Dedicated to

MY PARENTS

MY REGARDS
for them
is viscous and incompressible

DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
BANARAS HINDU UNIVERSITY

VARANASI-221 005

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In forwarding Mrs. Fouzia Rahman's thesis entitled "On MHD flows of viscous incompressible fluids" supplicated in fulfilment of the requirements for the degree of Doctor of Philosophy in Mathematics of the Banaras Hindu University, I hereby certify that she has completed the research work for the full period prescribed under clause 1 of the ordinances governing the award of the degree and that the thesis embodies the results of her investigations conducted during the period she worked as a research scholar under my supervision.

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PREFACE

The present thesis entitled "ON MHD FLOWS OF VISCOUS INCOMPRESSIBLE FLUIDS" is being presented for the award of the degree of Doctor of Philosophy in Mathematics. It is the outcome of my researches conducted in the Department of Mathematics, Banaras Hindu University during the years 1983-86 under the esteemed guidance of Dr. Newal Kishore, Reader in the Department of Mathematics, Banaras Hindu University, Varanasi, India.

The whole thesis consists of six chapters. The first chapter is introductory, giving the general description and fundamental equations of magnetohydrodynamics, free convection flow, flow through porous media, rotating fluid flow, oscillatory flow and flow with Hall currents. Lastly, a brief review of the past researches related to the thesis have been given. Throughout the work we are considering the flows of electrically conducting, viscous and incompressible fluids. The magnetic Reynolds number is assumed small for all the problems except the problems discussed in chapter two.

The second chapter has been divided into parts. Part A of this chapter deals with the flow between two infinite, non-conducting, parallel porous flat plates, when the lower plate is injecting fluid and the upper one is absorbing it. The flow is subjected to a uniform transverse magnetic field and the magnetic Reynolds number of the flow is sufficiently large so as to include the effect of induced magnetic field. The expressions for the velocity and induced magnetic fields have been obtained by using Laplace transform technique. The effect of the magnetic parameter M on the velocity and induced magnetic field has been studied. It is found that the velocity decreases with increase in M in the lower region between the plates and increases with increase in M in the upper region. The induced magnetic field decreases with increase in M. In part B of this chapter, the effect of a uniform transverse magnetic field on unsteady MHD free convective flow past an impulsively started infinite vertical non-conducting plate has been discussed. Here also, the magnetic Reynolds number is assumed to be sufficiently large to take account of the induced magnetic field. There is constant heat flux at the plate. Expressions for the velocity and induced magnetic have been obtained by Laplace transform technique. The effect of the different parameters on the flow have been discussed with the help of tables.

In part A of the third chapter, the effect of a uniform transverse magnetic field on the steady free convective flow through a porous medium, occupying a semi-infinite region of space and bounded by a steadily moving vertical porous plate has been studied. The flow is subjected to constant suction. Approximate solutions to the equations relevant to the problem have been obtained. The influence of the different parameters on the velocity and temperature fields have been discussed with the help of graphs and tables.

The problem considered in part B of this chapter is an extension of the problem considered in part A. Here, we have taken into account the effect of rotation on the flow. Due to rotation the flow becomes three dimensional. Approximate solutions to equations relevant to the problem have been obtained. Effects of the various parameters on the primary velocity, secondary velocity, the components of skin friction and the temperature have been discussed.

The fourth chapter is concerned with the unsteady free convective flow past an impulsively started infinite vertical porous plate in presence of a uniform transverse magnetic field. The free stream is assumed to oscillate in time about a constant mean. The flow is subjected to constant suction velocity and there is constant heat flux at the plate. Approximate solutions for the mean flow and transient flow have been obtained and the results have been discussed with the help of tables and graphs.

In the fifth chapter we have studied the effects of Hall currents on the unsteady MHD free convective flow past an impulsively started infinite vertical porous plate in presence of a uniform transverse magnetic field. The plate temperature is assumed to oscillate in time about a constant mean and the flow is subjected to constant suction at the plate. Approximate solutions for the mean flow and transient flow have been obtained. The influence of the various parameters on the mean and transient flows has been discussed with the help of tables and graphs.

In the last chapter, an attempt has been made to study the effects of rotation and Hall currents on the unsteady MHD free convective flow through a porous medium occupying a semi-infinite region of space and bounded by an infinite vertical porous plate in presence of a transversely applied uniform magnetic field. The plate is assumed to oscillate in time about a constant mean and there is constant heat flux at the plate. Approximate solutions for the mean flow and transient flow have been obtained and the results have been discussed with the help of graphs and tables.

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I have great pleasure to record my sincere and hearty gratitude to Dr. Newal Kishore, Reader, Department of Mathematics, Banaras Hindu University, under whose esteemed guidance I am privileged to work. I am greatly indebted to him for his valuable suggestions, personal cordiality and geneorisity that he bestowed upon me throughout the progress of this work.

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CONTENTS

CHAPTER – I

INTRODUCTION

Magnetohydrodynamics

Magnetohydrodynamics is that branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. It combines in a common framework the electromagnetic and fluid dynamic theories, to yield a description of the concurrent effects of the magnetic field on the flow and the flow on the magnetic field. Magnetohydrodynamic (MHD) phenomena result from the mutual effect of a magnetic field and a conducting fluid flowing across it. Thus, an electromagnetic force is produced in a fluid flowing across a transverse magnetic field, and the resulting current and magnetic field combine to produce a force that resists the fluid's motion. The current also generates its own magnetic field which distorts the original magnetic field.

Faraday[1] (1832) carried out experiments with the flow of mercury in glass tubes placed between poles of a magnet, and discovered that a voltage was induced across

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1. Faraday, M. Experimental researches in Electricity Phil., Trans. vol. 15, p. 175 (1832).

the tube due to the motion of the mercury across the magnetic field, perpendicular to the direction of flow and to the magnetic field. He observed that the current generated by this induced voltage interacted with the magnetic field to slow down the motion of the fluid, and this current produced its own magnetic field that obeyed Ampere's right hand rule and thus, in turn distorted the magnetic field.

The first astronomical application of the MHD theory occurred in 1899 when Bigalow suggested that the sun was a gigantic magnetic system. Alfven[1] (1942) discovered MHD waves in the sun. These waves are produced by disturbances which propagate simultaneously in the conducting fluid and the magnetic field.

The current trend for the application of magneto-fluid dynamics is toward a strong magnetic field (so that the influence of electromagnetic force is noticeable) and toward a low density of the gas (such as in space flight and in nuclear fusion research). Under these conditions the Hall current and ion slip become important.

Electromagnetic Equations:

Magnetohydrodynamic equations are the ordinary

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1. Alfven,H., On the existance of Electromagnetic- Hydrodynamic Waves, Arkiv F.Mat.Astro. O.Fysik Bd. Vol.29B,No.2, (1942).

electromagnetic and hydrodynamic equations modified to take account of the interaction between the motion of the fluid and electromagnetic field; formulation of electromagnetic theory in mathematical form is known as Maxwell's equations. Maxwell's basic equations show the relations of basic field quantities and their production. The basic laws of electromagnetic theory are all contained in special theory of relativity. But here we will always assume that all velocities are small in comparison with the speed of light.

Before writing down the MHD equation we should first of all know the ordinary electromagnetic equations and hydrodynamic equations.

First, we give the electromagnetic equations:[1]

Charge Continuity:

1.1 $\nabla . \bar{D} = \rho_e$

1.2

Current Continuity:

1.2 $\nabla . \bar{J} = -\frac{\partial \rho_e}{\partial t}$

Magnetic Field Continuity:

1.3 $\nabla . \bar{B} = 0$

Ampere's Law:

1.4 $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

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1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers and applied physicists. McGraw Hill Book Company, (1973), p. 38.

Faraday's Law

1.5 $$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

Constitutive Equations for $\overline{D}$ and $\overline{B}$

1.6 $$\overline{D} = \varepsilon \overline{E}$$

1.7 $$\overline{B} = \mu_e \overline{H}$$

Lorentz force on a charge

1.8 $$\overline{F}_P = q\,(\overline{E} + \overline{V}_P \times \overline{B})$$

Current Density

1.9 $$\overline{J} = \sigma\,(\overline{E} + \overline{V} \times \overline{B}) + \rho_e \overline{V}$$

The first five are the Maxwell's equations.

where

$\overline{D}$	electric displacement
ρ_e	charge density
$\overline{E}$	electric field
$\overline{H}$	magnetic field
$\overline{B}$	magnetic induction
$\overline{J}$	current density
$\frac{\partial \overline{D}}{\partial t}$	displacement current density
ε	the electrical permitivity of the medium
μ_e	the magnetic permeability of the medium
$\rho_e \overline{V}$	the convection current due to charges moving with the fluid.

$\overline{V}_P$ velocity of the charge

$\overline{V}$ velocity field.

Fundamental Equations of Fluid dynamics of Viscous Fluids[1]

In the study of fluid flow one determines the velocity distribution as well as the states of the fluid over the whole space for all time. There are six unknowns namely, the three components of velocity $\overline{V}$ (u, v, w), the temperature T the pressure p and the density ρ of the fluid, which are functions of spatial co-ordinates and time. In order to determine these unknown we have the following equations.

(a) Equation of state which connects the temperature, the pressure and the density of the fluid.

1.10 $$p = R \rho T$$

For an incompressible fluid the equation of state is simply

1.11 $$\rho = \text{constant}$$

(b) Equation of continuity which gives the relation of conservation of mass of the fluid. The equation of continuity for a viscous incompressible fluid is,

1.12 $$\nabla \cdot \overline{V} = 0$$

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1. Pai, S.I. Viscous flow Theory 1-Laminar flow, D.Van Nos-

(c) Equations of motion, known as the Navier-Stokes equations and which give the relations of the conservation of momentum of the fluid.

For a viscous incompressible fluid the equation of motion is

1.13 $$\rho \frac{\overline{D}\,\overline{V}}{D t} = \overline{F} - \nabla p + \mu \nabla^2 \overline{V}$$

where $\overline{F}$ is the body force per unit volume and the last term on the right hand side represents the force per unit volume due to viscous stresses and p is the pressure.

The operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

This is known as the material derivative or total derivative with respect to time, and it gives the variation of a certain quantity of the fluid particle with respect to time.

∇^2 is the Laplacian operator.

(d) Equation of energy which gives the relation of conservation of energy of the fluid.

For an incompressible fluid with constant viscosity and heat conductivity the energy equation is

1.15 $$\rho C_p \frac{DT}{Dt} = \frac{\partial Q}{\partial t} + k \nabla^2 T + \phi$$

where,

$\frac{\partial Q}{\partial t}$ is the rate of heat produced per unit volume by external agencies,

C_p is the specific heat at constant pressure,

k is the thermal conductivity of the fluid and ϕ is the dissipation function.

