Rate

For 8-PSK, the symbol-error probability can be obtained from:

$$P_s = 1 - \int_{\pi/8}^{-\pi/8} P_{\theta_r}(\theta_r)\, d\theta_r \tag{3.12}$$

Where,

$$p_{\theta_r}(\theta_r) = \frac{1}{2\pi} e^{-2\gamma_s \sin^2 \theta_r \int V e^{-\left(V-\sqrt{4\gamma_s}\right)^{2/2}} dV}$$

$$V = \sqrt{r_1^2 + r_2^2}$$

$$\theta_r = \tan^{-1}(r_2/r_1)$$

$$\gamma_s = \frac{E_s}{N_0}$$

$r_1 \sim 3\left(\sqrt{E_s}, N_0/2\right)$ and $r_2 \sim 3\left(0, N_0/2\right)$ jointly Gaussian random variables. This may be approximated for 8-PSK and high E_b/N_0 by:

$$P_s = 2Q\left(\sqrt{2\gamma_s} \sin\frac{\pi}{8}\right) \tag{3.13}$$

The bit-error probability for 8-PSK can only be determined exactly once the bit-mapping is known. However, when gray coding is used, the most probable error from one symbol to the next produces only a single bit-error and

$$P_b = \frac{1}{k} P_s \tag{3.14}$$

Higher-order modulations deliver a higher raw data-rate at the expense of higher error-rates; however bounds on the error rates of various digital modulation schemes can be computed with application of the union bound to the signal constellation.

16-Phase Shift Keying(16-PSK)

We can keep on subdividing the signal space into smaller regions one more time for 16-PSK. Doing so that each signal is now only 11.25^0 apart, gives us 16-PSK. This will give 16 signals or symbols, so each symbol can convey 4 bits. Bit rate is now four times that of BPSK for the same symbol rate. The following figures show the 16-PSK signal at various stages during modulation. We can keep on increasing bits per symbol this way. 16-PSK is bandwidth efficient but it has higher BER than a common modulation from the class of Quadrature Amplitude Modulation called 16-QAM which has the same bit efficiency. That is why; 16-PSK is rarely used.

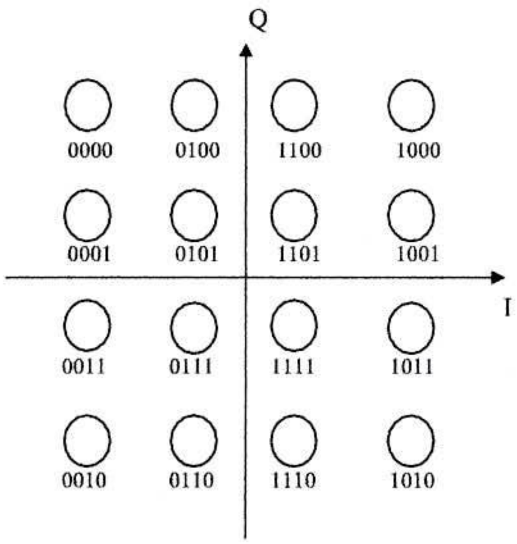

FIGURE 3.6: Constellation diagram for rectangular 16-PSK.

TABLE 3.4: Phase separation of 16-PSK modulation.

Symbol	Bits	Phase
S_1	0000	0^0
S_2	0001	22.5^0
S_3	0010	45^0
S_4	0011	67.5^0
S_5	0100	90^0
S_6	0101	112.5^0
S_7	0110	135^0
S_8	0111	157.5^0
S_9	1100	180^0
S_{10}	1101	202.5^0
S_{11}	1110	225^0
S_{12}	1111	247.5^0
S_{13}	1000	270^0
S_{14}	1001	292.5^0
S_{15}	1011	315^0
S_{16}	1010	337.5^0

Bit Error Rate

For 16-PSK, the symbol-error probability can be obtained from:

$$P_s = 1 - \int_{\pi/16}^{-\pi/16} P_{\theta_r}(\theta_r)\, d\theta_r$$

Where,

$$p_{\theta_r}(\theta_r) = \frac{1}{2\pi} e^{-2\gamma_s \sin^2 \theta_r} \int V e^{\ (V\ \ \sqrt{4\gamma_s})^{2/2}} dV$$

$$V = \sqrt{r_1^2 + r_2^2}$$

$$\theta_r = \tan^{-1}(r_2/r_1)$$

$$\gamma_s = \frac{E_s}{N_0}$$

$r_1 \sim 4\left(\sqrt{E_s}, N_0/2\right)$ and $r_2 \sim 4\,(0, N_0/2)$ jointly Gaussian random variables. This may be approximated for 16-PSK and high E_b/N_0 by:

$$P_s = 2Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{16}\right) \tag{3.15}$$

The BER for 16-PSK can be determined when the bit-mapping is known. However, when gray coding is used, the most probable error from one symbol to the next produces only a single bit-error and

$$P_b = \frac{1}{k} P_s$$

Higher-order modulations deliver a higher raw data-rate at the expense of higher error-rates; however bounds on the error rates of various digital modulation schemes can be computed with application of the union bound to the signal constellation.

32-Phase Shift Keying(32-PSK)

In 32-PSK, each signal is only 5.625^0 apart and this will give 32 signals or symbols, so each symbol can convey 5 bits. Bit rate is now five times that of BPSK for the same symbol rate. The following figures show the 32-PSK signal at various stages during modulation. We can keep on increasing bits per symbol this way. 32-PSK is bandwidth efficient but it has higher BER.

Bit Error Rate

For 32-PSK, the symbol-error probability can be obtained from:

$$P_s = 1 - \int_{\pi/32}^{-\pi/32} P_{\theta_r}(\theta_r)\, d\theta_r$$

Higher-order modulations deliver a higher raw data-rate at the expense of higher error-rates; however bounds on the error rates of various digital modulation schemes can be computed with application of the union bound to the signal constellation.

Where,

$$p_{\theta_r}(\theta_r) = \frac{1}{2\pi} e^{-2\gamma_s \sin^2 \theta_r \int V e^{-(V-\sqrt{4\gamma_s})^{2/2}} dV}$$

$$V = \sqrt{r_1^2 + r_2^2}$$

$$\theta_r = \tan^{-1}(r_2/r_1)$$

$$\gamma_s = \frac{E_s}{N_0}$$

$r_1 \sim 5\left(\sqrt{E_s}, N_0/2\right)$ and $r_2 \sim 5\left(0, N_0/2\right)$ jointly Gaussian random variables. This may be approximated for 32-PSK and high E_b/N_0 by:

$$P_s = 2Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{32}\right) \tag{3.16}$$

When gray coding is used, the most probable error from one symbol to the next produces only a single bit-error and BER for 32-PSK can be determined

$$P_b = \frac{1}{k}P_s$$

Higher-order modulations deliver a higher raw data-rate at the expense of higher error-rates; however bounds on the error rates of various digital modulation schemes can be computed with application of the union bound to the signal constellation.

3.5 Applications of PSK

Owing to PSK's simplicity, particularly when compared with its competitor quadrature amplitude modulation, it is widely used in existing technologies such as: wireless LAN standard, IEEE 802.11b-1999. BPSK is used in deep space telemetry. QPSK is used extensively in applications including Code Division Multiple Access (CDMA), cellular service, wireless local loop, Iridium (a voice/data satellite system) and Digital Video Broadcasting Satellite (DVB-S). 8-PSK is widely used in satellite, air-craft, and telemetry pilots for monitoring broadband video systems. 16-PSK has uses in microwave digital radio, modems, DVB-C and DVB-T. Terrestrial microwave and DVB-T uses 32-PSK. Because of simplicity BPSK is appropriate for low-cost passive transmitters, and is used in RFID standards such as ISO/IEC 14443 which has been adopted for biometric passports, credit cards such as American Express's ExpressPay, and many other applications. Bluetooth 2 will use $\pi/4$-DQPSK at its lower rate (2 Mbit/s) and 8-DPSK at its higher rate (3 Mbit/s) when the link between the two devices is sufficiently robust. Bluetooth 1 modulates with Gaussian minimum-shift keying, a binary scheme, so either modulation choice in version 2 will yield a higher data-rate. A similar technology, IEEE 802.15.4 (the wireless standard used by ZigBee) also relies on PSK. IEEE 802.15.4 allows the use of two frequency bands: 868915 MHz using BPSK and at 2.4 GHz using

OQPSK. Notably absent from these various schemes is 8-PSK. This is because its error-rate performance is close to that of 16-QAM it is only about 0.5 dB better but its data rate is only three-quarters that of16-QAM. Thus 8-PSK is often omitted from standards and, as seen above, schemes tend to 'jump' from QPSK to16-QAM (8-QAM is possible but difficult to implement). Included among the exceptions is Hughes Net satellite ISP. For example, the model HN7000S modem uses 8-PSK modulation.

Chapter 4

Equalizer Techniques

4.1 Introduction

In wireless communication, an equalizer is a device that tends to reverse the distortion experienced by a signal transmitted through a channel. In MIMO wireless communication, its objective is to recover the transmitted symbols by reducing intersymbol interference (ISI) and to improve the BER characteristic by maintaining a good SNR. It may be a simple linear filter or a complex algorithm. In this chapter, an overview of equalizer types is provided. Section 4.2 describes the classification of equalizer techniques. Following that, Section 4.3, 4.4 and 4.5 reviews some existing equalizers that will be useful for designing optimum equalizer. Lastly, Section 4.6 covers proposed Modified MMSE equalizer that will perform approximately identical or even better than existing optimal ML equalizer and reduces implementation cost at the same time and section 4.7 describes maximal ratio combining (MRC) technique which is needed in order to improve the performance of proposed Modified MMSE equalizer further.

4.2 Classification of Equalizers

Equalizer may be a simple linear filter or a complex algorithm. In wireless communications, equalizer may be classified as the following types:

Linear Equalizer

Linear equalizer processes the incoming signal with a linear filter. The linear equalizer is also classified by the following categories:

i Zero Forcing (ZF) equalizer

ii Minimum Mean Square Error (MMSE) equalizer

iii Decision Feedback Equalizer (DFE)

iv Blind equalizer

v Adaptive equalizer

vi Viterbi equalizer

vii Turbo equalizer

Non-linear Equalizer

Non-linear equalizer may be classified as

a. Maximum Likelihood (ML) equalizer

b. Successive Interference Cancellation (SIC)

c. Successive Interference Cancellation with optimal ordering (SIC-Sort)

Linear equalizers such as ZF, MMSE, non-linear equalizers SIC, SIC-Sort, combination of linear and non-linear equalizers is concentrated for further discussions in the following sections.

4.3 ZF Equalizer

The ZF equalizer is a linear equalizer that inverse the channel frequency response to the received signal, to restore the signal after the channel. This form of equalizer was first proposed by Robert Lucky. The name ZF corresponds to bringing down the ISI to zero in a noise free environment. It is widely used in communication systems that reduce computational complexity but experiences noise enhancement. This will be useful when ISI is more predominant than noise. The solution of the ZF equalizer is given by:

$\hat{x} = \left(H^H H\right)^{-1} Hx = H^+ x$, Where, $()^+$ represents the pseudo-inverse. The ZF receiver converts the joint decoding problem into C single stream, thereby significantly reducing receiver complexity. However, this complexity reduction comes, at the expense of noise enhancement which results in significant performance degradation compared to the ML equalizer. The diversity order achieved by each of the individual data streams equals D C + 1.

Consider a 2×2 MIMO wireless channel with a transmission sequence of $x_1 x_2 \ldots x_C$. In conventional transmission, we will send x_1 in the first time slot, x_2 in the second time slot, and so on. In the present case, we have 2 transmitter antennas, so we may group the symbols into groups of two. In the first time slot, from the first and second antenna $x_1 x_2$ are sent. In second time slot, from the first and second antenna $x_3 x_4$are sent and so on. Since we are grouping two symbols and sending them in one time slot, we need only c/2 time slots to complete the transmission. It is the composition of a possible MIMO transmission scheme containing 2×2 antennas.

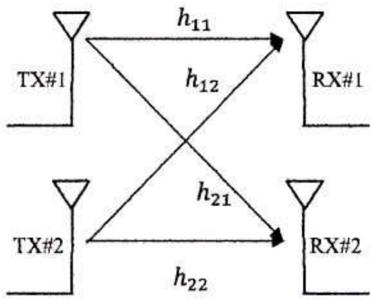

FIGURE 4.1: 2×2 wireless channel.