For an incompressible fluid

$$1.16 \quad \phi = 2\mu \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{1}{2}\left(Y_{xy}^2 + Y_{yz}^2 + Y_{zx}^2\right)\right]$$

where,

$$Y_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$Y_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$Y_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

MHD Approximations:[1]

The electromagnetic equations as given from 1.1 – 1.9 are not usually applied in their present form and require interpretation and several assumptions to provide the set to be used in MHD. In MHD we consider a fluid that is grossly neutral. The charge density ρ_e in Maxwell's equations must

1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers and applied physicists McGraw Hill Book Co. (1973), p.72.

then be interpreted as an excess charge density which is generally not large. If we disregard the excess charge density then we must disregard the displacement current. In most problems the displacement current, the excess charge density and the current due to convection of the excess charge are small.

The electromagnetic equations to be used are then the following:

1.17 $$\nabla \cdot \bar{D} = 0$$

1.18 $$\nabla \cdot \bar{J} = 0$$

1.19 $$\nabla \cdot \bar{B} = 0$$

1.20 $$\nabla X \bar{H} = \bar{J}$$

1.21 $$\nabla X \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

1.22 $$\bar{D} = \varepsilon \bar{E}$$

1.23 $$\bar{B} = \mu_e \bar{H}$$

1.24 $$\bar{J} = \sigma (\bar{E} + \bar{V} X \bar{B})$$

<u>MHD Equations:</u>

We shall now modify the equations of fluid dynamics suitably to take account of the electromagnetic phenomena.

(a) The MHD equation of continuity for viscous incompressible electrically conducting fluid remains the same

1.25 $$\nabla . \bar{V} = 0$$

(b) The MHD momentum equation for a viscous incompressible and electrically conducting fluid is[1]

1.26 $$\rho \frac{D\bar{V}}{Dt} = \bar{F} - \nabla p + \mu \nabla^2 \bar{V} + (\bar{J} \times \bar{B})$$

where $\bar{F}$ is the body force term per unit volume and the new term $\bar{J} \times \bar{B}$ is the force on the fluid per unit volume produced by the interaction of the current and the magnetic field (called a $\bar{J} \times \bar{B}$ force or Lorentz force).

The MHD energy equation for a viscous incompressible electrically conducting fluid is

1.27 $$\rho C_p \frac{DT}{Dt} = \frac{\partial Q}{\partial t} + k \nabla^2 T + \phi + \frac{J^2}{\sigma}$$

The new term $\frac{J^2}{\sigma}$ is the Joule heating and is due to the resistance of the fluid to the flow of current.

From equations 1.20, 1.21 and 1.24 we have an equation for the magnetic field viz.

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1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers and applied physicists, McGraw Hill Book Co. (1973), p.73.

$$\frac{\partial \bar{H}}{\partial t} = \nabla \times (\bar{V} \times \bar{H}) - \nu_H \nabla \times (\nabla \times \bar{H})$$

1.28 or, $$\frac{\partial \bar{H}}{\partial t} = (\bar{H}.\nabla)\ \bar{V} - (\bar{V}.\nabla)\ \bar{H} + \nu_H \nabla^2 \bar{H}$$

$(\because\ \nabla.\bar{H} = o$ and $\nabla.\bar{V} = o$ for incompressible fluid)

where $\nu_H = \dfrac{1}{\mu_e \sigma}$ is the magnetic diffusivity.

In some problems it is of interest to write the MHD momentum and energy equations in terms of the magnetic field, hence eliminating J from 1.26 and 1.27 we get respectively.

1.29 $$\rho \frac{D\bar{V}}{Dt} = \bar{F} - \nabla p + \mu \nabla^2 \bar{V} + \mu_e (\bar{H}.\nabla)\ \bar{H} - \frac{1}{2} \mu_e \nabla (H^2) .$$

and

1.30 $$\rho C_p \frac{DT}{Dt} = \frac{\partial Q}{\partial t} + k \nabla^2 T + \phi + (\nabla \times \bar{H}).[\nu_H \mu_e (\nabla \times \bar{H}) - \mu_e \bar{V} \times \bar{H}]$$

THE IMPORTANT NON-DIMENSIONAL PARAMETERS OF FLUID DYNAMICS AND MAGNETOHYDRODYNAMICS

We define here some important non-dimensional parameters used in the present investigation:

Reynolds Number Re.

It is the most important parameter of fluid dynamics of a viscous fluid. It represents the ratio of the inertial force to viscous force and is defined as

1.31 $$R_e = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho U^2 L^2}{\mu U L} = \frac{UL}{\nu}$$

where U, L, ρ and μ are the characteristic values of velocity, length, density and coefficient of viscosity of the fluid respectively. When the Reynolds number of the system is small the viscous force is predominant and the effect of viscosity is important in the whole velocity field. When the Reynolds number is large the inertial force is predominant, and the effect of viscosity is important only in a narrow region near the solid wall or other restricted region which is known as boundary layer. If the Reynolds number is enormously large, the flow becomes turbulent.

Prandtl number Pr:

The Prandtl number is the ratio of kinematic viscosity to thermal diffusivity and may be written as follows

1.32 $$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{\frac{k}{\rho C_p}}$$

The value of ν shows the effect of viscosity of the fluid. The smaller the value of ν is, the narrower is the region which is affected by viscosity and which is known as the boundary layer region when ν is very small. The value of $\frac{k}{\rho C_p}$ shows the thermal diffusivity due to heat conduction.

The smaller the value of $\frac{k}{\rho C_p}$ is, the narrower is the region which is affected by heat conduction and which is known as thermal boundary layer when $\frac{k}{\rho C_p}$ is small. Thus the Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. For a gas the Prandtl number is of order of unity.

Peclet number Pe

Peclet number is defined as:

$$1.33 \qquad P_e = \frac{UL\, C_p \rho}{k} = P_r \cdot R_e$$

It is the product of Reynolds number and Prandtl number.

Eckert Number E.

The Eckert number can be interpreted as the addition of heat due to viscous dissipation and is very small for incompressible fluid and for low motion. It may be defined as follows:

$$1.34 \qquad E = \frac{U^2}{C_p (T_w - T_\infty)}$$

where U is some reference velocity and $T_w - T_\infty$ is the difference between two reference temperatures.

Magnetic Pressure Number R_H:

It is the ratio of the magnetic pressure to the dynamic pressure and is given by

$$R_H = \frac{\mu_e H_o^2}{\rho U^2} \tag{1.35}$$

It is a measure of the effect of the magnetic field on the fluid. Only when R_H is the order of unity, will the flow be influenced noticeably by the magnetic field, and if it is very small, all the magnetic effects can be disregarded.

Magnetic Reynolds Number R_σ:

It is the ratio of the fluid flux to the magnetic diffusivity and is given by

$$R_\sigma = \frac{UL}{\frac{1}{\mu_e \sigma}} \tag{1.36}$$

It is one of the most important parameters of MHD. The magnetic Reynolds number determines the diffusion of the magnetic field along the streamlines. R_σ is a measure of the effect of the flow on the magnetic field. If it is very small compared to unity, the magnetic field is not distorted by the flow. When it is very large the magnetic field moves with the flow and is called frozen in.

Magnetic Number R_m:

It is the ratio of the magnetic force to the inertial force and is given by

1.37 $$R_m = \mu_e H_o \left(\frac{\sigma L}{\rho U} \right)^{1/2} = \left(R_H R_\sigma \right)^{1/2}$$

When R_σ is very small R_m is also used to measure the electro-magnetic effects on the flow.

Magnetic Prandtl Number P_m

The magnetic Prandtl number is the ratio of the viscous diffusivity to the magnetic diffusivity and is given by

1.38 $$P_m = \frac{\nu}{\nu_H} = \frac{R_\sigma}{R_e}$$

P_m is generally small and is a measure of the relative magnitude of the fluid boundary layer thickness to the magnetic boundary layer thickness. However when the magnetic Reynolds number is large, the magnetic boundary layer thickness is small and is of nearly the same size as the viscous boundary layer thickness.

In this case P_m is not small.

MHD Boundary Layer Assumptions:[1]

Boundary layer phenomenon occurs when the influence of a physical quantity is restricted to small regions near confining boundaries. This phenomenon occurs when the non-dimensional diffusion parameters - the Reynolds number, Peclet number or magnetic Reynolds number are large. The boundary layers are then the velocity and thermal or magnetic boundary layers, and each thickness is inversely proportional to the square root of the associated diffusion number. Prandtl fathered classical fluid-dynamic boundary theory by observing, from experimental flows, that for large Reynolds number, the viscosity and thermal conductivity appreciably influenced the flow only near a wall. When distant measurements in the flow direction are compared with a characteristic dimension in that direction, transverse measurements compared with the boundary layer thickness, and velocities compared with the free stream velocity, the Navier-Stokes and energy equations can be considerably simplified by neglecting small quantities. The number of component equations is reduced to those in the flow direction and pressure changes across the boundary layer are negligible. The pressure is then only a function of the flow direction and can be determined from the inviscid flow solution. Also the number of viscous terms is reduced to the

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1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers and applied physicists, McGraw Hill Book Co. (1973), p.141.

dominant term and the heat conduction in the flow direction in negligible.

MHD boundary layer flows are separated into two types by considering the limiting cases of a very large or a negligibly small magnetic Reynolds number. When the magnetic field is oriented in an arbitrary direction relative to a confining surface and the magnetic Reynolds number is very small, the flow direction component of the magnetic interaction and corresponding Joule heating is only a function of the transverse magnetic field component and the local velocity in the flow direction. Changes in the transverse magnetic field component and pressure across the boundary layer are negligible. The thickness of the magnetic boundary layer is very large and the induced magnetic field is negligible. However, when the magnetic Reynolds number is very large, the magnetic boundary layer thickness is small and is of nearly the same size as the viscous and thermal boundary layers and then the MHD boundary layer equations must be solved simultaneously. In this case, the magnetic field moves with the flow and is called frozen in.

Two Dimensional Flow:

If the velocity distribution in a moving fluid depends on only two coordinates (x and y say) and the velocity is everywhere parallel to the x-y plane, the flow is said to be two dimensional.

The MHD Boundary Layer Equations for two-dimensional Flow in case of Small Magnetic Reynolds Number:

With constant fluid properties, transversely applied uniform magnetic field H_o and x-axis along the direction of flow, the MHD boundary-layer equations for incompressible fluid flow under the boundary layer assumptions are as follows:[1]

1.39 $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

1.40 $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}\mu_e^2 H_o^2 u$$

1.41 $$\frac{\partial p}{\partial y} = 0$$

1.42 $$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma\mu_e^2 H_o^2 u^2$$

MHD Boundary-Layer Equations for Two-dimensional Flow in case of Large Magnetic Reynolds Number:

When the magnetic Reynolds number is large we cannot neglect the induced magnetic field. With constant fluid properties, transversely applied uniform magnetic field H_o and x-axis along the flow direction the MHD boundary layer equations for incompressible fluid under the boundary layer assumptions are[2] as follows:

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1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers applied Physicists, McGraw Hill Book Co. (1973), p. 149.
2. Pai, S.I. Magnetogasdynamics and Plasma dynamics Wein Springer Verlog (1962), p.67.

1.43 $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

1.44 $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\mu_e}{\rho} H_o\frac{\partial H_x}{\partial y}$$

1.45 $$\frac{\partial}{\partial y}\left(p + \mu_e\frac{H_x^2}{2}\right) = 0$$

1.46 $$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\sigma\rho C_p}\left(\frac{\partial H_x}{\partial y}\right)^2$$

1.47 $$\frac{\partial H_x}{\partial t} + u\frac{\partial H_x}{\partial x} + v\frac{\partial H_x}{\partial y} = H_x\frac{\partial u}{\partial x} + H_o\frac{\partial u}{\partial y} + \nu_H\frac{\partial^2 H_x}{\partial y^2}$$

where H_x is the induced magnetic field.

Suction and Injection:

For ordinary boundary layer flows with adverse pressure gradients, the boundary layer flow will eventually separate from the surface. Separation of the flow causes many undesirable features over the whole field; for instance if separation occurs on the surface of an airfoil, the lift of the airfoil will decrease and drag will enormously increase. In some problems we wish to maintain laminar flow without separation. Various means have been proposed to prevent the separation of boundary layer flows, suction and injection are two of them.

Injection of Fluid:

Fluid is injected from the body into the boundary layer so as to increase the kinetic energy of the fluid in the boundary layer and to delay the separation.

Suction:

The retarded fluid in the boundary layer is sucked into the body. The point of suction is near the point of separation, either slightly ahead or behind so that no back flow will occur.

Suction is a very effective means for avoiding separation. Suction of the fluid along the surface of the body is able to keep the boundary layer laminar, because the boundary layer is kept so thin that the transition from a laminary boundary layer flow to a turbulent one is avoided.

Free and Forced Convection:[1,2]

The problem of heat transfer due to convection may be divided into two cases, free convection and forced convection. By free convection we mean flows in which the motion is caused by the effect of gravity on heated fluids of variable density, by forced convection we mean flows in which the velocities arising from variable density distribution,

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1. Pai, S.I. Viscous Flow theory 1- Laminar flow D.Van Nostrand Company Inc (1956),p.99.
2. Tritton,D.J. Physical Fluid Dynamics. Van Nostrand Reinhold company Ltd. (1979), p. 127.

arising from the effect of gravity are negligible in comparison with the velocities of the main or forced flows . The temperature variations within a convective flow give rise to variations in the properties of the fluid. An analysis including the full effects of these is so complicated. that some approximations become essential. The equations are commonly used in a form known as Boussinesq approximation. In the Boussinesq approximation, variations of all fluid properties other than the density are ignored completely. Variations of the density are ignored except in so far they give rise to a gravitational force, i.e. the density variation with temperature is considered only in the body force term, the influence of density variations in other terms of the momentum and energy equations are considered negligible.

In free convection, a body force term viz.

$$F_e = g \rho \beta (T-T_o)$$

appears in the equations of motion where g is the acceleration due to gravity, β is the coefficient of thermal expansion and $T-T_o$ is the excess temperature of the heated parts of the fluid over the parts which remain cold. The non-dimensional parameter characterizing free convection is known as Grashoff number and may be defined as

1.48 $$G = \frac{\nu g \beta (T-T_o)}{U_o^3}$$

where T, T_o are two representative temperatures and U_o is some characteristic velocity.

The Boundary Layer Equations of Motion of MHD Free Convection Flow:

The continuity and energy equations remain the same in cases of free and forced convection. In free convection flow we have a body force term in the momentum equation.

The two dimensional boundary layer momentum equation of MHD steady free convection flow[1] in absence of pressure gradient is

1.49 $$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T-T_\infty) - \frac{\sigma B_o^2 u}{\rho}$$

where the flow is in the x-direction and magnetic field is acting along y-direction.

1. Cramer, K.R. and Pai, S.I. Magnetofluid dynamics for engineers and applied Physicists, McGraw Hill Book Co., p. 165 (1973).

Porous Medium:

One may be tempted to define Porous Media as solid bodies that contain "pores" it being assumed as intuitively quite clear what is meant by a pore. However it is much more difficult to give an exact geometrical definition of what is meant by the notion of a pore. A special effort must therefore be made to obtain a proper description.

Intuitively "pores" are void spaces which must be distributed more or less frequently through the material if the latter is to be called "porous". Extremely small voids in a solid are called "molecular interstices" very large ones are called "caverns" Pores are void spaces intermediate between caverns and molecular interstices; the limitation of their sizes is therefore intuitive and rather indefinite.

The pores in a porous medium may be inter-connected or non-interconnected. Flow of fluids through porous medium is possible only if at least part of the pore space is inter-connected. The interconnected part of the pore system is called the effective pore space of the porous medium.

According to the above description the following are some examples of porous media:

towers packed with pebbles, porous rocks such as lime stone, fibrous aggregates such as cloth, filter paper etc. and finally catalytic particles, containing extremely fine 'micro' pores.

A Model of Flow Through a Porous Medium:

Bear and Bachmat[1] proposed a model of flow through a porous medium in which the restriction of the fluid transport, in well defined channels is an essential feature; because of the immediate presence of the walls of the solid matrix, the velocity of a fluid particle at a point in the void space is essentially in the direction parallel to the walls, and not normal to them. They visualise the void space of a porous medium as composed of a spatial network of interconnected random passages (Channels or tubes) and junctions. Channels are of varying length, cross-section and orientation, a junction is a place where channels meet.

Permeability:

Permeability is the term used for conductivity of the porous medium with respect to permeation by a Newtonian fluid. This is a property that measures the ability of the porous medium to transmit fluid through it.

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1. Bear, I. and Bachmat, Y.: IASH Symp. Artificial Recharge and Management of Aquifers Haifa, Israel, IASH, p.72(1967).

Darcy's Law:

Darcy[1,2] described his experiments on the seepage of water through sand. The experiments were on unidirectional flows only, and the main result is that the mean velocity is directly proportional to permeability K of the medium having dimension of area, to the grad of $-(p + \rho\Omega)$, where Ω is the potential of grivational attraction on sand and equal to g times elevation, and inversely proportional to the viscosity μ of the fluid. Generalized to three-dimensional flow Darcy's law has the Cartesian form

$$u_i = -\frac{K}{\mu} \frac{\partial}{\partial x_i} (p + \rho\Omega) \tag{1.50}$$

provided ρ is constant. It should be emphasized that u_i is the ith component of the meanvelocity taken over a volume containing many grains of the porous material.

Equations of Motion of Viscous Incompressible Fluid Through Porous Medium:

The porous medium is in fact a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid which has dynamical

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1. Darcy, H.P.G.: Les Fontaines Publique de la ville de Dijon Paris (1956).
2. ———: Researches experimentates relatives an movement de leau dans les tuyaux, Paris (1957).

properties equal to the local averages ofthe original non-homogeneous continuum. Hence one can study the flow of a hypothetical homogeneous fluid under the action of properly averaged external forces and so, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some additional resistance.

The MHD equation of motion for a viscous incompressible electrically conducting fluid through a porous medium is

1.51 $$\rho \frac{D\bar{V}}{Dt} = \bar{F} - \nabla p + \mu\nabla^2\bar{V} - \frac{\mu\bar{V}}{K} + (\bar{J}\times\bar{B})$$

where K is the permeability of the medium.

Equations of Motion in Rotating Co-ordinates[1].

If one takes a body of fluid and rotates its boundaries at a constant angular velocity $\vec{\omega}$ then at any time sufficiently long after starting the rotation, the whole body is rotating with this angular velocity, moving as if it were a rigid body. There are no viscous stresses acting within the fluid. Any disturbance i.e. anything that would produce a motion in a non-rotating system, will produce motion relative to this rigid body rotation. This relative motion can be considered as the flow pattern; it is the pattern that will be observed by an observer fixed to the rotating boundaries.

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1. Tritton, D.J.: Physical Fluid Dynamics, Van Nostrand Reinhold Company Ltd., p. 163 (1979).

The effect of using a rotating frame of reference is well known from the mechanics of solid systems, ther are accelerations associated with the use of a non-inertial frame that can be taken into account by introducing centrifugal and coriolis forces. The statement may be expressed in a form appropriate to fluid systems by –

$$1.52 \quad \left(\frac{D\bar{V}}{Dt}\right)_I = \left(\frac{D\bar{V}}{Dt}\right)_R + \bar{\Omega} \times (\bar{\Omega} \times \bar{r}) + 2\bar{\Omega} \times \bar{V}_R$$

The subscripts I and R refer to inertial and rotating frames of reference. $\left(\frac{D\bar{V}}{Dt}\right)_I$ is thus the acceleration that the fluid particle is experiencing and so $\rho\left(\frac{D\bar{V}}{Dt}\right)_I$ is the quantity to be equated with the sum of the various forces acting on the fluid particle. $\left(\frac{D\bar{V}}{Dt}\right)_R$ is the acceleration relative to the rotating frame and can thus be expanded in the usual way

$$1.53 \quad \left(\frac{D\bar{V}}{Dt}\right)_R = \frac{\partial \bar{V}_R}{\partial t} + (\bar{V}.\nabla\bar{V})_R$$

Dropping the subscript R as all velocities will be referred to the rotating frame the equation of motion is

$$1.54 \quad \frac{\partial \bar{V}}{\partial t} + \bar{V}.\nabla\bar{V} = -\frac{1}{\rho}\nabla p - \bar{\Omega} \times (\bar{\Omega} \times \bar{r}) - 2\bar{\Omega} \times \bar{V} + \nu\nabla^2\bar{V}$$

 (27)

The second and third terms on the right hand side of equation 1.54 are respectively the centrifugal and co-riolis forces.

In many problems the centrifugal force is unimportant. This is because it can be expressed as the gradient of scalar quantity.

1.55 $$\overline{\Omega} \times (\Omega \times \overline{r}) = - \nabla \left(\frac{1}{2} \Omega^2 r'^2\right)$$

where r' is the distance from the axis of rotation. Hence replacing pressure p by

1.56 $$p - \frac{1}{2} \rho \Omega^2 r'^2 = P \text{ (say)}$$

the equation of motion reduces to

1.57 $$\frac{D\overline{V}}{Dt} = - \frac{1}{\rho} \nabla P - 2 \overline{\Omega} \times \overline{V} + \nu \nabla^2 \overline{V}$$

Two important dimensionless parameters appearing in rotating fluid are

the Ekman number E

1.58 $$E = \frac{\nu}{\Omega L^2}$$

and the Rossby number

1.59 $$\epsilon = \frac{U}{\Omega L}$$

where L is some characteristic length.

Boundary Layer Equations in Rotating Co-ordinates:

An important practical type of rotating boundary layer flow is the flow over rotating blades, occurring in turbines, helicopters and propellers. In this case, the centrifugal and coriolis forces due to rotation, combine with pressure gradients and viscous forces, cause the flow to be three dimensional.

We consider a blade rotating about the z-axis with angular velocity ω and fix the axes with respect to the rotating blade. Let the y-axis be along the span of the blade and x-axis be the third axis so as to form a right-handed cartesian system.