From the figure, two symbols which are interfered with each other can be solved by mathematical analysis. In the first time slot, the received signal on the first receiver antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{4.1}$$

The received signal on the second receiver antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{4.2}$$

Where, y_1 , y_2 are the received symbol on the first and second antenna respectively, $h_{1,1}$ is the channel from 1^{st} transmitter antenna to 1^{st} receiver antenna, $h_{1,2}$ is the channel from 2^{nd} transmitter antenna to 2^{nd} treceiver antenna, $h_{2,1}$ is the channel from 1^{st} transmitter antenna to 2^{nd} receiver antenna, $h_{2,2}$ is the channel from 2^{nd} transmitter antenna to 2^{nd} receiver antenna, x_1, x_2 are the transmitted symbols and n_1, n_2 is the noise on 1^{st}, 2^{nd} receiver antenna. We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{2,1}$ and $h_{2,2}$. The receiver also knows y_1, and y_1. The unknowns are x_1 and x_2. For ease, the above equation can be represented in matrix formation as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{4.3}$$

Hence,

$$y = Hx + n \tag{4.4}$$

To solve x we need to find a matrix W which satisfies WH=I. The ZF linear equalizer for meeting this constraint is given by,

$$Z_{ZF} = \left[H^H H\right]^{-1} H^H \tag{4.5}$$

Where, W_{ZF} - Equalization matrix and H - Channel matrix. This matrix is known as the pseudo inverse for a general $C \times D$ matrix. Where,

$$H^H H = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \tag{4.6}$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{1,2} + h_{2,1}h_{2,2} \\ h_{1,2}h_{1,1} + h_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \tag{4.7}$$

If the off diagonal terms in the matrix $H^H H$ are not zero, ZF equalizer tries to null out the interfering terms when performing equalization, i.e. when solving x_1 the interference from $x_2$is tried to be nulled and vice versa. While doing so, there can be augmentation of noise. Hence ZF equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement.

Linear ZF equalizer may be combined with non-linear equalizer and takes the following forms:

i. ZF with successive interference cancellation (ZF-SIC)

ii. ZF successive interference cancellation with optimal ordering (ZF-SIC-Sort)

ZF with Successive Interference Cancellation (ZF-SIC)

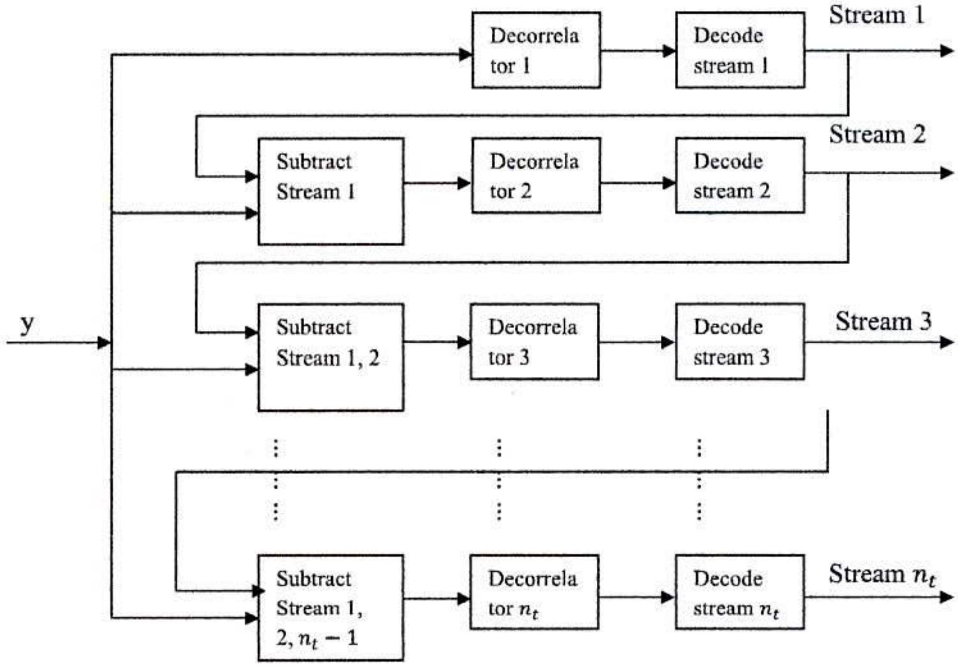

FIGURE 4.2: Structure of ZF–SIC equalizer.

Successive interference cancellation (SIC) is a non-linear process that allows a receiver to decode data packets simultaneously. Since SIC is known to be sensitive to error propagation, a careful adjustment of the data rate at each spatial layer is compulsory and decoding of already detected spatial layers is required prior to an interference cancelation step. The performance of the ZF approaches can be improved significantly. This can be accomplished by the following procedure: after the signal of the first data stream is restored, it can be subtracted from the received signal; then we get the signal with the suppressed interference and after this the procedure repeats for the next data streams.

The second stream has to deal with interference only for $N_C - 2$ streams. This procedure is called ZF-SIC. However, it has certain drawbacks. It works properly only when the data streams are successfully decoded. Otherwise, the error spreads to all the remaining streams. This effect is called error propagation. This is the reason why the detection order is very important for the system performance. In Fig. 4.2 the ZF-SIC design is shown.

Using the ZF equalization approach and 2×2 MIMO, the receiver can obtain an estimate of the two transmitted symbols x_1 and x_2 i.e.

$$\begin{bmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \end{bmatrix} = \left[H^H H\right]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{4.8}$$

Taking one of the estimated symbols for example $\widetilde{x}_1$ and subtract its effect from the received vector y_1 and y_2 , i.e.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} \ \ \tilde{x}_2 \\ y_2 - h_{2,2} \ \ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1}x_1 + n_1 \\ h_{2,1}x_1 + n_2 \end{bmatrix} \tag{4.9}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{4.10}$$

$$r = hx_1 + n \tag{4.11}$$

The above equation is identical with receive diversity equation. The equalized symbol is,

$$\tilde{x}_1 = \frac{h^H r}{h^H h}$$

This forms the simple explanation for ZF-SIC approach.

ZF-SIC with Optimal Ordering (ZF-SIC-Sort)

In conventional SIC, the receivers arbitrarily take one of the estimated symbols, and subtract its effect from the received symbol y_1 and y_2. However, we are more decisive in choosing whether we should subtract the effect of $\tilde{x}_1$ or $\tilde{x}_2$ first. For this reason, we have to find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver. The received power at both the antennas corresponding to the transmitted symbol x_1 is,

$$P_{x_1} = |h_{1,1}|^2 + |h_{2,1}|^2 \tag{4.12}$$

The received power at both the antennas corresponding to the transmitted symbol x_2 is

$$P_{x_2} = |h_{1,2}|^2 + |h_{2,2}|^2 \tag{4.13}$$

If $P_{x_1} > P_{x_2}$ then the receiver decides to remove the effect of $\tilde{x}_1$ from the received vector y_1 and y_2 and re-estimate $\tilde{x}_2$.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,1} & \tilde{x}_1 \\ y_2 - h_{2,1} & \tilde{x}_1 \end{bmatrix} = \begin{bmatrix} h_{1,2}x_2 + n_1 \\ h_{2,2}x_2 + n_2 \end{bmatrix} \tag{4.14}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,2} \\ h_{2,2} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$r = hx_2 + n \tag{4.15}$$

Once the effect of either $\tilde{x}_1$ or $\tilde{x}_2$ is removed, the new channel becomes one transmit antenna, two receive antenna configuration and the symbol on the other spatial dimension can be optimally equalized by maximal ratio combining (MRC). The equalized symbol is,

$$\tilde{x}_2 = \frac{h^H r}{h^H h}$$

Else if $P_{x_1} \leq P_{x_2}$ the receiver decides to subtract effect of $\tilde{x}_2$ from the received vector y_1 and y_2 then re-estimate $\tilde{x}_1$.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} & \tilde{x}_1 \\ y_2 - h_{2,2} & \tilde{x}_1 \end{bmatrix} = \begin{bmatrix} h_{1,1}x_2 + n_1 \\ h_{2,1}x_2 + n_2 \end{bmatrix}$$

Expressing in matrix form,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$r = hx_1 + n \tag{4.16}$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply MRC. The equalized symbol is,

$$\tilde{x}_1 = \frac{h^H r}{h^H h}$$

Successive interference cancellation with optimal ordering (SIC-Sort) ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous interference cancellation. Hence gives lower error rate than simple successive interference cancellation. But noise is still an obstacle that has to minimize in order to achieve good performance in terms of BER.

4.4 Maximum Likelihood (ML) Equalizer

Maximum Likelihood (ML) equalizer is a non linear equalizer in which a search is performed over all possible symbols and the most likelihood one is chosen. It calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. No noise enhancement takes place and numerical issues are virtually not present, as no matrix inversions or divisions are necessary. The ML equalizer tries to search the most likelihood one which minimizes,

$$J = |y - Hx|^2 \tag{4.17}$$

$$J = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2 \tag{4.18}$$

Let A and C denote a set of signal constellation points and a number of transmit antennas, respectively. ML equalizer is the method which compares the received signals with all possible transmitted signal that is modified by channel matrix H and estimates transmit symbol vector x according to the maximum likelihood principle, which is shown as:

$$\hat{x}_{ML} = arg_{x \in A^C} \left[min \left\| y - Hx \right\|^2 \right] \tag{4.19}$$

Where, x is the estimated symbol vector.

In case of QPSK modulation the possible values are 4^2=16. In order to find the maximum likelihood one, we have to find the minimum from all the sixteen combinations of

$$J_{11,11} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 11 \\ 11 \end{bmatrix} \right\|^2$$

$$J_{11,10} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 11 \\ 10 \end{bmatrix} \right\|^2$$

$$J_{11,01} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 11 \\ 01 \end{bmatrix} \right\|^2$$

$$J_{11,00} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 11 \\ 00 \end{bmatrix} \right\|^2$$

$$J_{01,11} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 01 \\ 11 \end{bmatrix} \right\|^2$$

$$J_{01,10} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 01 \\ 10 \end{bmatrix} \right\|^2$$

$$J_{01,01} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 01 \\ 01 \end{bmatrix} \right\|^2$$

$$J_{01,00} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 01 \\ 00 \end{bmatrix} \right\|^2$$

$$J_{10,11} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 10 \\ 11 \end{bmatrix} \right\|^2$$

$$J_{10,10} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} \right\|^2$$

$$J_{10,01} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 10 \\ 01 \end{bmatrix} \right\|^2$$

$$J_{10,00} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 10 \\ 00 \end{bmatrix} \right\|^2$$

$$J_{00,11} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 00 \\ 11 \end{bmatrix} \right\|^2$$

$$J_{00,10} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 00 \\ 10 \end{bmatrix} \right\|^2$$

$$J_{00,01} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 00 \\ 01 \end{bmatrix} \right\|^2$$

$$J_{00,00} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 00 \\ 00 \end{bmatrix} \right\|^2$$

The estimate of the transmitting symbol is chosen based on the minimum value from the above sixteen values i.e.

$$\text{if the minimum is,} J_{11,11} \Rightarrow \begin{bmatrix} 11 & 11 \end{bmatrix}$$

$$\text{if the minimum is,} J_{11,10} \Rightarrow \begin{bmatrix} 11 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{11,01} \Rightarrow \begin{bmatrix} 11 & 01 \end{bmatrix}$$

$$\text{if the minimum is,} J_{11,00} \Rightarrow \begin{bmatrix} 11 & 00 \end{bmatrix}$$

$$\text{if the minimum is,} J_{10,00} \Rightarrow \begin{bmatrix} 10 & 00 \end{bmatrix}$$

$$\text{if the minimum is,} J_{10,01} \Rightarrow \begin{bmatrix} 10 & 01 \end{bmatrix}$$

$$\text{if the minimum is,} J_{10,10} \Rightarrow \begin{bmatrix} 10 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{10,11} \Rightarrow \begin{bmatrix} 10 & 11 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,11} \Rightarrow \begin{bmatrix} 01 & 11 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,10} \Rightarrow \begin{bmatrix} 01 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,10} \Rightarrow \begin{bmatrix} 01 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,10} \Rightarrow \begin{bmatrix} 01 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,01} \Rightarrow \begin{bmatrix} 01 & 01 \end{bmatrix}$$

$$\text{if the minimum is,} J_{01,00} \Rightarrow \begin{bmatrix} 01 & 00 \end{bmatrix}$$

$$\text{if the minimum is,} J_{00,11} \Rightarrow \begin{bmatrix} 00 & 11 \end{bmatrix}$$

$$\text{if the minimum is,} J_{00,10} \Rightarrow \begin{bmatrix} 00 & 10 \end{bmatrix}$$

$$\text{if the minimum is,} J_{00,01} \Rightarrow \begin{bmatrix} 00 & 01 \end{bmatrix}$$

$$\text{if the minimum is,} J_{00,00} \Rightarrow \begin{bmatrix} 00 & 00 \end{bmatrix}$$

Although ML detection offers optimal error performance, it suffers from complexity issues. It has exponential complexity in the sense that the receiver has to consider $|A|^C$ possible symbols.

4.5 Minimum Mean Square Error (MMSE) Equalizer

A more balanced linear equalizer is the MMSE equalizer, which does not eliminate ISI entirely but minimizes total noise power and ISI components in the output. In wireless communications, MMSE equalizer approach minimizes the mean square error (MSE), which is a common measure of estimator quality. Let X is an unknown random variable, and Y is a known random variable. An estimator $\widetilde{X}_y$ is any function of the measurement Y, and its MSE is given by

$$MSE = E\left\{\left(\widetilde{X} - X\right)^2\right\} \tag{4.20}$$

Where, the expectation is taken over both X and Y. When it is not possible to determine a closed form for the MMSE equalizer then minimize the MSE within a particular class, such as the class of linear equalizers. The linear MMSE equalizer is the equalizer achieving minimum MSE among all the equalizers of the form AY+b. Where, Y is a random vector, A is a matrix and b is a vector.