If we apply the boundary layer approximations to equation 1.54, the distance of the boundary layer in the z-direction is of the order of δ , which is much smaller than the characteristic length in the x or y-direction and if the velocity component w is much smaller than u or v we have the boundary layer equations of motion in rotating coordinates.[1]

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1. Pai, S.I.: Viscous Flow Theory, I-Laminar Flow D.Van Nostrand Company Inc., p. 146 (1956).

1.60 $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2v\omega - x\omega^2 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}$$

1.61 $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2u\omega - y\omega^2 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2}$$

1.62 $$\frac{\partial p}{\partial z} = 0$$

Oscillatory Flow:

Fluctuations in a stream incident on a body are known to occur, and it is important to understand how the the boundary layer reacts to the oscillation of the stream. For example, in the occurrence of flutter of air on air-craft the boundary layer effects may be considerable. The effects of free oscillations on the flow past horizontal bodies were studied by Moore[1], Lighthill[2] and others. A simple case of oscillation treated by Lighthill[2] is the one in which the free stream oscillates in magnitude but not in direction. Owing to the mathematical difficulties there are sometimes restrictions on the amplitude and frequency of oscillations. After the pioneering initiation by Lighthill

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1. Moore,F.K.: NACA 2471 (1951).
2. Lighthill,M.J.: Proc. Roy. Soc. Lond. A224,1 (1954).

there have been many work on the subject of laminar boundary layer which have regular fluctuating flow superimposed on the mean steady flow. The most general case in which the stream fluctuates both in direction and magnitude has been studied by Gibson.

Equations of Motion of MHD Oscillatory Flow:

The two dimensional MHD boundary layer equations of motion with transversely applied uniform magnetic field B_o and x-axis along the flow direction, assuming the flow to be at small magnetic Reynolds number are—

1.63 $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_o^2 u$$

1.64 $$\frac{\partial p}{\partial y} = 0$$

If the free stream oscillates in magnitude only and is a function of time, i.e. $\overline{U} = \overline{U}(t)$ where $\overline{U}$ is the velocity of the free stream. We have from (1.63) for the free stream,

1.65 $$\frac{dU}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma}{\rho} B_o^2 U$$

Eliminating $-\frac{\partial p}{\partial x}$ between 1.63 and 1.65 we get the equation of oscillatory MHD flow as

1.66 $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_o^2 (u - U)$$

Hall Currents:

It is known that in an ionized gas where the density is low or the magnetic field is strong the conductivity of the fluid becomes a tensor. Due to the free spiralling of electrons and ions about the magnetic lines of force, before suffering collisions with other particles, the conductivity transverse to the magnetic lines of force is reduced and a current is induced in a direction normal to both electric and magnetic fields. The flow of such currents is known as Hall Currents. Due to the presence of these currents the efficiency of the MHD generator or accelerator is reduced.

The generalized Ohm's law taking Hall Current into account in the absence of electric field is of the form[1].

1.67 $$\bar{J} + \frac{w_e \tau_e}{H_o} \bar{J} \times \bar{H} = \sigma (\mu_e \bar{V} \times \bar{H} + \frac{1}{e n_e} \nabla p_e)$$

where H_o is the constant transverse magnetic field,

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1. Cowling, T.G.: Magnetohydrodynamics Interscience Publ.Inc. New York, p. 101 (1957).

σ — the electrical conductivity

μ_e — the magnetic permeability

w_e — the cyclotron frequency

τ_e — the collision time of electrons with ions

e — the electric charge

n_e — the number density of electrons

p_e — the electron pressure.

The boundary layer equations of MHD with Hall currents[1]: Let us consider xz to be the plane of the plate, the positive x-axis being in the direction of flow, y-axis is taken perpendicular to the plate. A uniformly distributed strong magnetic field H_o is acting in the y-direction. The effect of Hall current gives rise to a force in the z-direction, which induces a flow in that direction. Hence the flow becomes 3-dimensional. The plate is considered to be non-conducting.

The fundamental equations of incompressible MHD flow with generalized Ohm's Law are,

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1. Katagiri, M.: J. Phys. Soc. Jpn. 27, 1051 (1969).

1.68 $$\nabla . \overline{V} = 0$$

1.69 $$\frac{\partial \overline{V}}{\partial t} + (\overline{V}.\nabla)\overline{V} = -\frac{1}{\rho} \nabla p + \nu\nabla^2 \overline{V} + \frac{1}{\rho} \overline{J} X \overline{B}$$

1.70 $$\overline{J} + \frac{w_e \tau_e}{H_o} (\overline{J} X \overline{H}) = \sigma (\mu_e \overline{V} X \overline{H} + \frac{1}{en_e} \nabla p_e)$$

Since the plate is infinite in extent all the physical quantities except pressure are functions of y and t only. Assuming the magnetic Reynolds number to be small we neglect the induced magnetic field in comparison with the applied magnetic field.

Using the relation $\nabla . \overline{H} = 0$ for the magnetic field $\overline{H} = (H_x, H_y, H_z)$ we obtaine $H_y = H_o$ (H_o is a constant) everywhere in the fluid.

From the relation $\nabla . \overline{J} = 0$ for current density $\overline{J} = (J_x, J_y, J_z)$ we have J_y = constant. Since the plate is non-conducting $J_y = 0$ at the plate and hence zero everywhere.

By applying the usual boundary layer approximations, to equation 1.69 the basic equations under the above assumptions are.

1.71 $$\frac{\partial v}{\partial y} = 0$$

1.72 $$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\mu_e}{\rho} J_z H_o$$

1.73 $$\frac{\partial p}{\partial y} = 0$$

1.74 $$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\frac{\partial^2 w}{\partial y^2} + \frac{\mu_e}{\rho} J_x H_o$$

where J_x and J_z are obtained from 1.70.

Under the usual assumptions, that the electron pressure (for a weakly ionized gas), the thermoelectric pressure and the ion slip are negligible we have from 1.70,

1.75 $$J_x - w_e\tau_e J_z = -\sigma\mu_e H_o w$$

1.76 $$J_z + w_e\tau_e J_x = \sigma\mu_e H_o u$$

from 1.75 and 1.76 we get

1.77 $$J_x = \frac{\sigma\mu_e H_o}{1+m^2}(mu - w)$$

1.78 $$J_z = \frac{\sigma\mu_e H_o}{1+m^2}(u + mw)$$

A BRIEF DESCRIPTION OF PAST RESEARCHES RELEVANT TO THE THESIS:

Couette Flow:

The Couette flow of a viscous incompressible and electrically conducting fluid between two infinite parallel plates in the presence of a magnetic field when one of the plates starts impulsively from rest, was studied by Katagiri[1]. He presented his analysis by taking the magnetic lines of force fixed relative to the fluid.

Singh and Kumar[2] have considered Katagiri's problem by taking magnetic lines of force fixed relative to the moving plate. The Laplace transform technique has been used to solve the equation.

Suction and Injection:

Berman[3] has studied the problem of viscous flow in the annular space bounded by two concentric circular cylinders when the inner cylinder is discharging fluid and the outer one is absorbing it.

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1. Katagiri, M.: J. Phys. Soc. Jpn. 17, 393 (1962).
2. Singh, A.K. and Kumar, N. Wear 89, 125 (1983).
3. Berman, A.S.: J. Appl. Phys. 29, 71 (1958).

Satya Prakash[1] investigated the problem of unsteady incompressible viscous flow under a time varying pressure gradient in a straight channel with two parallel porous flat walls when one wall is discharging fluid and the other wall absorbing it.

Kishore et al[2] have extended the work of Satya Prakash to magnetohydrodynamic case. The magnetic Reynolds number of the flow is assumed to be small so that the induced magnetic field has been neglected.

Muhuri[3] has described the flow of an electrically conducting, viscous and incompressible fluid between two parallel porous walls when one of the walls moves with uniform acceleration and there is uniform suction and injection, in presence of a uniform transverse magnetic field. The magnetic lines of force are assumed fixed relative to the fluid.

Free Convection:

Soundalgekar and Patil[4] have studied the unsteady free convection flow of an electrically conducting, viscous and incompressible fluid past an impulsively started infinite vertical plate with constant heat flux at the plate.

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1. Prakash, S.: Proc.natn.Inst.Sci.India, 35A, 123 (1969).
2. Kishore, N., Tejpal, S. and Katiyar, H.K.: Ind.J.Pure Appl. Math. 12 (11) 1372 (1981).
3. Muhuri, P.K.: J.Phys.Soc.Jpn. 18, 1671 (1963).
4. Soundalgekar, V.M. and Patil, M.R.: Astrophys.Space Sci.70, 179 (19..

Nanousis et al[1] have studied the effect of a uniform transverse magnetic field on unsteady free convection flow of a viscous incompressible and electrically conducting fluid past an impulsively started infinite non-conducting vertical porous plate when the fluid is subjected to constant suction velocity. The magnetic lines of force are assumed fixed relative to the fluid.

Singh[2] has modified the problem of Nanousis et al[2] by assuming the magnetic lines of force to be fixed relative to the plate.

Raptis and Tzivanidis[3], have studied the effect of a magnetic field on steady free convection flow past an infinite vertical limiting surface. The limiting surface is unmoving and is subjected to constant suction velocity and there is constant heat flux at the surface. The magnetic Reynolds number is not small so that the induced magnetic field has been taken into account.

Soundalgekar and Wavre[4] have studied the two dimensional unsteady free convective flow in the presence of foreign mass past an infinite vertical porous plate, when the

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1. Nanousis,N.D., Georgantopoulos,G.A. and Papaioannous,A.I. Astrophys.Space Sci. 70, 377(1980).
2. Singh,A.K.: Astrophys. Space Sci.,87,455(1982).
3. Raptis,A.A. and Tzivanidis,G.J.: Astrophys.Space Sci.,94,311 (1983).
4. Soundalgekar,V.M. and Wavre,P.D.: Int.J.Heat Mass Transfer 20, 1363(1977).

plate temperature oscillates in time about a constant mean. Assuming constant suction at the plate, approximate solutions to the coupled non-linear equations have been obtained.

Flow Through Porous Media:

Ahmadi and Manvi[1] have derived the equations of motion of viscous flow through a rigid porous medium.

Varshney[2] studied the hydrodynamic fluctuating flow of a viscous incompressible fluid through a porous medium bounded by a porous plate.

A theoretical analysis of two dimensional free convective flow through a porous medium bounded by a porous and steady temperature plate was presented by Raptis et al.[3]

Megahed[4] has studied the unsteady two dimensional flow of a viscous incompressible and electrically conducting fluid through a porous medium bounded by an infinite porous horizontal plate and subjected to uniform external magnetic field, assuming low magnetic Reynolds number. Two cases have been studied by him:

(i) At time $t>0$ the plate starts moving with velocity $u(t)$ and the flow is subjected to time dependent suction velocity $v_o(t)$,

(ii) The fluid is subjected to constant suction velocity at the plate surface and the free stream velocity is assumed as any given arbitrary function of time.