Now, we consider the case where two symbols are interfered with each other. In the first time slot, the received signals on the first and second receiving antennas are,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

In matrix form, the above two equations can be expressed as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

The above wireless channel is modulated by the theorem, y=Hx+n. The MMSE approach tries to find a coefficient W which minimizes the criterion,

$$E\left\{[W_{y-x}]\,[W_{y-x}]^H\right\}$$

To solve x we need to find a matrix W which satisfies WH=I. The MMSE equalizer for satisfying this constraint is given by,

$$W = \left[H^H H + N_0 I\right]^{-1} H^H$$

So the solution of the MMSE equalizer is given by the following equation

$$\widetilde{x} = \left[H^H H + N_0 I\right]^{-1} H^H x \tag{4.21}$$

Where W- equalization matrix and H - channel matrix. This matrix is known as the pseudo inverse for a general $C \times D$ matrix. Where,

$$\begin{aligned} H^H H &= \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \\ &= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{1,2} + h_{2,1}h_{2,2} \\ h_{1,2}h_{1,1} + h_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \end{aligned}$$

In fact, when the noise term is zero, the MMSE equalizer tends to ZF equalizer and this model can be extended to $C \times D$ antenna configuration.

4.6 MMSE Estimation

Consider a linear combination of random variables N_1, N_2, N_3 to estimate another random variable N_4 using $\widetilde{N}_4$. If the random variables $N = [N_1, N_2, N_3]$ are real Gaussian random variables with zero mean and covariance matrix given by

$$Cov\,(x) = E\left[NN^T\right] \tag{4.22}$$

We will estimate the vector X_4 and find coefficients a_i such that the estimate $\hat{N}_4 = \sum_{i=1}^{3} a_i N_i$ is an optimal estimate of N_4. We will use the autocorrelation matrix, R and the cross correlation matrix, C to find vector A, which consists of the coefficient values that will minimize the estimate. The autocorrelation matrix R is easily determined.

4.7 MMSE-SIC

MMSE-SIC proceeds in the same way as ZF-SIC perform. But in MMSE-SIC, noise term is added before the inversion step of the equalization matrix. Using the MMSE equalization approach, the receiver can obtain an estimate of the two transmitted symbols x_1 and x_2 i.e.

$$\begin{bmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \end{bmatrix} = \left[H^H H + N_0 I\right]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{4.23}$$

Taking one of the estimated symbols for example $\tilde{x}_1$ and subtract its effect from the received vector y_1 and y_2 , i.e.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} & \tilde{x}_2 \\ y_2 - h_{2,2} & \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1}x_1 + n_1 \\ h_{2,1}x_1 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$r = hx_1 + n$$

The above equation is identical with receive diversity equation. The equalized symbol is,

$$\tilde{x}_1 = \frac{h^H r}{h^H h}$$

This forms the simple explanation for MMSE equalizer with successive interference cancellation (MMSE-SIC) approach. It performs better than MMSE but cannot exceed the performance of ML equalizer.

4.8 Maximal Ratio Combining (MRC)

MRC is a special form of diversity combining where multiple replicas of the same signal, received over different diversity branches, are combined in order to maximize the instant SNR at the combined output. Before adding the signals of every receive branch the symbols on all receive antennas are weighted. The weight factor corresponds with the complex conjugated channel coefficient of each receive branch. MRC is the optimum combiner for independent additive white Gaussian noise (AWGN) channels and can restore a signal to its original shape.

The signal received on each antenna is given by

$$y_i = h_i x + n_i, \text{i=1,2,}\ldots\text{,}N_D \tag{4.24}$$

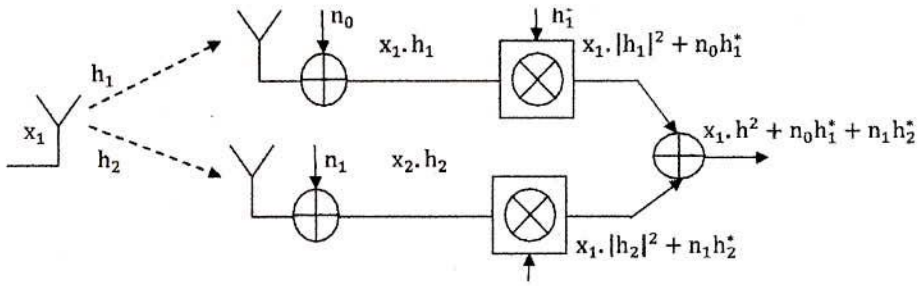

FIGURE 4.3: Maximal ratio combining using two antennas.

Where, h_i and n_i are the channel coefficients and the noise experienced by antenna i, respectively, x is the transmitted signal, and N_D is the number of receive antennas. Besides, it is considered that the antennas are sufficiently spaced from each other and the channel coefficients, affected by fading, can be assumed to be independent. The weighted combination for the input antennas to be taken into account is expressed as follows:

$$y = \sum_{i=1}^{N_D} \beta_i y_i = x \sum_{i=1}^{N_D} \beta_i h_i + \sum_{i=1}^{N_D} \beta_i n_i \tag{4.25}$$

From this combination, the SNR of the channel is given by

$$SNR(h_1, h_1, ..., h_{N_D}) = \frac{E\left|x \sum_{i=1}^{N_D} \beta_i h_i\right|^2}{E\left|\sum_{i=1}^{N_D} \beta_i h_i\right|^2} = \frac{\left|\sum_{i=1}^{N_D} \beta_i h_i\right|^2}{\left|\sum_{i=1}^{N_D} \beta_i\right|^2} \tag{4.26}$$

The process described above is shown in figure 4.3, where an example of a receiver with dual antenna diversity is depicted. The signal is sent over a channel with transmission coefficients h_1 and h_2, and reaches both receive antennas with some added noise. Then, the process consists of multiplying the signal in each receive branch by the corresponding conjugated channel coefficient, and at the end, the signals from both branches are added.

The BER of PSK in a Rayleigh fading channel can be determined by MRC technique. The probability of BER for PSK P_b can be determined by using the formula given below:

$$P_b = \frac{1}{\pi} \int_0^{(A-1)\pi/A} \left[det\left(\frac{\sum sin^2(\pi/A)}{sin^2\theta}\right) + I\right]^{-1} d\theta \tag{4.27}$$

For multiple Eigen values the formula can be written as:

$$P_b = \frac{1}{\pi}\int_0^{(A-1)\pi/A}\sum_{n=1}^{N}\sum_{s=1}^{m_n} r_{s,n}\left[det\left(\frac{\hat{\gamma}_n\lambda_n \sum sin^2(\pi/A)}{sin^2\theta}\right)+I\right]^{-s} d\theta \tag{4.28}$$

We obtain the following result for P_b

$$P_b = \frac{A-1}{A} - \sum_{n=1}^{N}\sum_{s=1}^{m_n} r_{s,n}\frac{1}{\pi}\sqrt{\frac{c}{1+c}}\left(\pi/2+\tan^{-1}\alpha\right)\times\sum_{k=0}^{s-1}\binom{2k}{k}\frac{1}{\left[4(1+c)\right]^k} - \sum_{n=1}^{N}\sum_{s=1}^{m_n} r_{s,n}\frac{1}{\pi}\sqrt{\frac{c}{1+c}}\sin\tan^{-1}\alpha\sum_{k=1}^{s-1}\sum_{i=1}^{k}\frac{T_{i,k}}{(1+c)^k}\cos\left(\tan^{-1}\alpha\right)^{2(k-i)+1} \tag{4.29}$$

Where,$c = \hat{\gamma}_n log_2 A.\lambda_n sin^2\pi/A$ considering that $\lambda_n$is the average SNR per bit on each channel; $r_{s,n}$ is as defined $\alpha = \sqrt{\frac{c}{1+c}}\cot\frac{\pi}{A}$

$$T_{i,k} = \frac{\binom{2k}{k}}{\binom{2(k-i)}{k-i}4^i\left[2(k-i)+1\right]} \tag{4.30}$$

Chapter 5

Proposed Modified MMSE Equalizer

5.1 Introduction

In this chapter, traditional MMSE equalizer is modified in order to achieve the improved performance and later it is compared with other existing equalizers. Proposed equalizer termed as Modified MMSE is formed by following matrix ordering and adjusted it with the optimum power of the transmitted signal to form modified vector that eliminates the interference relentlessly and provides superior performance. This process is same as MMSE-SIC equalizer but matrix is updated by modified vector of the transmitted signal. The limitations of specific equalizers are mentioned in order to eliminate the problems of the transmitted signal and design the proposed equalizer. The structure and performance of the Modified MMSE equalizer along with other existing equalizers is studied for MIMO wireless communication system with the purpose of designing a new optimum equalizer. The BER performance of the system is presented by increasing the order of modulation. The system discussed above is designed using PSK modulation and the performance is evaluated by MatLab simulation. Different antenna configurations such as 2×2, 2×3, 2×4 and proposed Modified MMSE equalizer are used to show the optimum performance in terms of BER vs. SNR over the other existing equalizers. The analysis has been done for Rayleigh fading channel. Results have been presented for different antenna configurations over different fading channels using different modulation levels. In the first section, structure of proposed Modified MMSE and flowchart diagram of ZF, ML, MMSE and Modified MMSE are provided. Then simulation performances are analyzed by step by step for different equalizers.

The optimal equalizer design and performance of MIMO system is evaluated using the following process:

i. BER analysis of different equalizers and proposed Modified MMSE equalizer by using different types of PSK modulation for different orders of MIMO configuration

ii. Comparison of graphical analysis and conclude about the optimum equalizer that exhibits the superior performance.

5.2 Problems of Specific Equalizer

Some of the major problems of the existing equalizers are given below:

i. Noise Amplification- ZF equalizer

ii. ISI Problem- MMSE equalizer

iii. Compound Hardware Structure and Computation Complexity- ML equalizer

By considering all of the problems, the proposed Modified MMSE equalizer is designed that performs superior among all of the existing equalizers as well as keeping complexity at a reasonable level.

5.3 Proposed Modified MMSE Equalizer

In traditional SIC-Sort, the receivers remove successive interference by optimal power of the transmitted symbol and subtract its effect from the received symbol. As it has to consider power criterion for each of the transmitted symbol so its performance cannot exceed the ML equalizer. However, in proposed Modified MMSE, successive interference is cancelled based on prescribed ordered matrix and updated modified vector, which is termed as Modified MMSE. The process is same as MMSE-SIC equalizer but ordered matrix is updated by modified vector of the transmitted symbol. This method offers even the greater performance compared to ML equalizer. In general, the performance of the linear MMSE detection method is worse than that of other nonlinear equalizer techniques. However, linear detection methods require a low complexity of hardware implementation. We can improve their performance significantly without increasing the complexity by MMSE-SIC with modified vector called Modified MMSE method. It is a group of linear receivers, each of which detects one of the parallel data streams, with the detected signal components successively cancelled from the received signal at each stage. More specifically, the detected signal in each stage is subtracted from the received signal so that the remaining signal with the reduced interference can be used in the subsequent stage. Figure 5.1 illustrates the Modified MMSE signal detection process for n spatial streams.

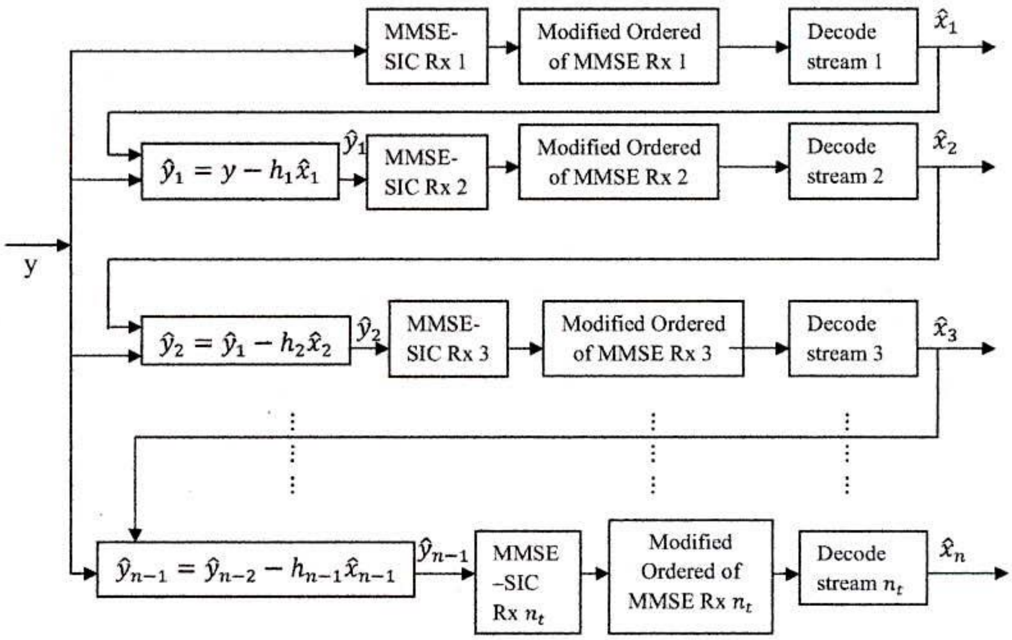

FIGURE 5.1: Structure of proposed Modified MMSE equalizer.

The proposed Modified MMSE detection is composed of c stages, as in the MMSE-SIC. C modified vectors of length $1, 2, \ldots, c$ are selected at stages $1, 2, \ldots, c$. In order to simplify the description of the algorithm, without loss of generality, it is assumed that the optimal ordering is $1, 2, \ldots, c$. Thus, the C modified selections for $[x_1 x_2 \ldots x_i]^T$ in the i-th stage ($i = 1, 2, \ldots, c$) are performed as follows:

Step 1: matrix ordering

The matrix ordering is based on h_k^2, $k = 1, 2, \ldots$,c

Step 2: $C \times |L|$ arbitrary vector generation

In the first stage, (i = 1), $|L|$ arbitrary vectors are generated; in the other stages,$(1, 2, \ldots$,c), $C \times |L|$ vectors are generated. C modified vectors are available for $[x_1, x_2, \ldots, x_i]^T$ as obtained in the $(i-1)$-th stage. According to the received signal y=Hx+n, the interference is subtracted from each arbitrary vector in $S_{i-1}(j)$, $j = 1, 2, \ldots$,c and as in the first stage, all the constellation points in L are tried, as x_i, for each $x_1, x_2, \ldots, x_C$ is obtained using the MMSE-SIC method. As a result, $C \times |L|$ vectors of length c are obtained.

Step 3: C modified vector selection of length c

For each vector obtained in the step 2, the corresponding ML metric is calculated, and the best C modified vectors of length c are chosen.

Step 4: truncation to obtain in C modified vectors of length i

The C modified vectors in step 3 are truncated to length i. The C modified vectors of length i are stored as S_i, and $S_i(j)$, $j = 1, 2, \ldots$,c is the j-th element in the set.

Step 5: signal detection

Signal is detected in the receiver stages for C stages of length i

Let x_i denote the symbol to be detected in the i-th order, which may be different from the transmit signal at the i-th antenna, since x_i depends on the order of detection. Let $\widehat{x}_i$ denote a portion of x_i. For Modified MMSE, MMSE method in equation (4.23) can be used for symbol estimation. Suppose that the MMSE method is used in the following discussion. The 1^{st} stream is estimated with the 1^{st} row vector of the MMSE weight matrix in equation (4.23). After estimation and segmentation to produce $\widehat{x}_1$, the remaining signal in the first stage is formed by subtracting it from the received signal, that is,

$$\hat{y}_1 = y - h_1\hat{x}_1 = h_1(x_1 - \hat{x}_1) + h_2x_2 + \cdots + h_cx_c + k \tag{5.1}$$

If $x_1 = \widehat{x}_1$, then the interference is successfully canceled in the course of estimating x_2; however, if $x_1 \neq \widehat{x}_1$, then error propagation is incurred because the MMSE weight that has been designed under the condition of $x_1 = \widehat{x}_1$ is used for estimating x_2.

Due to the error transmission caused by incorrect decision in the previous stages, the order of detection has significant influence on the overall performance of Modified MMSE detection. As a consequence, SINR method of detection ordering is described below.

SINR Based Ordering

Signals with a higher post-detection signal-to-interference-plus-noise-ratio (SINR) are detected first. Consider the linear MMSE detection with the following post-detection SINR:

$$SINR_i = \frac{E_x \left|W_{i,MMSE} h_i\right|^2}{E_x \sum_{l \neq i} \left|W_{i,MMSE} h_l\right| + \sigma_z^2 \left|W_{i,MMSE}\right|^2}, \text{i=1,2,...,C} \tag{5.2}$$

Where E_x is the energy of the transmitted signals, $W_{i,MMSE}$ is the i-th row of the MMSE weight matrix in Equation (4.23), and h_i is the i-th column vector of the channel matrix. Here, mean-square error (MSE) is minimized and the post-detection SINR is maximized by the MMSE detection. Once C SINR values, $\{SINR_i\}_{i=1,i\neq l}^{C}$ are calculated by using the MMSE weight matrix of equation (4.23), we choose the corresponding layer with the highest SINR. In the course of choosing the second-detected symbol, the interference due to the first detected symbol is canceled from the received signals. Suppose that (1) =l (i.e., the 1^{st} symbol has been canceled first). Then, the channel matrix in equation (11.7) is modified by deleting the channel gain vector corresponding to the 1^{st} symbol as follows:

$$H^1 = \begin{bmatrix} h_1 & h_2 & \cdots & h_{l-1} & h_{l+1} & \cdots & h_C \end{bmatrix} \tag{5.3}$$

Using equation (4.26) in place of H in equation (4.23), the MMSE weight matrix is recalculated. Now, (C-1) SINR values, are calculated to choose the symbol with the highest SINR. The same process is repeated with the remaining signal after canceling the next symbol with the highest SINR. In Modified MMSE, the total number of SINR values to be calculated is

$$\sum_{j=1}^{C} j = \frac{C\,(C+1)}{2} \tag{5.4}$$

5.4 ZF Equalizer Flowchart Diagram

In the ZF equalizer, symbols are generated based on constellation points for different PSK modulation. The symbols are grouped and transmit-receive matrix are formed. Then the symbols are multiplied with the channel and Gaussian noise is added. The received symbols are equalized with ZF criterion and hard decision is performed and BER are counted. This process is repeated for multiple values of SNR and results are plotted in graphical figure. The flowchart of ZF equalizer identify the processes clearly which is shown in the figure 5.2.

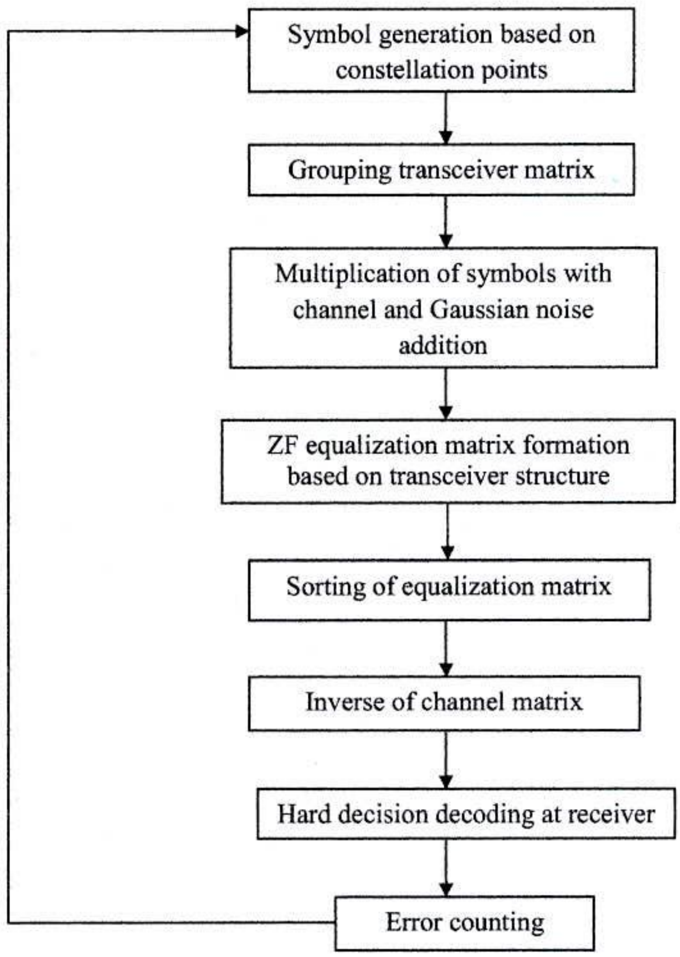

FIGURE 5.2: ZF equalizer flowchart diagram.

5.5 ML Equalizer Flowchart Diagram

In ML equalizer, symbols are generated based on constellation points for different PSK modulation. The symbols are grouped and transmit-receive matrix are formed. Then the symbols are multiplied with the channel and Gaussian noise is added. Minimum possible transmit symbol among all the transmit symbol combinations is selected. Finally, minimum possible transmit symbol to bits are converted and BER is counted. This process is repeated for multiple values of SNR and results are plotted in graphical figure. The flowchart of ML equalizer is shown in the figure 5.3.

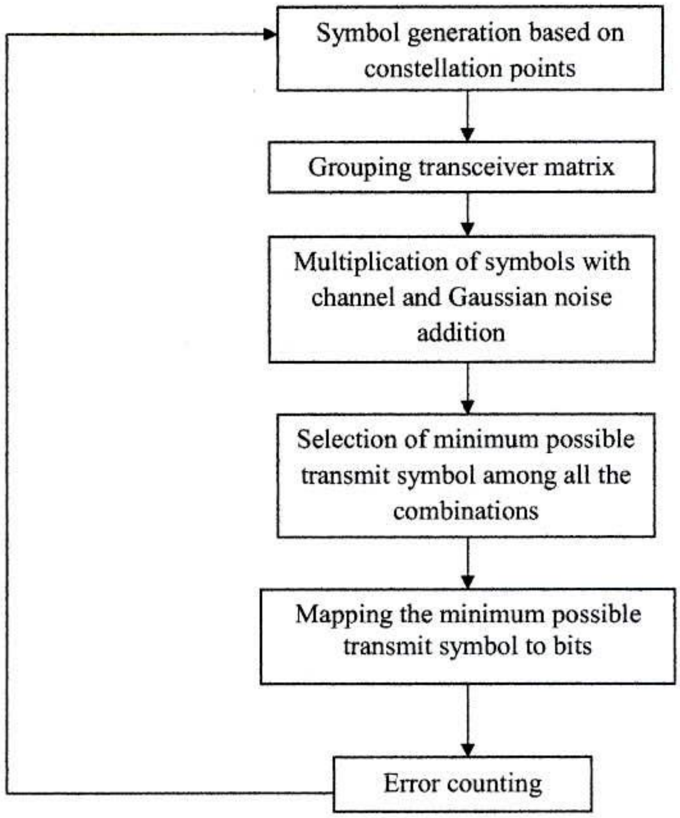

FIGURE 5.3: ML equalizer flowchart diagram.

5.6 MMSE Equalizer Flowchart Diagram

In MMSE equalizer, symbols are generated based on constellation points for different PSK modulation. The symbols are grouped and transmit-receive matrix are formed. Then the symbols are multiplied with the channel and Gaussian noise is added. The received symbols are equalized with MMSE criterion and hard decision is performed and BER are counted. This process is repeated for multiple values of SNR and results are plotted in graphical figure. The flowchart of MMSE equalizer identify the processes clearly which is shown in the figure 5.4.

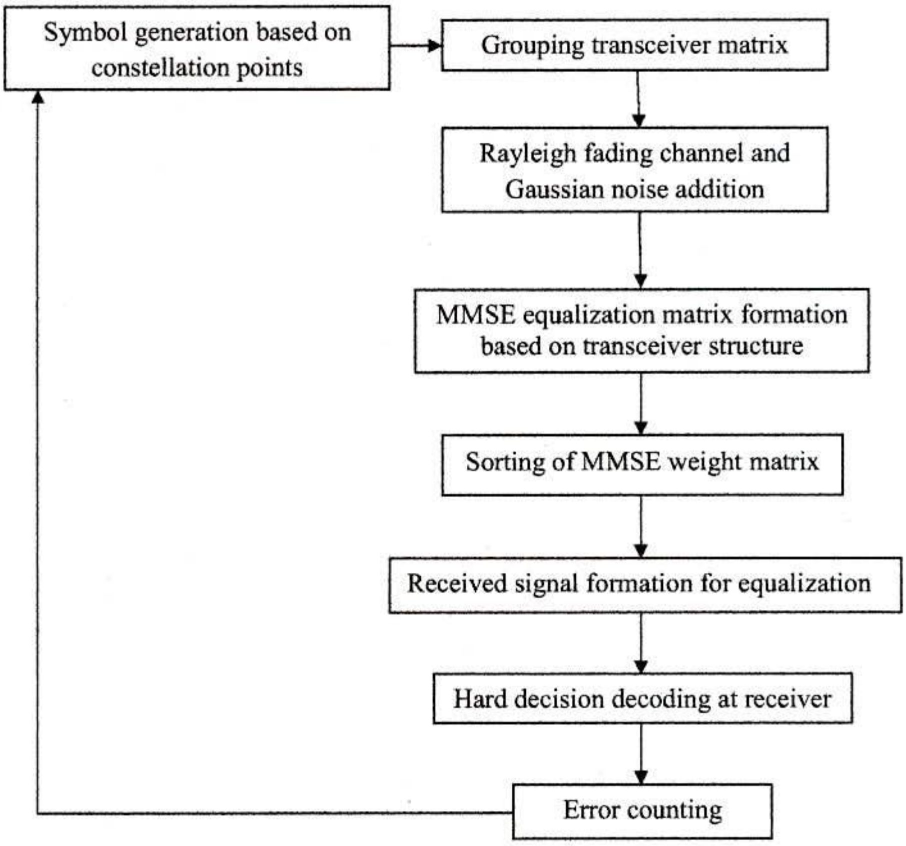

FIGURE 5.4: MMSE equalizer flowchart diagram.