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1. Ahmadi,G. and Manvi,R.: Ind. J. Tech.,9,441 (1971).
2. Varshney, C.L.: Ind. J. Pure Appl. Math.,10, 1958 (1979).
3. Raptis, A.A., Perdikis, C. and Tzivanidis, G.J.: Jl. Phys.,D. Appl. Phys. 14, L99 (1981).
4. Megahed,A.A. : Ind. Jl. Pure Appl. Math., 15(10), 1140(1984).

Raptis et al[1] have investigated the free convection and mass transfer steady flow of a viscous incompressible fluid through a porous medium, occupying a semi-infinite region of space bounded by an infinite vertical porous plate when the flow is subjected to constant suction velocity and the heat flux at the plate is constant.

Flow in Rotating Fluids:

Chawla[2], Singh and Sathi[3], Soundalgekar and Pop[4] studied the effect of rotation on Rayleigh's problem in non-magnetic case. Interesting conclusions have been derived in these problems.

Debnath and Mukherjee[5] have studied the unsteady boundary layer flow of an incompressible homogeneous viscous rotating fluid bounded by an infinite porous plate with uniform suction or blowing. They have discussed the structure of the steady and the unsteady flow fields including the nature of the associated boundary layers induced by the non-torsional oscillation of the plate.

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1. Raptis,A.A., Kafousias,N.G. and Massalas,C.V.: ZAMM 62, 489 (1982).
2. Chawla,S.S.: J.Phys.Soc.Jpn.,23,663(1967).
3. Singh,M.P. and Sathi,H.L.: J.Math.Mech.,18,193(1968).
4. Soundalgekar,V.M. and Pop,I.: Bull.Math.,14(62)375(1971).
5. Debnath,L. and Mukherjee,S.: Phys.Fluids,16,1418(1973).

Debnath[1] has investigated the unsteady boundary layer flow in the semi-infinite expanse of an electrically conducting rotating viscous fluid bounded by an infinite non-conducting porous plate with uniform suction or blowing in the presence of a transverse uniform magnetic field. The structure of the steady and unsteady flow fields and the associated hydromagnetic multiple boundary layer have been studied.

The free convective flow past an infinite vertical isothermal plate started impulsively in motion in its own plane in a viscous incompressible and electrically conducting fluid in presence of a transverse uniform magnetic field has been presented by Singh[2] in a rotating system. The governing equations of the flow have been solved by Laplace transform technique.

Oscillatory Flows:

The effect of free stream oscillations on the flow past horizontal bodies were stdied by Moore[3] and Lighthill[4]. Their oscillations were based on small amplitude of oscillations.

Georgantopoulos[5] has discussed the free convection effects on oscillating flow in the stokes problem past an infinite porous vertical plate with constant suction.

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1. Debnath, L.: Phys. Fluids, 17 (9), 1704 (1974).
2. Singh, A.K.: Astrophys Space Sci., 95, 283 (1983).
3. Moore, F.K.: NACA, 2471 (1951).
4. Lighthill, M.J.: Proc. Roy. Soc., London, A224, 1 (1954).
5. Georgantopoulos, G.A.: Astrophys. Space Sci., 65, 433 (1979).

Kafousias, et al[1] have extended the above problem in the presence of a uniform transverse magnetic field without taking into account the induced magnetic field.

Georgantopoulos and Koullias[2] have studied the free convection and mass transfer effects on the hydromagnetic oscillatory flow past an infinite vertical porous plate, in case of small magnetic Reynolds number,

The unsteady two-dimensional free convection, hydromagnetic oscillatory flow past an infinite vertical porous limiting surface was investigated by Kafousais[3] when the limiting surface is moved impulsively with a constant velocity. The magnetic Reynolds number of the flow is not taken to be small so that the induced magnetic field is not negligible. With viscous dissipative heat and Joule heating taken into account, approximate solutions to the governing equations are obtained.

Flow with Hall Currents:

Katagiri[4] has discussed the effects of Hall currents on the steady boundary layer flow of an electrically conducting, viscous and incompressible fluid past a semi-infinite plate in the presence of a constant transverse magnetic field.

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1. Kafousias, N.G., Massalas, C.V., Raptis, A.A., Tzivanidis, G.J., Georgantopoulos, G.A. and Goudas, G.L.: Astrophys. Space Sci., 68, 99 (1980).
2. Georgantopoulos, G.A. and Koullias, J.: Astrophys. Space Sci., 74, 357 (1981).
3. Kafousias, N.G.: Astrophys. Space Sci., 76, 133 (1981).
4. Katagiri, M.J.: Phys. Soc. Jpn., 27, 1051 (1969).

Hall effects on Couette flow between two parallel plates for both cases of impulsives as well as unifirmly accelerated motion of one of the plates has been discussed by Jana, and Datta[1]. Expressions for the shear stress components are obtained in terms of two non-dimensional parameters, the Hartmann number and Hall parameter.

Singh[2] has studied the Hall effects on the MHD free convection flow of an incompressible, viscous and electrically conducting fluid past an impulsively started infinite vertical porous plate in the case of small magnetic Reynolds number. Exact solution have been obtained by defining a complex velocity with the help of Laplace transform technique.

Hall effects on the hydromagnetic free convection flow past an impulsively started infinite vertical porous plate has been analysed by Singh[3] when the free stream oscillates in magnitude.

The effects of Hall currents on MHD free convective flow past an infinite vertical porous flat plate has been studied by Agrawal et al[4] when the fluid and the plate

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1. Jana, R.N. and Datta, N.: Int.J.Engg.Sci., 15, 35(1977).
2. Singh, A.K.: Astrophys.Space Sci., 93, 177(1983).
3. Singh, A.K.: Astrophys.Space Sci., 93, 1(1983).
4. Agrawal, H.L., Ram, P.C. and Singh, V.: J.Natn.Acad.Math.India, 1,65 (1983).

are in a state of rigid body rotation. The free stream is assumed to oscillate in time about a constant mean value and the applied magnetic field is perpendicular to the plate.

Agrawal et al[1,2] have analysed the effects of Hall currents on the combined effects of thermal and mass diffusion flow of an electrically conducting, viscous and incompressible fluid past an infinite vertical porous plate in presence of a uniform externally applied magnetic field. The free stream is assumed to oscillate in time about a constant mean.

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1. Agrawal,H.L., Ram,P.C. and Singh,V.: Astrophys Space Sci.,91,445(1983).
2. Agrawal,H.L., Ram,P.C. and Singh,V.: Astrophys Space Sci., 94, 383(1983).

CHAPTER – II

PART 'A'

UNSTEADY MHD FLOW BETWEEN TWO PARALLEL POROUS FLAT PLATES

Introduction:

The flow of a viscous incompressible and electrically conducting fluid between two infinite parallel plates in presence of a magnetic field, when one of the plates starts moving impulsively from rest was studied by Katagiri (1962).

Muhuri (1963) has studied the flow of a viscous incompressible and electrically conducting fluid between two porous walls, when one of the walls moves with uniform acceleration and there is uniform suction and injection. Katagiri (1962) and Muhuri (1963) presented their analysis by assuming the magnetic Reynolds number to be small so that the induced magnetic field is neglected.

In this part of the chapter we have reviewed Muhuri's (1963) problem under different conditions. In our problem the magnetic Reynolds number is taken to be sufficiently large so as to include the effect of the induced magnetic field. The lower plate is assumed to be moving with velocity $U_o e^{a't'}$. The Laplace transform technique is used to solve for the velocity field and induced magnetic field. The effect of the magnetic parameter on the velocity and induced magnetic fields is discussed with the help of graphs.

<u>Mathematical Analysis</u>:

We consider the two-dimensional flow of the fluid between two parallel, non-conducting infinite porous flat plates at $y' = 0$ and $y' = d$. At time $t' \leq 0$ the fluid and the plates are at rest. At time $t' > 0$ the lower plate begins to move in its own plane in the x'-direction with velocity $U_o e^{a't'}$. A uniform magnetic field of strength H_o is acting perpendicularly to the plates. The magnetic Reynolds number of the flow is not small so that the induced magnetic field has been taken into account. Fluid is being injected into the flow region with constant velocity w_o through the plate at $y' = 0$ and is being sucked away with the same velocity through the plate at $y' = d$. The flow is in the x'-direction and y'-axis is normal to the plates. Since the plates are infinite in extent all physical quantities are functions of y' and t' only. In our problem we assume the pressure gradient to be zero. The equations of motion taking into account the induced magnetic field are (Pai (1962)).

Momentum Equation:

$$2.1 \quad \rho \frac{D\overline{V}'}{Dt'} - \mu_o (\overline{H}'.\nabla)\ \overline{H}' = -\nabla\left(p' + \frac{\mu_o H'^2}{2}\right) + \mu\ \nabla^2 \overline{V}'$$

Equation for $\overline{H}'$:

$$2.2 \quad \frac{\partial \overline{H}'}{\partial t'} + (\overline{V}'.\nabla)\overline{H}' = (\overline{H}'.\nabla)\overline{V}' + \frac{1}{\sigma\mu_o} \nabla^2\ \overline{H}'$$

Equation of Continuity:

$$2.3 \qquad \nabla.\overline{V}' = 0$$

where $\overline{V}' = (u', v', w')$

$\overline{H}' = (H'_x,\ H'_y,\ H'_z)$

According to the condition of our problem, equations 2.1 – 2.3 give the following differential equations

$$2.4 \qquad \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\mu_o H_o}{\rho} \quad \frac{\partial H'_x}{\partial y'}$$

$$2.5 \qquad \frac{\partial H'_x}{\partial t'} + v' \frac{\partial H'_x}{\partial y'} = H_o \frac{\partial u'}{\partial y'} + \frac{1}{\sigma\mu_o} \frac{\partial^2 H'_x}{\partial y'^2}$$

$$2.6 \qquad \frac{\partial v'}{\partial y'} = 0$$

where

H'_x —— the induced magnetic field.

$H'_y = H_o$ —— the constant externally applied transverse magnetic field.

μ_o —— the magnetic permeability.

σ —— the electrical conductivity.

the other quantities have their usual meanings.

Equation 2.6 integrates to

2.7 $$v' = w_o \quad (w_o > 0)$$

Where w_o is the constant velocity of injection at the lower plate and constant suction velocity at the upper plate, the velocity being normal to the plates.