5.7 Proposed Modified MMSE Equalizer Flowchart Diagram

In proposed Modified MMSE equalizer, symbols are generated based on constellation points for different PSK modulation. The symbols are grouped and transmitting-receiving matrix are formed. Then the symbols are multiplied with the channel and Gaussian noise is added.

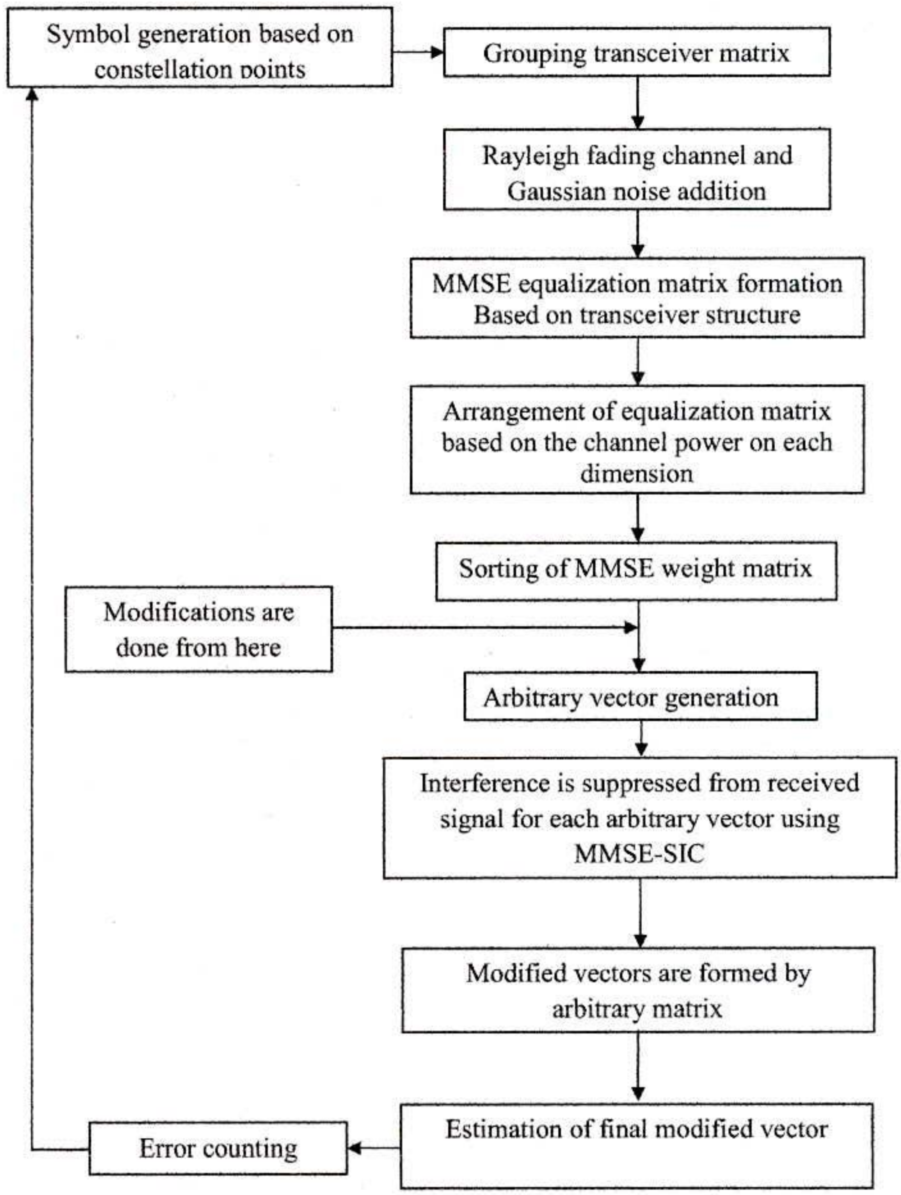

FIGURE 5.5: Proposed Modified MMSE equalizer flowchart diagram.

The received symbols are equalized with MMSE criterion. Then, noise effect is removed based on higher power and new received signal is formed by matrix ordering and modified vector estimation. Finally, hard decision is performed and BER is counted. This process is repeated for multiple values of SNR and results are plotted in graphical figure. The flowchart of proposed Modified MMSE equalizer identify the processes clearly which is shown in the figure 5.5.

For simulation purpose, the simulation parameters those are used throughout implementing and evaluating optimum equalizer are given in the table 5.2.

TABLE 5.1: Simulation Parameters.

Equalizers Applied	ML, MMSE, ZF, proposed Modified MMSE
SNR (dB)	$0 \sim 25$
Channel Characteristics	Rayleigh flat fading varying randomly with every frame
Modulation and Demodulation applied	M=2, 4,8,16,32
Antenna Configurations (*transmitting* $\times$ *receiving*)	2×2, 2×3, 2×4

5.8 BER Analysis of Proposed Modified MMSE Equalizer

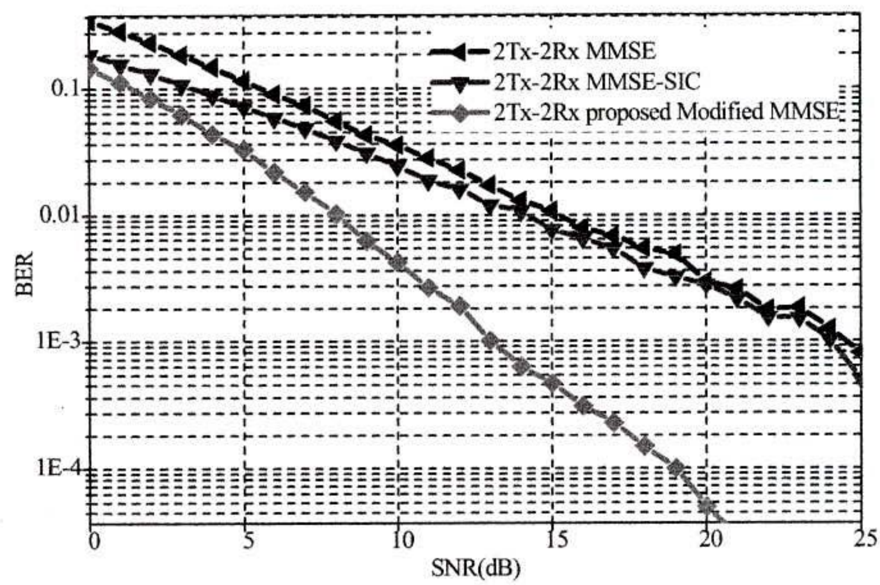

FIGURE 5.6: BER vs. SNR characteristics for 2×2 MIMO and BPSK with MMSE, MMSE-SIC and proposed Modified MMSE equalizers.

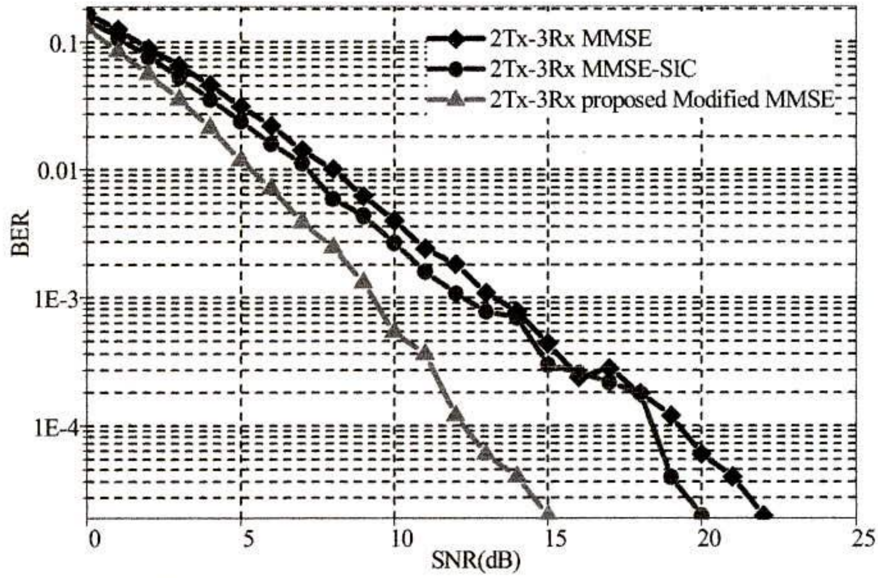

FIGURE 5.7: BER vs. SNR characteristics for 2 × 3 MIMO and BPSK with MMSE, MMSE-SIC and proposed Modified MMSE equalizers.

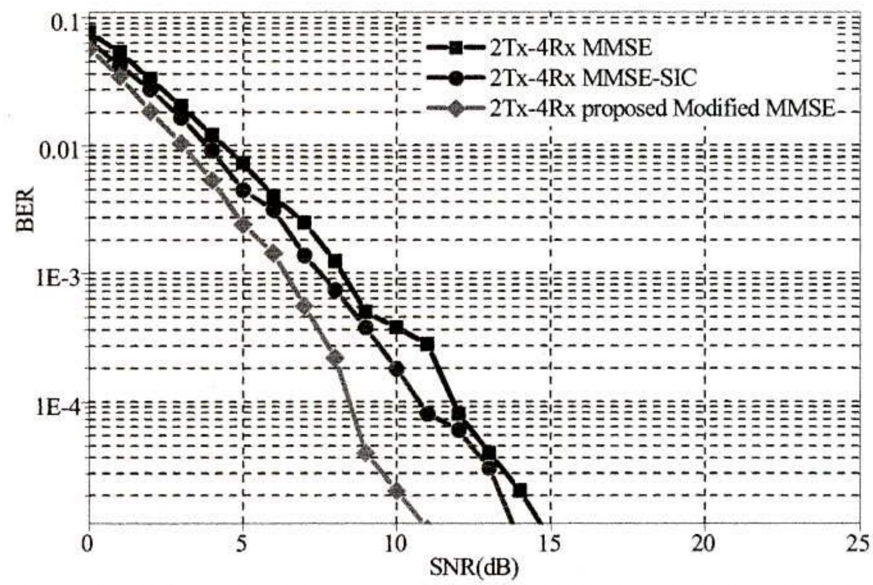

FIGURE 5.8: BER vs. SNR characteristics for 2 × 4 MIMO and BPSK with MMSE, MMSE-SIC and proposed Modified MMSE equalizers.

Figure 5.6, 5.7, 5.8 shows the BER vs. SNR characteristic curves for MIMO wireless communications. 2 × 2, 2 × 3 and 2 × 4 antenna configurations are used for simulation of proposed modified MMSE equalizer to that of existing MMSE equalizers. Different PSK modulations are used for implementation of the proposed Modified MMSE equalizer. From all of the three graphical analyses, it is evident that BER tends to decrease dramatically for proposed Modified MMSE equalizer compared to MMSE and

MMSE-SIC equalizers.

TABLE 5.2: BER improvement for proposed modified MMSE equalizer at SNR=10dB.

Different MIMO Configuration	BER
2×2 MIMO	$4.2E-3$
2×3 MIMO	$5.36E-4$
2×4 MIMO	$2E-5$

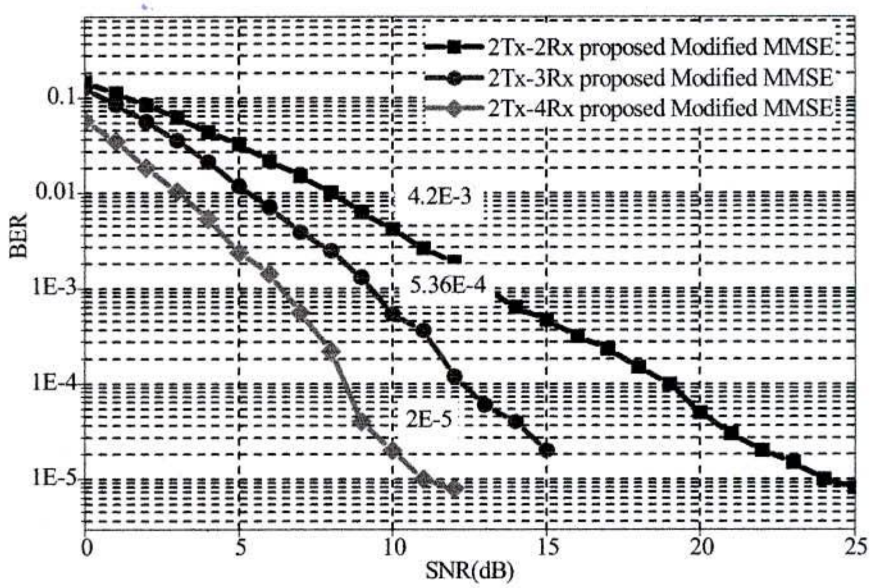

FIGURE 5.9: BER vs. SNR characteristics for proposed Modified MMSE equalizers with BPSK.

The utility of using proposed Modified MMSE equalizer is shown in table 5.2 in the form of BER for PSK modulation in presence of Rayleigh fading channel. As, we step toward higher order antenna configurations, the BER will keeps on decreasing for proposed Modified MMSE. The BER improvement varies from one modulation level to another due to random noise and fading effect. Lastly from figure 5.9 it is obvious that for higher order MIMO and PSK modulation, the proposed Modified MMSE shows superior performance over conventional MMSE.