As the plates are non-conducting the initial and boundary conditions of the flow are,

2.8 $$t' \leq 0 : u' = 0, \qquad H'_x = 0 \quad \text{for } 0 \leq y' \leq d$$

2.9 $$t' > 0 : u' = U_o e^{a't'}, \; H'_x = 0 \quad \text{at } y' = 0$$

$$u' = 0, \qquad H'_x = 0 \quad \text{at } y' = d.$$

Introducing the following non-dimensional quantities

$$u = \frac{u'}{U_o}, \qquad y = \frac{y'}{d}, \qquad H = \left(\frac{\mu_o}{\rho}\right)^{1/2} \frac{H'_x}{U_o}$$

2.10 $$t = \frac{t' w_o}{d}, \qquad a = \frac{a' d}{w_o}, \qquad R = \frac{w_o d}{\nu}$$

$$M = \left(\frac{\mu_o}{\rho}\right)^{1/2} \frac{H_o}{w_o}, \qquad P_m = \nu \sigma \mu_o$$

in equations 2.4 and 2.5 we have

2.11 $$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = \frac{1}{R} \frac{\partial^2 u}{\partial y^2} + M \frac{\partial H}{\partial y}$$

2.12 $$\frac{\partial H}{\partial t} + \frac{\partial H}{\partial y} = M \frac{\partial u}{\partial y} + \frac{1}{R.P_m} \frac{\partial^2 H}{\partial y^2}$$

The initial and boundary conditions become in non-dimensional form:

2.13 $$t \leq 0 : \; u = 0, \quad H = 0, \quad \text{for } 0 \leq y \leq 1$$

2.14 $$t > 0 : \; u = e^{at}, \; H = 0, \quad \text{at } y=0$$

$$u = 0, \quad H = 0 \quad \text{at } y=1$$

where,

R — Reynolds number

P_m — Magnetic Prandtl number

M — Magnetic parameter

We will use Laplace transform technique to solve the coupled equations 2.11 and 2.12 and assume that $P_m = 1$.

Taking Laplace transform of equations 2.11 and 2.12 and using 2.13 we have respectively

2.15 $$\frac{1}{R}\frac{d^2\bar{u}}{dy^2} - \frac{d\bar{u}}{dy} + M\frac{d\bar{H}}{dy} - s\bar{u} = 0$$

2.16 $$\frac{1}{R}\frac{d^2\bar{H}}{dy^2} - \frac{d\bar{H}}{dy} + M\frac{d\bar{u}}{dy} - s\bar{H} = 0$$

where $$\bar{u}(y,s) = \int_0^\infty e^{-st}\,u(y,t)\,dt$$

$$\bar{H}(y,s) = \int_0^\infty e^{-st}\,H(y,t)\,dt$$

The boundary condition 2.14 is transformed to

$$\bar{u} = \frac{1}{s-a}, \quad \bar{H} = 0 \quad \text{at } y=0$$

2.17 $$\bar{u} = 0, \quad \bar{H} = 0 \quad \text{at } y=1$$

In order to uncouple equations 2.15 and 2.16 we add them and subtracting 2.16 from 2.15 we get,

2.18 $$\frac{1}{R}\frac{d^2X}{dy^2} - (1-M)\frac{dX}{dy} - sX = 0$$

2.19 $$\frac{1}{R}\frac{d^2Q}{dy^2} - (1+M)\frac{dQ}{dy} - sQ = 0$$

where $X = \bar{u} + \bar{H}$ and $Q = \bar{u} - \bar{H}$

subject to boundary conditions.

2.20

$$X = \frac{1}{s-a}, \qquad Q = \frac{1}{s-a} \qquad \text{at } y = 0$$

$$X = 0, \qquad Q = 0 \qquad \text{at } y = 1$$

Solutions of the equations 2.18 and 2.19 under boundary condition 2.20 yield,

2.21

$$\bar{u} = \frac{1}{2} e^{k_1 y} \sum_{n=o}^{\infty} \left[\frac{e^{-a_1 (b_1^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2 (b_1^2+s_1)^{\frac{1}{2}}}}{s_1} \right]$$

$$+ \frac{1}{2} e^{k_2 y} \sum_{n=o}^{\infty} \left[\frac{e^{-a_1 (b_2^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2 (b_2^2+s_1)^{\frac{1}{2}}}}{s_1} \right]$$

and

2.22

$$\bar{H} = \frac{1}{2} e^{k_1 y} \sum_{n=o}^{\infty} \left[\frac{e^{-a_1 (b_1^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2 (b_1^2+s_1)^{\frac{1}{2}}}}{s_1} \right]$$

$$- \frac{1}{2} e^{k_2 y} \sum_{n=o}^{\infty} \left[\frac{e^{-a_1 (b_2^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2 (b_2^2+s_1)^{\frac{1}{2}}}}{s_1} \right]$$

where,

$$k_1 = \frac{R(1-M)}{2}, \qquad k_2 = \frac{R(1+M)}{2}$$

$$a_1 = (2n+y)\ R^{\frac{1}{2}}, \qquad a_2 = (2n+2-y)\ R^{\frac{1}{2}}$$

$$b_1^2 = \frac{R(1-M)^2+4a}{4}, \qquad b_2^2 = \frac{R(1+M)^2+4a}{4}$$

$$s_1 = s - a$$

Using tables of Inverse Laplace Transform of Bateman (1954) we get the expression for u and H from 2.21 and 2.22 respectively.

as

2.23
$$u = \frac{e^{(at+k_1y)}}{4} \sum_{n=0}^{\infty} \Big[e^{-a_1b_1} \operatorname{erfc}\left(\frac{a_1t^{-\frac{1}{2}}}{2} - b_1t^{\frac{1}{2}}\right) +$$

$$+ e^{a_1b_1} \operatorname{erfc}\left(\frac{a_1t^{-\frac{1}{2}}}{2} + b_1t^{\frac{1}{2}}\right) -$$

$$- \left\{ e^{-a_2b_1} \operatorname{erfc}\left(\frac{a_2t^{-\frac{1}{2}}}{2} - b_1t^{\frac{1}{2}}\right) + e^{a_2b_1} \operatorname{erfc}\left(\frac{a_2t^{-\frac{1}{2}}}{2} + b_1t^{\frac{1}{2}}\right) \right\} \Big] +$$

$$+ \frac{e^{(at+k_2y)}}{4} \sum_{n=0}^{\infty} \Big[e^{-a_1b_2} \operatorname{erfc}\left(\frac{a_1t^{-\frac{1}{2}}}{2} - b_2t^{\frac{1}{2}}\right) +$$

$$+ e^{a_1b_2} \operatorname{erfc}\left(\frac{a_1t^{-\frac{1}{2}}}{2} + b_2t^{\frac{1}{2}}\right) -$$

$$- \left\{ e^{-a_2b_2} \operatorname{erfc}\left(\frac{a_2t^{-\frac{1}{2}}}{2} - b_2t^{\frac{1}{2}}\right) + e^{a_2b_2} \operatorname{erfc}\left(\frac{a_2t^{-\frac{1}{2}}}{2} + b_2t^{\frac{1}{2}}\right) \right\} \Big]$$

2.24

$$H = \frac{e^{(at+k_1y)}}{4} \sum_{n=0}^{\infty} \Big[e^{-a_1b_1} \operatorname{erfc}\left(\frac{a_1t^{-1/2}}{2} - b_1t^{1/2}\right) +$$

$$+ e^{a_1b_1} \operatorname{erfc}\left(\frac{a_1t^{-1/2}}{2} + b_1t^{1/2}\right) -$$

$$-\left\{ e^{-a_2b_1} \operatorname{erfc}\left(\frac{a_2t^{-1/2}}{2} - b_1t^{1/2}\right) + e^{a_2b_1} \operatorname{erfc}\left(\frac{a_2t^{-1/2}}{2} + b_1t^{1/2}\right) \right\}$$

$$- \frac{e^{(at+k_2y)}}{4} \sum_{n=0}^{\infty} \Big[e^{-a_1b_2} \operatorname{erfc}\left(\frac{a_1t^{-1/2}}{2} - b_2t^{1/2}\right) +$$

$$+ e^{a_1b_2} \operatorname{erfc}\left(\frac{a_1t^{-1/2}}{2} + b_2t^{1/2}\right) -$$

$$-\Big\{ e^{-a_2b_2} \operatorname{erfc}\left(\frac{a_2t^{-1/2}}{2} - b_2t^{1/2}\right)$$

$$+ e^{a_2b_2} \operatorname{erfc}\left(\frac{a_2t^{-1/2}}{2} + b_2t^{1/2}\right) \Big\} \Big]$$

Discussion of the Results:

In order to get physical insight into the problem numerical calculations have been carried out for the velocity u and induced magnetic field H, corresponding to different