5.9 BER vs. SNR Characteristics for BPSK Modulation

Fig 5.10, 5.11, 5.12 describes the performance of proposed Modified MMSE equalizer with the existing ML, ZF, ZF-SIC, ZF-SIC-Sort, MMSE and MMSE-Sort equalizers. The equalizers are compared on the basis of BER vs. SNR for 2×2, 2×3 and 2×4

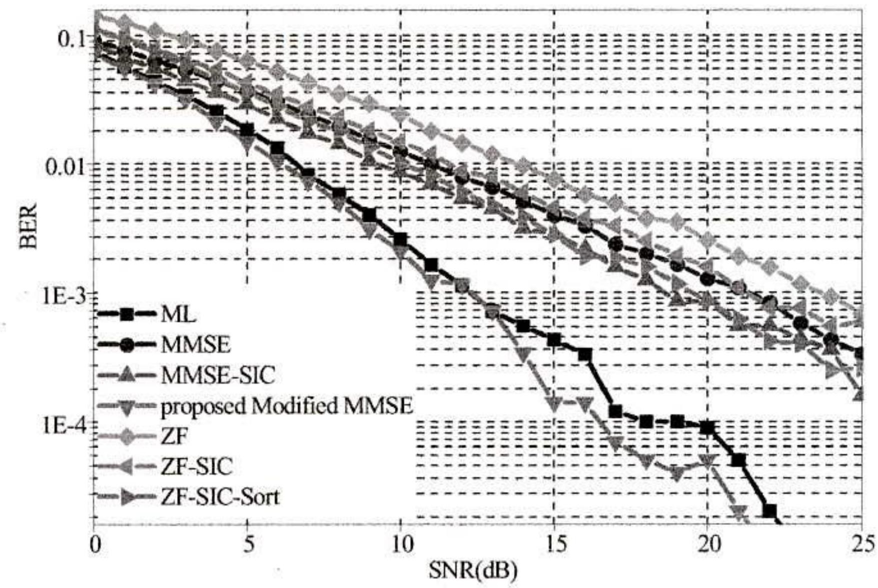

FIGURE 5.10: BER vs. SNR characteristics for 2 × 2 MIMO and BPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

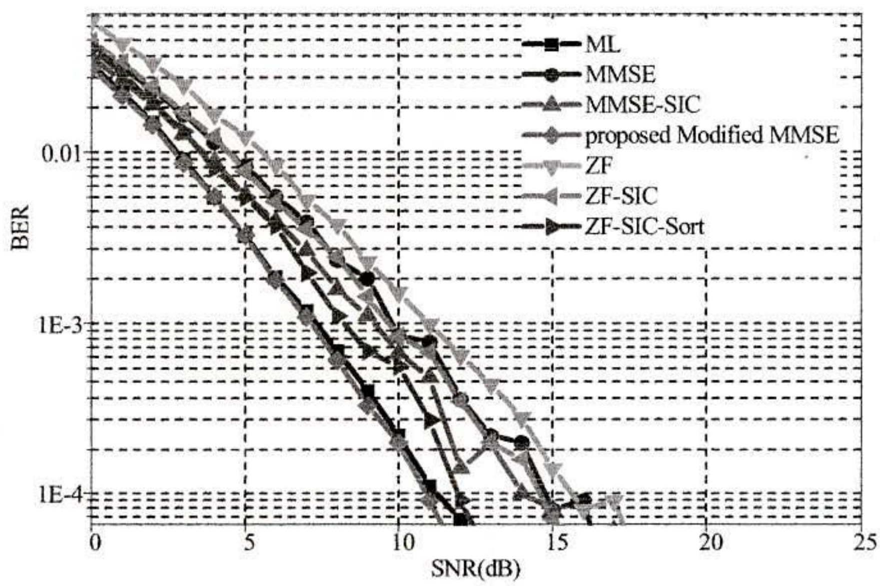

FIGURE 5.11: BER vs. SNR characteristics for 2 × 3 MIMO and BPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

MIMO wireless channel. Here, BPSK modulation is applied for performance evaluation. From the figures, it is observed that with higher order MIMO combinations the BER performance is improved further than that of lower order combinations. It is also evident that proposed Modified MMSE outperforms all of the existing equalizers and performs even better than the existing optimal ML approach. From the graphical representations, it is obvious for all these configurations for proposed Modified MMSE the BER is lowest

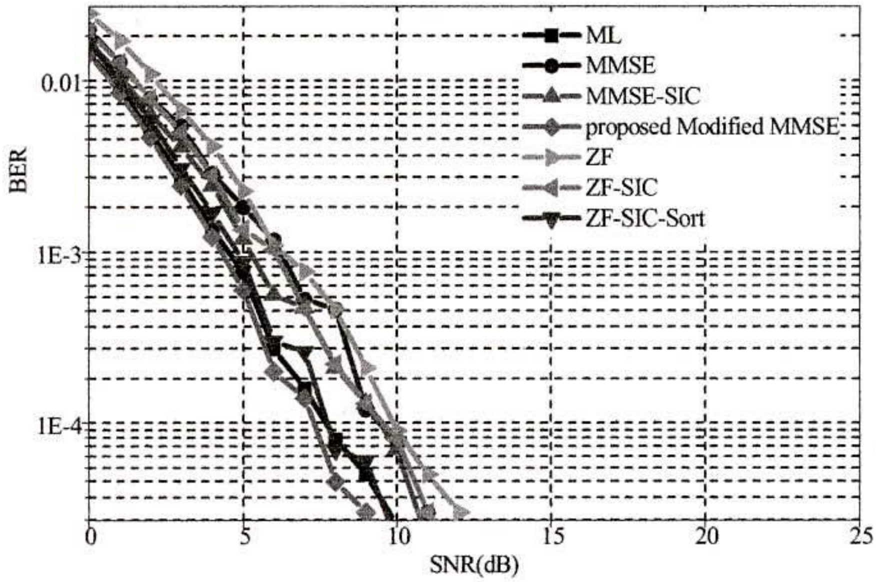

FIGURE 5.12: BER vs. SNR characteristics for 2 × 4 MIMO and BPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

among all the equalizers that have been used for this simulation analysis.

5.10 BER vs. SNR Characteristics for QPSK Modulation

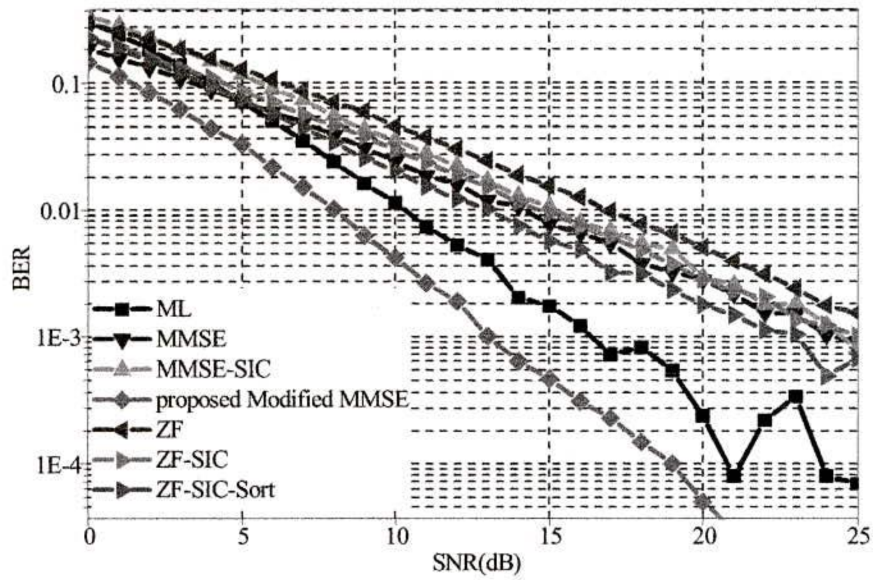

FIGURE 5.13: BER vs. SNR characteristics for 2 × 2 MIMO and QPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

Performance of proposed Modified MMSE equalizer over the existing optimum ML equalizers as well as other linear and non-linear equalizers has been described by the figure

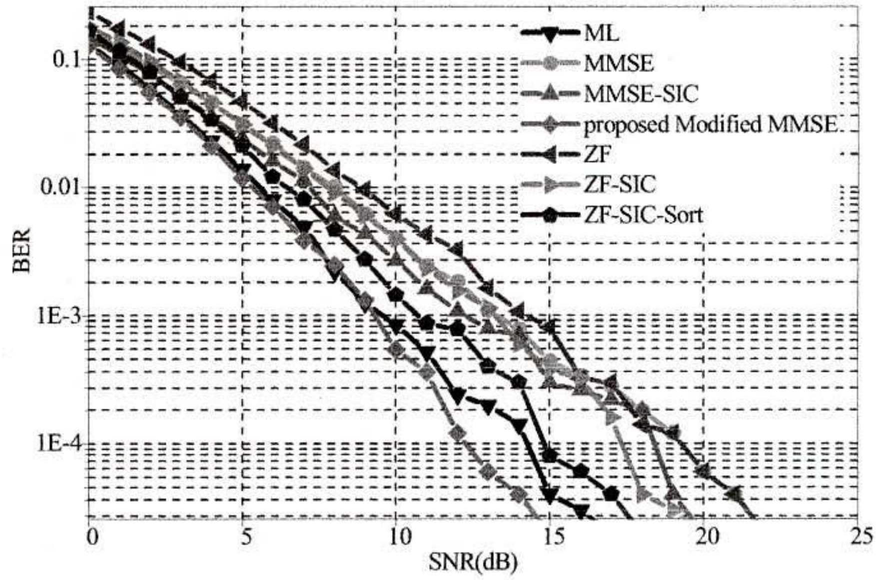

FIGURE 5.14: BER vs. SNR characteristics for 2 × 3 MIMO and QPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

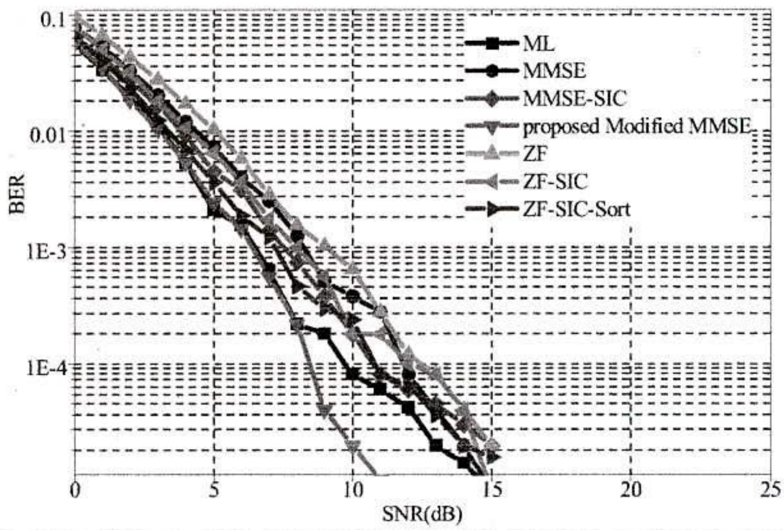

FIGURE 5.15: BER vs. SNR characteristics for 2 × 4 MIMO and QPSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

5.13, 5.14 and 5.15. QPSK modulation and 2×2, 2×3 and 2×4 MIMO configurations are applied for the performance analysis.From the graphical representations, it is obvious for all these configurations for proposed Modified MMSE the BER is lowest among all the equalizers that have been used for this simulation analysis.

5.11 BER vs. SNR Characteristics for 8-PSK Modulation

Fig 5.16, 5.17 and 5.18 represents the simulation performance of proposed Modified MMSE equalizer over other existing equalizers for 2×2, 2×3 and 2×4 MIMO wireless channel by 8-PSK modulation. In 8-PSK the performance of linear, non-linear and combination of both are implied so that we can predict the equalizer which is optimum among all of the equalizers.