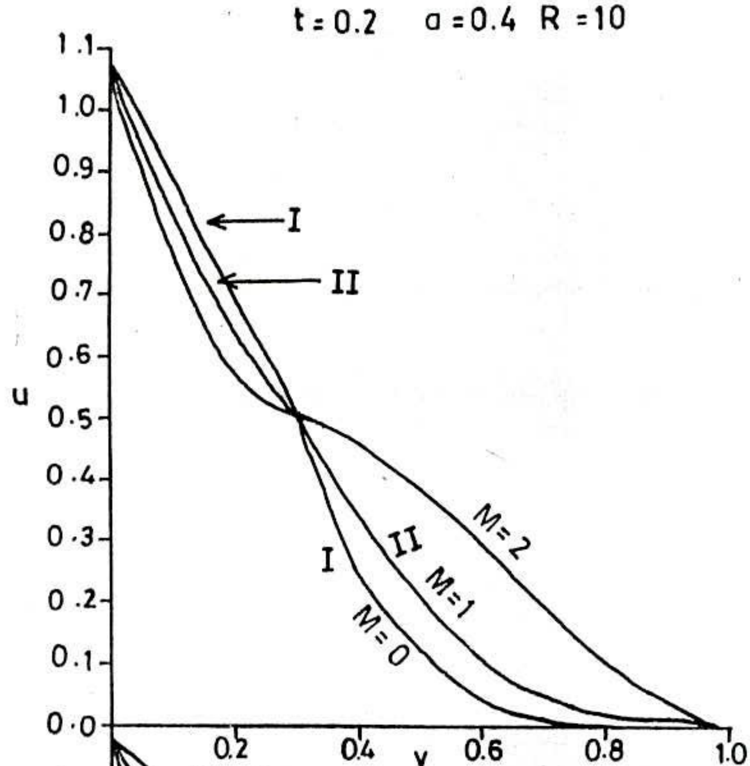

Fig. 2.1 Velocity variations with distance for M=0,1,2

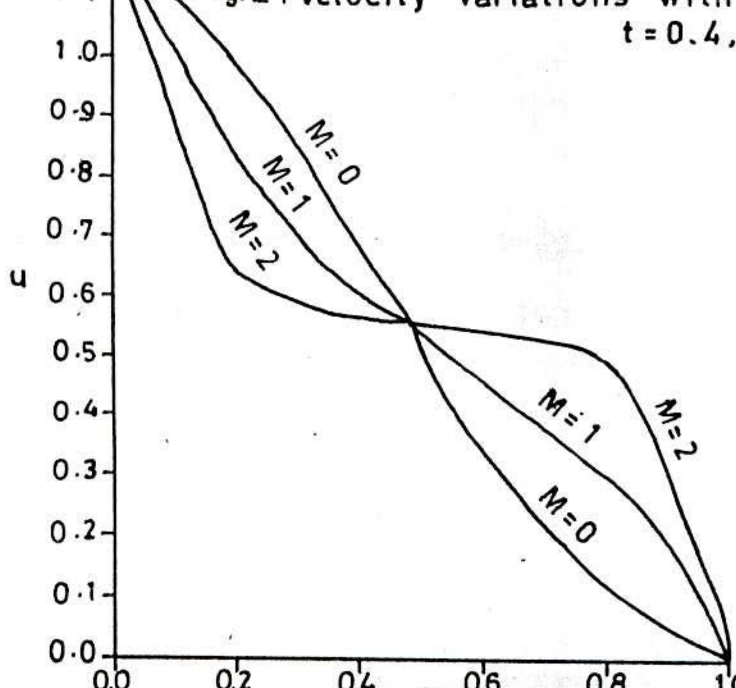

Fig. 2.2 Velocity variations with distance for M=0,1,2

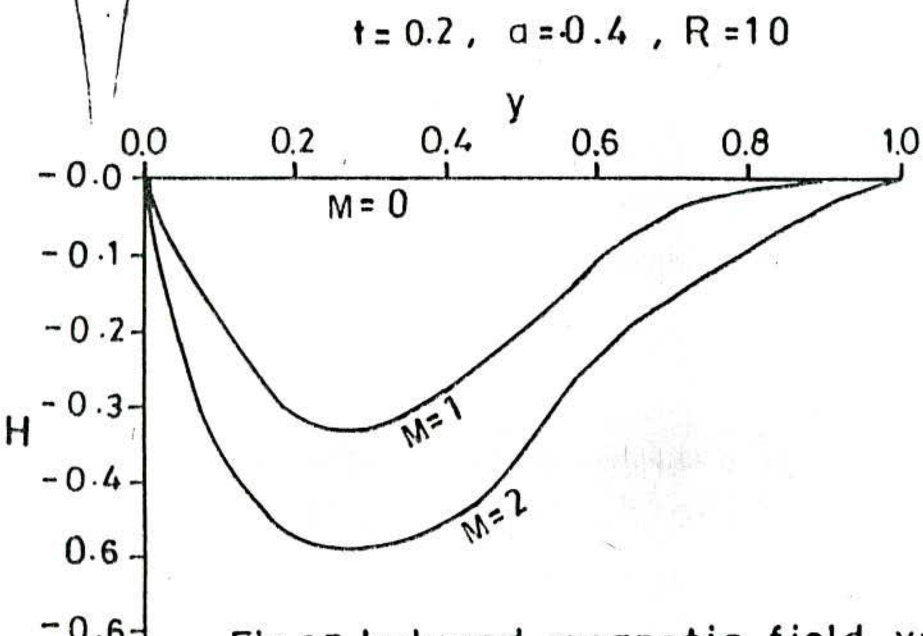

Fig.2.3 Induced magnetic field variations with distance for M=0,1 and 2

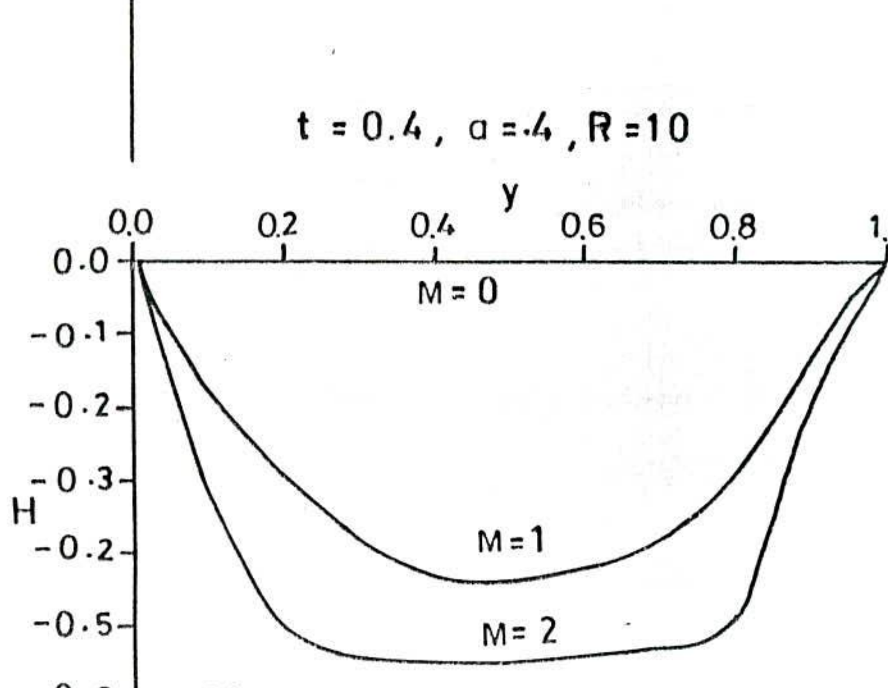

Fig.2.4 Induced magnetic field variations with distance for M= 0,1,2

values of the magnetic parameter M and time t. In the entire calculation we have taken R = 10 and a = 0.4.

The profiles for the velocity versus distance have been displayed in figures 2.1 and 2.2 for t = 0.2 and t = 0.4 respectively. From both the figures it is clear that the velocity u decreases with increase in M in the lower region between the plates whereas it increases with increase in M in the upper region.

The profiles for the induced magnetic field versus distance have been displayed in figures 2.3 and 2.4 for t = 0.2 and t = 0.4 respectively. It is seen from the figures that H takes negative values. It decreases with increase in M.

Curves corresponding to M = 0 represent the non-magnetic case.

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PART -'B'

UNSTEADY MHD FREE CONVECTION FLOW OF AN INCOMPRESSIBLE VISCOUS FLUID WITH CONSTANT HEAT FLUX

Introduction:

The effect of transverse magnetic field on steady free convection flow of a viscous incompressible and electrically conducting fluid past an unmoving infinite porous vertical limiting surface with constant heat flux at the surface has been carried out by Raptis and Tzivanidis (1983). The flow is subjected to constant suction at the plate. The effect of transverse magnetic field on the unsteady free convection flow past an impulsively started infinite vertical limiting surface, without constant heat flux at the surface has been carried out by Kafousias and Georgantopoulos (1982). In the above two problems the magnetic Reynolds number is not taken to be small, so that the induced magnetic field has been taken into account. The object of this part of the present chapter is to study the effect of transverse magnetic field on unsteady free convective flow of an incompressible viscous and electrically conducting fluid past an impulsively started infinite non-conducting vertical plate; there is constant heat flux at the plate. The magnetic Reynolds number is not small so that the induced magnetic field has been taken into account. The Laplace transform technique has been used to obtain the expressions for the velocity field and induced magnetic field.

Mathematical Analysis:

The unsteady MHD free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical plate has been considered. At time $t' \leq 0$ the fluid and the plate are assumed to be at rest. At time $t' > 0$ the plate starts moving in its own plane with uniform velocity U_o and heat is also started supplied to the plate at a constant rate. A uniform magnetic field of strength H_o is acting perpendicular to the plates. The magnetic Reynolds number of the flow is not taken to be small so that the induced magnetic field has been taken into account. The flow is assumed to be in the x'-direction which is taken along the vertical plate in the upward direction and y'-axis is normal to the plate. The pressure gradient is assumed to be zero.

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered nonly in the body force term. Under the above conditions the flow is governed by the following equations (Kafousias and Georgantopoulos(1982))

2.25 $$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\,(T' - T'_\infty) + \frac{\mu_o H_o}{\rho} \frac{\partial H'_x}{\partial y'}$$

2.26 $$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}$$

2.27 $$\frac{\partial H'_x}{\partial t'} = H_o \frac{\partial u'}{\partial y'} + \frac{1}{\sigma \mu_o} \frac{\partial^2 H'_x}{\partial y'^2}$$

with the initial and boundary conditions,

2.28 $$t' \leq 0 : \quad u'=0, \quad H'_x=0, \quad T'= T'_\infty \quad \forall\ y'$$

2.29 $$t' > 0 : \quad u'=U_o, \quad \frac{\partial T'}{\partial y'} = -\frac{q'}{k}, \quad H'_x=0 \text{ at } y'=0$$

$$u'=0, \quad T'=T'_\infty, \quad H'_x = 0 \quad \text{at } y'= \infty$$

where T' — the temperature of the fluid near to the plate

T'_∞ — the temperature of the fluid far away from the plate.

k — the thermal conductivity.

μ_o — the magnetic permeability.

q' — the constant heat flux per unit area at the plate.

β — the coefficient of thermal expansion.

C_p — the specific heat at constant pressure.

H'_x — the induced magnetic field.

The other variables have their usual meanings. Introducing the following non-dimensional quantities,

$$u = \frac{u'}{U_o}, \quad y = \frac{y'U_o}{\nu}, \quad T = \frac{T' - T'_\infty}{\frac{q'\nu}{kU_o}}$$

$$t = \frac{t'U_o^2}{\nu}, \quad G = \frac{\nu^2 g\beta q'}{kU_o^4}, \quad P = \frac{\rho\nu C_P}{k}$$

2.30 $$P_m = \sigma \mu_o \nu, \; H = \left(\frac{\mu_o}{\rho}\right)^{\frac{1}{2}} \frac{H'_x}{U_o}$$

$$M = \left(\frac{\mu_o}{\rho}\right)^{\frac{1}{2}} \frac{H_o}{U_o}$$

in equations 2.25 — 2.27 we get

2.31 $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + GT + M\frac{\partial H}{\partial y}$$

2.32 $$\frac{\partial T}{\partial t} = \frac{1}{P} \frac{\partial^2 T}{\partial y^2}$$

2.33 $$\frac{\partial H}{\partial t} = \frac{1}{P_m} \frac{\partial^2 H}{\partial y^2} + M\frac{\partial u}{\partial y}$$

The initial and boundary conditions becomes in non-dimensional form

2.34 $$t \leq 0 : \quad u=0, \quad T=0, \quad H=0 \quad \forall \quad y$$

2.35 $$t > 0 : \quad u=1, \quad \frac{\partial T}{\partial y} = -1, \quad H=0 \quad \text{at } y=0$$

$$u=0, \quad T=0, \quad H=0 \quad \text{at } y=\infty$$

In order to solve the equations 2.31 to 2.33 we will use Laplace transform technique and assume that $P=P_m=1$

Applying Laplace transformation to equations 2.31 to 2.33 and using 2.34 we get respectively,

2.36 $$s\bar{u} = \frac{d^2\bar{u}}{dy^2} + G\bar{T} + M\frac{d\bar{H}}{dy}$$

2.37 $$s\bar{T} = \frac{d^2\bar{T}}{dy^2}$$

2.38 $$s\bar{H} = \frac{d^2\bar{H}}{dy^2} + M\frac{d\bar{u}}{dy}$$

where $\bar{u}$, $\bar{H}$ and $\bar{T}$ are respectively the Laplace transforms of u, H and T.