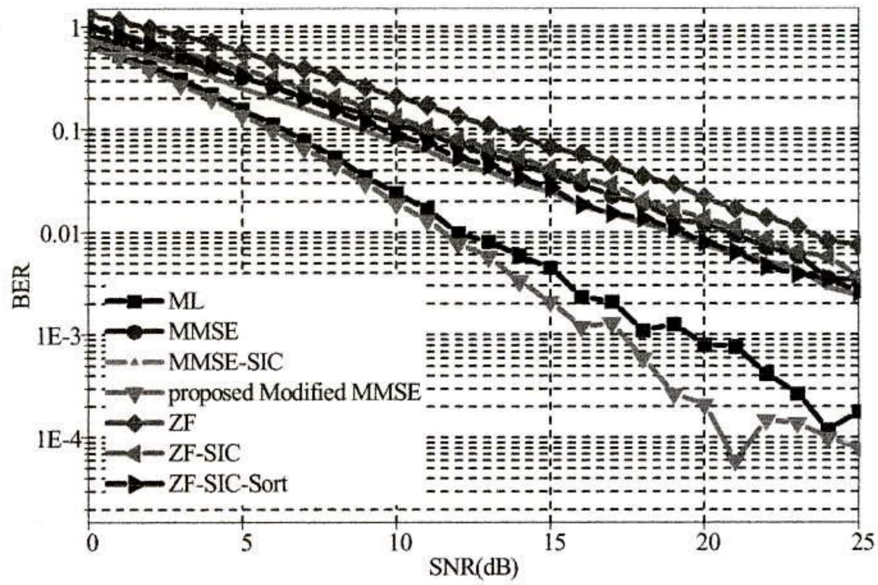

FIGURE 5.16: BER vs. SNR characteristics for 2×2 MIMO and 8-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

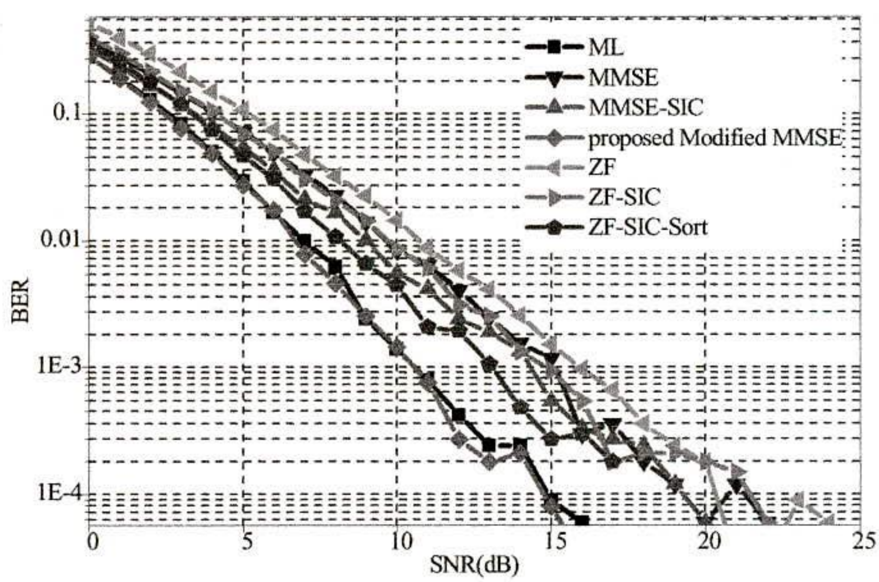

FIGURE 5.17: BER vs. SNR characteristics for 2×3 MIMO and 8-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

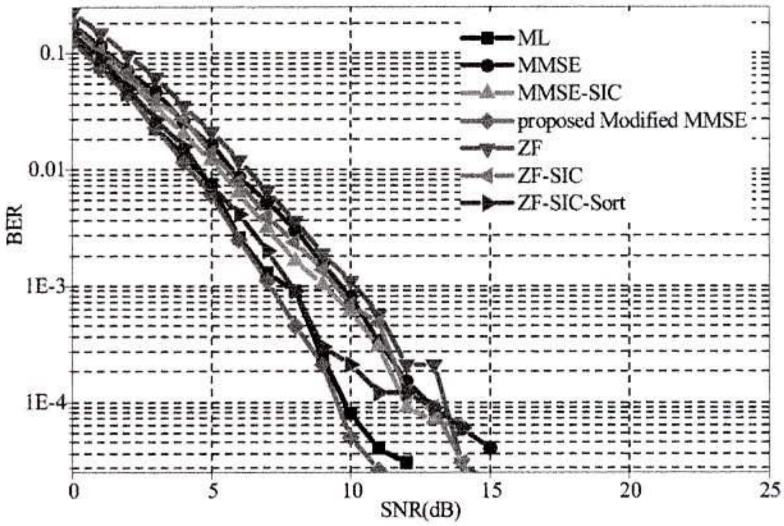

FIGURE 5.18: BER vs. SNR characteristics for 2 × 4 MIMO and 8-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

From the BER vs. SNR plot, BER is decreasing with increasing SNR and for proposed Modified MMSE the BER is approximately identical even less than that of ML equalizer. For higher order MIMO configurations the BER decreases swiftly than the lower order MIMO configurations.

5.12 BER vs. SNR Characteristics for 16-PSK Modulation

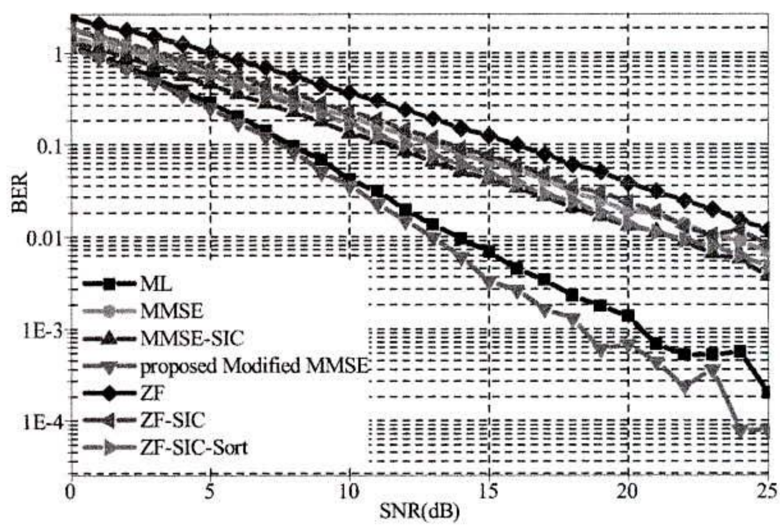

FIGURE 5.19: BER vs. SNR characteristics for 2 × 2 MIMO and 16-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

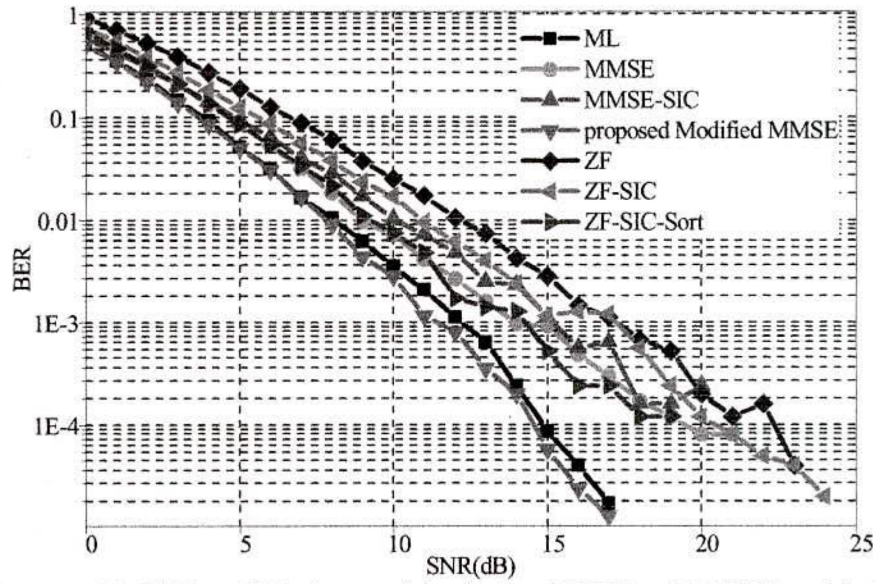

FIGURE 5.20: BER vs. SNR characteristics for 2 × 3 MIMO and 16-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

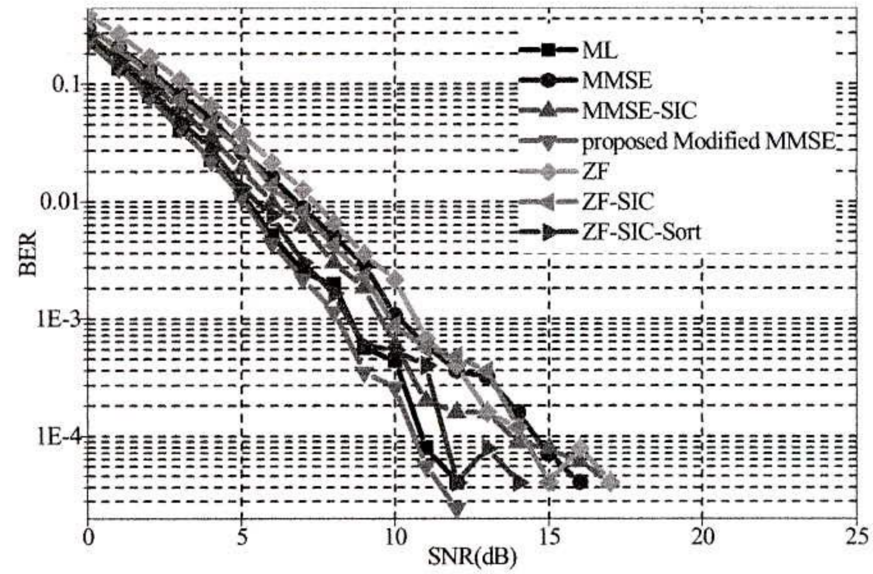

FIGURE 5.21: BER vs. SNR characteristics for 2 × 4 MIMO and 16-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

Fig 5.19, 5.20 and 5.21 represents the graphical analysis of proposed Modified MMSE equalizer over other equalizers for 16-PSK modulation. BER vs. SNR performance has been analyzed by using 2 × 2, 2 × 3 and 2 × 4 MIMO configurations. In case of 16-PSK, 16 phases are used to implement the simulation performance analysis. These phases are equispaced and their respective symbols are used to convey wireless signal.In these cases 4 bits are transmitted each time. From the BER vs. SNR plot, BER is decreasing with increasing SNR for all the equalizers. But for 16-PSK the BER rate of proposed Modified MMSE is least among all of the equalizers.

5.13 BER vs. SNR Characteristics for 32-PSK Modulation

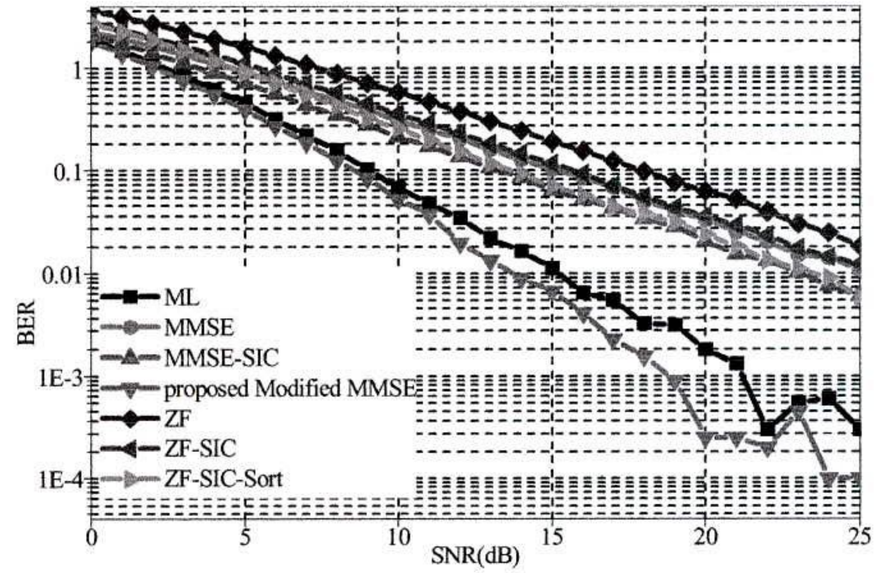

FIGURE 5.22: BER vs. SNR characteristics for 2 × 2 MIMO and 32-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

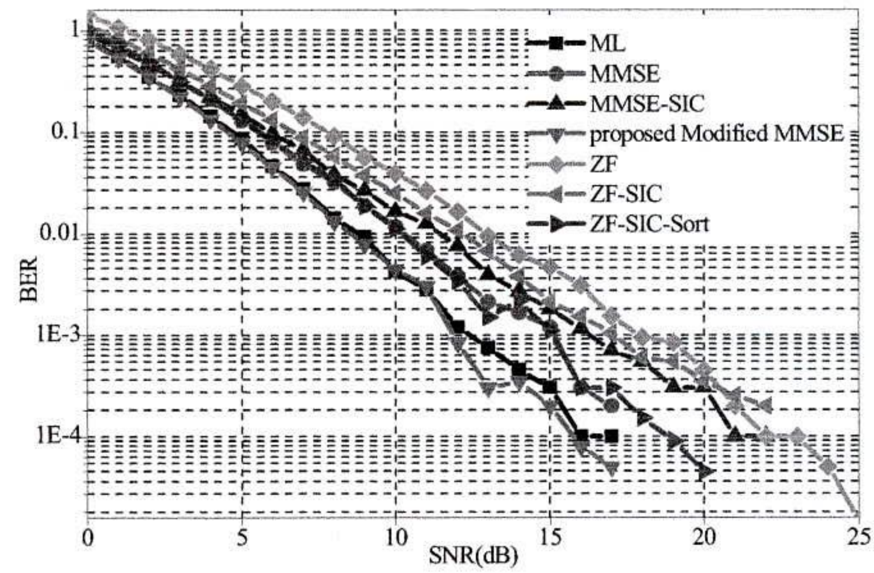

FIGURE 5.23: BER vs. SNR characteristics for 2 × 3 MIMO and 32-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

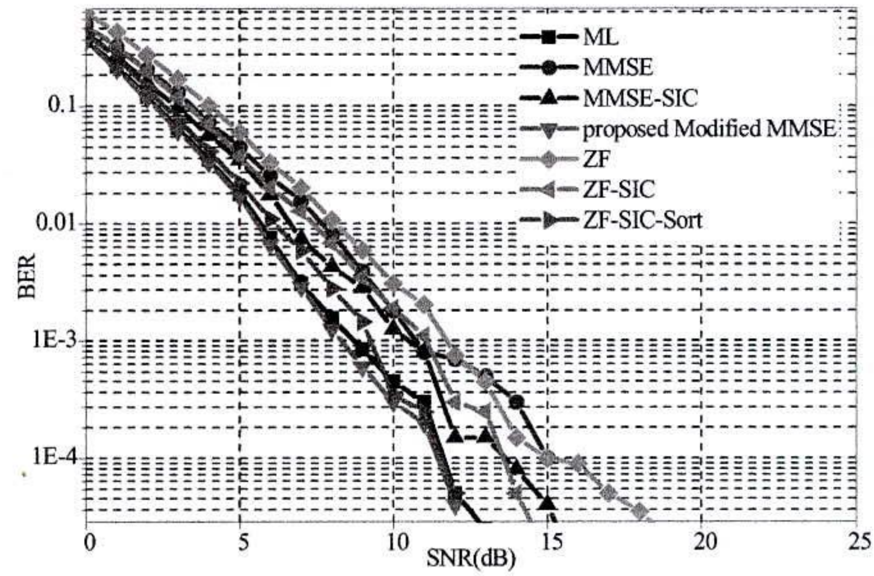

FIGURE 5.24: BER vs. SNR characteristics for 2 × 4 MIMO and 32-PSK modulation with proposed Modified MMSE equalizer over the existing equalizers.

Fig 5.22, 5.23 and 5.24 represents the BER vs. SNR analysis of proposed Modified MMSE equalizer over other equalizers for 2 × 2, 2 × 3 and 2 × 4 MIMO wireless channel by 32-PSK modulation. From the BER vs. SNR plot, BER is decreasing with increasing SNR and for proposed Modified MMSE the performance is better even that of ML equalizer.

TABLE 5.3: Diversity order for different equalizers at the expense of SNR loss.

Equalizer	Diversity Order	SNR Loss
ZF	$M_D - M_C+1$	High
ZF-SIC	$\approx M_D - M_C+1$	Low
ZF-SIC-Sort	$\approx M_D - M_C+1$	Low
MMSE	$\approx M_D - M_C+1$	Low
MMSE-SIC	$\approx M_D - M_C+1$	Low
Proposed Modified MMSE	$\geq M_D - M_C + 1, \leq M_D$	Zero
ML	M_D	Zero

Table 5.3 depicts the diversity that is achieved for different equalizers and it also shows that the SNR loss in the process of achieving high BER. Proposed Modified MMSE has lower diversity than ML equalizer while its performance exceeds ML.

5.14 BER vs. SNR Characteristics for Different Equalizers with Maximal Ratio Combining (MRC)

Figure 5.25, 5.26 and 5.27 below describes the BER vs. SNR characteristics of proposed Modified MMSE equalizer as well as other equalizers along with MRC are plotted. For different MIMO transmit-receive diversity and PSK modulation is used as implementation and simulation purpose. As proposed Modified MMSE is worked under MMSE with interference cancellation by ordered successive process, it has the same combination as 1TX-NRx so it finally reduces BER further by maximal ratio combining (MRC). The MRC completes the optimization target by reducing BER further and performance is optimized.

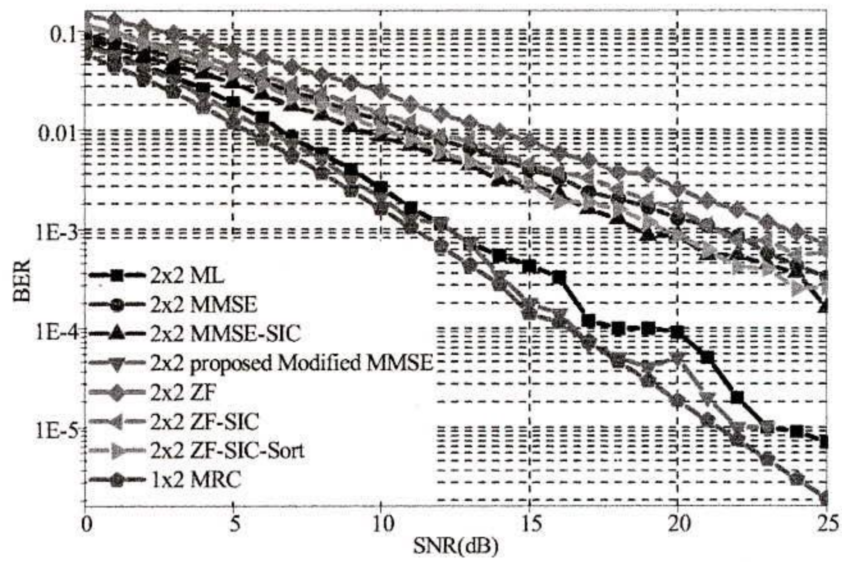

FIGURE 5.25: BER vs. SNR characteristic for BPSK modulation and different equalizers with MRC.

From all of the performance evaluation and graphical analysis, we are able to select an optimum equalizer which is different from existing optimum ML equalizer. The new optimum equalizer is that of modified version of MMSE equalizer which is expressed as proposed Modified MMSE. The proposed Modified MMSE outperforms all of the existing equalizers even its performance is greater than ML equalizer which is considered as optimal among the existing equalizers. From theoretical and practical analysis of present thesis, it is established that proposed Modified MMSE is the optimum equalizer that performs better than all the existing equalizers while keeping computation and implementation complexity simple. As proposed Modified MMSE is derived from MMSE, so

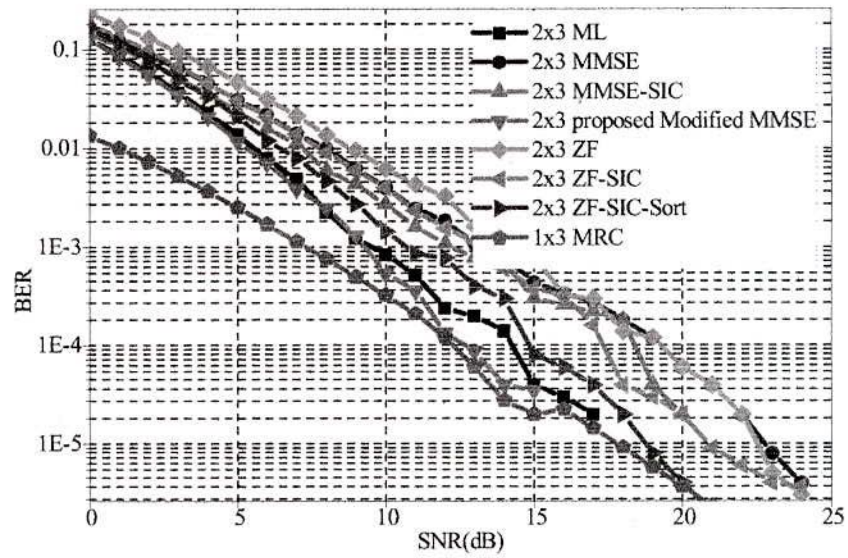

FIGURE 5.26: BER vs. SNR characteristic for BPSK modulation and different equalizers with MRC.

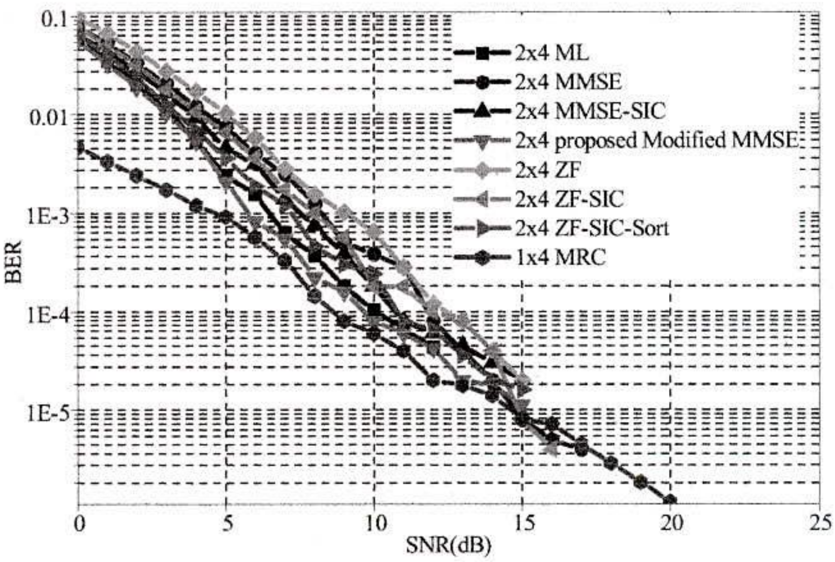

FIGURE 5.27: BER vs. SNR characteristic for BPSK modulation and different equalizers with MRC.

it incorporates a nonlinear process difference from that of ML equalizer which comprises exponential process. So its iteration process is less time consuming than that of ML. Hence, considering all the facts, it is recognized that Modified MMSE is the optimum among all of the existing equalizers for PSK modulation which is the outcome of this thesis.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, an idea about the optimum equalizer for MIMO wireless communication system at higher modulation levels and for different antenna configurations is presented. The performance are analysed in the forms of PSK modulation under Rayleigh fading environment. MIMO wireless system can be implemented using existing equalizers to achieve large data rate. But there is a problem that BER and implementation cost will not support to achieve desired performance at the same time i.e. if BER is lower for an specific equalizer then implementation cost will be higher for that equalizer which is the major cause of concern. This research proposes a novel equalizer that will offer optimum performance both aspects and increases data rate at the same time. The performance of MIMO wireless communication system are analysed via linear and non-linear equalizers to design optimum equalizer for MIMO channel. The linear equalizers are ZF and MMSE and non-linear equalizers include ML, ZF-SIC, ZF-SIC-Sort, MMSE-SIC and proposed Modified MMSE. The performances are analysed for 2×2, 2×3 and 2×4 MIMO configurations and for different PSK-modulation. The PSK modulation orders that are applied for selection of optimum equalizer are BPSK, QPSK, 8-PSK, 16-PSK and 32-PSK. ZF equalizer removes all ISI and is ideal only when the channel is noiseless. When the channel is noisy, it has a tendency to amplify the noise and is much suited for static channels with high SNR. ML equalizer has superior performance over the existing equalizers but it has high computational complexity. On the contrary, MMSE is a balanced linear equalizer and minimizes the total noise power and ISI components in the output but it does not eliminate ISI completely. So a novel equalizer is required that will perform superior regarding both aspects i.e. in terms of BER and computational complexity over the existing equalizers. This thesis develops a novel equalizer which is termed as Modified MMSE that outperforms even ML equalizer in terms of data rate and BER performance characteristics. While both equalizers are non-linear but ML equalizer tends to be exponential with higher transmitting antennas and higher order modulation. The complexity of the proposed Modified MMSE equalizer reduces exponential to linear. So proposed Modified MMSE is the optimum equalizer that offers superior performance over the existing equalizers for MIMO wireless communication system in presence of increasing number of bits in a signal.

6.2 Future Work

In this research, we have studied about optimum equalizer for MIMO wireless communication system using PSK modulation. However, in MIMO wireless communication due to the increasing number of bits in a signal, the challenge is that the optimum performance as well as reduced complexity is not always achievable at the same time. As spatial diversity and spatial multiplexing are the two major issues of MIMO wireless communication which can impede the increment of high data at the same time keeping reduced complexity. As a result, how the BER can be increased further by newer equalizer that may be an active area of investigation. At the same time finding newer optimal technique that will exceed that of Modified MMSE equalizer with regard to performance criteria may be an area of further research.

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List of Publications

Referred International Journals

1. Sabuj Sarkar and Mohammad Shaifur Rahman, "Optimal BER in MIMO Rayleigh Fading Channel from QPSK Modulation: Modified MMSE Versus ML Equalizer Evaluation," *International Journal of Electronics & Informatics (IJEI)*, ISSN: 2186-0114, vol. x, no. x, pp. xxx-xxx, 2013.(Accepted for Publication)
2. Sabuj Sarkar and Mohammad Shaifur Rahman, "An Unique Equalizer in MIMO Wireless Multipath Communication Channel: BER Optimization by Modified MMSE Equalizer," *IOSR Journal of Electrical and Electronics Engineering (IOSRJEEE)*, India, ISSN: 2278-1676, vol. x, no. x, pp. xx-xx, Feb. 2013. (Accepted for Publication)
3. Sabuj Sarkar and Mohammad Shaifur Rahman, "A Novel Equalizer to Optimize BER in MIMO Wireless Multipath Fading Channel: An Extensive Study of Modified MMSE vs. Existing Equalizers," *International Journal of Emerging Sciences (IJES)*, ISSN: 2222-4254, vol. x, no. x, pp. xxx-xxx, 2013.(Submitted)

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