The boundary conditions are,

2.39
$$\bar{u} = \frac{1}{s}, \quad \frac{\partial \bar{T}}{\partial y} = -\frac{1}{s}, \quad \bar{H} = 0 \qquad \text{at } y = 0$$

$$\bar{u} = 0, \quad \bar{T} = 0, \quad \bar{H} = 0 \qquad \text{at } y = \infty$$

The solution of equation 2.37 under boundary condition 2.39 is given by,

$$\bar{T} = \frac{e^{-s^{\frac{1}{2}} y}}{s^{3/2}} \tag{2.40}$$

In order to uncouple the equations 2.36 and 2.38 we add them and substracting 2.38 from 2.36 we get,

$$\frac{d^2 X}{dy^2} + M \frac{dX}{dy} - sX = -G\bar{T} \tag{2.41}$$

and

$$\frac{d^2 Q}{dy^2} - M \frac{dQ}{dy} - sQ = -G\bar{T} \tag{2.42}$$

where $X = \bar{u} + \bar{H}$ and $Q = \bar{u} - \bar{H}$

subject to boundary condition

$$\left.\begin{array}{lll} X = \frac{1}{s}, & Q = \frac{1}{s} & \text{at } y=0 \\ X = 0, & Q = 0 & \text{at } y=\infty \end{array}\right. \tag{2.43}$$

Solutions of the equations 2.41 and 2.42 under boundary condition 2.43 yield

$$\bar{u} = \frac{e^{\frac{My}{-2}}}{2} \left[\frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s} - \frac{G}{M} \frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s^2} \right] +$$

2.44

$$+ \frac{e^{\frac{My}{2}}}{2} \left[\frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s} + \frac{G}{M} \frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s^2} \right]$$

and,

2.45 $$\bar{H} = \frac{e^{-\frac{My}{2}}}{2} \left[\frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s} - \frac{G}{M} \frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s^2} \right] -$$

$$- \frac{e^{\frac{My}{2}}}{2} \left[\frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s} + \frac{G}{M} \frac{e^{-(\frac{M^2}{4}+s)^{1/2}y}}{s^2} \right] +$$

$$+ \frac{G}{M} \frac{e^{-s^{1/2}y}}{s^2}$$

Using tables of inverse Laplace transform of Bateman (1954) we get the expression for u, T and H from 2.44, 2.40 and 2.45 respectively as,

2.46 $$u = \frac{1}{4} [e^{-My} \text{erfc} (\frac{y}{2}t^{-1/2} - \frac{Mt}{2}^{1/2}) +$$

$$+ \text{erfc} (\frac{yt}{2}^{-1/2} - \frac{Mt}{2}^{1/2}) -$$

$$- \frac{2G}{M^2} t^{1/2} \{ -(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) \operatorname{erfc}(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) e^{-My}$$

$$+ (\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2}) \operatorname{erfc}(\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2}) \}]$$

$$+ \frac{1}{4} [\operatorname{erfc}(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) + e^{My} \operatorname{erfc}(\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2})$$

$$+ \frac{2G}{M^2} t^{1/2} \{ -(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) \operatorname{erfc}(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2})$$

$$+ (\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2}) \operatorname{erfc}(\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2}) e^{My} \}].$$

2.47 $$T = 2t^{1/2} (e^{-y^2/4t} \pi^{-1/2} - \frac{y}{2} t^{-1/2} \operatorname{erfc} \frac{y}{2} t^{-1/2})$$

2.48 $$H = \frac{1}{4} [e^{-My} \operatorname{erfc}(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) + \operatorname{erfc}(\frac{y}{2} t^{-1/2} + \frac{M}{2} t^{1/2})$$

$$- \frac{2G}{M^2} t^{1/2} \{ -(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) \operatorname{erfc}(\frac{y}{2} t^{-1/2} - \frac{M}{2} t^{1/2}) e^{-My}$$

$$+ \left(\frac{y}{2}t^{-1/2} + \frac{M}{2}t^{1/2} \right) \operatorname{erfc}\left(\frac{y}{2}t^{-1/2} + \frac{M}{2}t^{1/2} \right) \Big\} \Big]$$

$$- \frac{1}{4} \Big[\operatorname{erfc}\left(\frac{y}{2}t^{-1/2} - \frac{M}{2}t^{1/2} \right) + e^{My} \operatorname{erfc}\left(\frac{y}{2}t^{-1/2} + \frac{M}{2}t^{1/2} \right)$$

$$+ \frac{2G}{M^2}t^{1/2} \Big\{ - \left(\frac{y}{2}t^{-1/2} - \frac{M}{2}t^{1/2} \right) \operatorname{erfc}\left(\frac{y}{2}t^{-1/2} - \frac{M}{2}t^{1/2} \right)$$

$$+ \left(\frac{y}{2}t^{-1/2} + \frac{M}{2}t^{1/2} \right) \operatorname{erfc}\left(\frac{y}{2}t^{-1/2} + \frac{M}{2}t^{1/2} \right) e^{My} \Big\} \Big]$$

$$+ \frac{Gt}{M} \Big[\left\{ 1 + 2 \left(\frac{y}{2}t^{-1/2} \right)^2 \right\} \operatorname{erfc} \frac{y}{2}t^{-1/2}$$

$$- y(\pi t)^{-1/2} e^{-\left(\frac{y}{2}t^{-1/2}\right)^2} \Big].$$

Table 2.1

Values of the velocity u

t	M	G	y →	0.0	0.5	1.0	1.5	2.0	2.5
0.2	0.4	3	u →	1.0000	0.4651	0.1266	0.0201	0.0018	0.0001
	0.7	3		1.0000	0.4663	0.1301	0.0215	0.0021	0.0001
	0.4	10		1.0000	0.5476	0.1526	0.0239	0.0021	0.0001
	0.7	10		1.0000	0.5486	0.1562	0.0255	0.0024	0.0001
0.4	0.4	3		1.0000	0.6879	0.3357	0.1227	0.0339	0.0071
	0.7	3		1.0000	0.6867	0.3415	0.1294	0.0377	0.0084
	0.4	10		1.0000	0.9496	0.4974	0.1832	0.0500	0.0102
	0.7	10		1.0000	0.9471	0.5030	0.1907	0.0543	0.0117

Table 2.2

Values of induced magnetic field H.

t	M	G	y →	0.0	0.5	1.0	1.5	2.0	2.5
0.2	0.4	3	H	0.0000	-0.0436	-0.0237	-0.0056	-0.0007	-0.0000
	0.7	3		-0.0000	-0.0761	-0.0415	-0.0099	-0.0012	-0.0001
	0.4	10		00.0000	-0.0455	-0.0257	-0.0061	-0.0007	-0.0000
	0.7	10		0.0000	-0.0794	-0.0450	-0.0108	-0.0013	-0.0001
0.4	0.4	3		0.0000	-0.0584	-0.0571	-0.0313	-0.0115	-0.0030
	0.7	3		0.0000	-0.1014	-0.0994	-0.0551	-0.0206	-0.0055
	0.4	10		0.0000	-0.0609	-0.0675	-0.0385	-0.0142	-0.0037
	0.7	10		0.0000	-0.1057	-0.1176	-0.0676	-0.0254	-0.0067

Table 2.3

Values of Temperature T

t	y	T
	0.0	0.5046
	0.5	0.1546
	1.0	0.0307
0.2	1.5	0.0037
	2.0	0.0003
	2.5	0.0000
	3.0	0.0000
	0.0	0.7136
	0.5	0.3223
	1.0	0.1184
0.4	1.5	0.0346
	2.0	0.0079
	2.5	0.0014
	3.0	0.0002

Conclusions:

In order to get physical insight into the problem, we have calculated the values of u, H and T for different values of magnetic parameter M, Grasholf number G and time t.

Values of the velocity are given in Table 2.1. From table we conclude that the velocity increases with increase in G and t. For $t=0.2$ the velocity increases with increase in M while for $t=0.4$ the effect of M is to decrease the velocity in a thin fluid layer near the plate and increase beyond it.

Values of the induced magnetic field H are displayed in table 2.2. From the table we see that H takes negative values and it decreases with increase in G, M and t.

In table 2.3 variation of temperature T is shown for different values of t. The temperature increases with time.

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CHAPTER – III

PART 'A'

STEADY MHD FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM BOUNDED BY AN INFINITE VERTICAL POROUS PLATE

Introduction:

Raptis et al (1982) have studied the steady free convective and mass transfer flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous plate, with constant heat flux at the plate. The flow is subjected to constant suction velocity at the plate.

The present part of the chapter is a modification of the above problem, in the sense that we have applied it to MHD case, but have neglected the effects of mass transfer and constant heat flux. Approximate solutions to the coupled non-linear equations are derived for the velocity and temperature fields. Effects of the various parameters occuring in the problem have been discussed with the help of graphs and tables.

The study of flows through porous medium is of great importance in many scientific and engineering applications. The porous medium is in fact a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid which has dynamical properties equivalent to those of non-homogeneous continuum. Thus we can study the flow of a hypothetical homogeneous fluid under the action of properly averaged external forces and so, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance.

Mathematical Analysis:

We consider the free convective flow of an electrically conducting viscous incompressible fluid through a porous medium occupying a semi-infinite region of space, bounded by an infinite vertical porous plate. The plate is assumed to be moving steadily in the vertically upward direction along which x'-axis is chosen and y'-axis is perpendicular to the plate. A uniform magnetic field of strength H_o is acting transverse to the plate. The magnetic Reynolds number of the flow is assumed to be small so that the induced magnetic field has been neglected. The flow is subjected to constant suction velocity at the plate and the pressure gradient is assumed to be zero.

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term. The two dimensional boundary layer equations which govern the steady MHD free convective flow through a porous medium are given by

3.1 $$\frac{dv'}{dy'} = 0$$

3.2 $$v'\frac{du'}{dy'} = g\beta(T'-T'_\infty) + \frac{\nu d^2u'}{dy'^2} - \frac{\sigma\mu_e^2 H_o^2 u'}{\rho} - \frac{\nu u'}{K'}$$

3.3 $$v\frac{dT'}{dy'} = \frac{k}{\rho C_p}\frac{d^2T'}{dy'^2} + \frac{\nu}{C_p}\left(\frac{du'}{dy'}\right)^2$$

where,

u' — velocity of the fluid in x'-direction

v' — velocity of the fluid in y'-direction.

T' — temperature of the fluid in the boundary layer.

T'_∞ — temperature of the fluid outside the boundary layer.

K' — permeability of the porous medium.

k — thermal conductivity of the fluid.

C_p — specific heat at constant pressure

β — the coefficient of thermal expansion.

The boundary conditions are:

$$u' = U_o, \quad T' = T'_\omega \quad \text{at } y' = 0$$

3.4 $$u' \to 0, \quad T' \to T'_\infty \quad \text{as } y' \to \infty$$

Equation (3.1) integrates to:

3.5 $$v' = -v_o \quad (v_o > 0)$$

where the negative sign indicates that there is suction at the plate.

Introducing the following dimensionless quantities,

$$y = \frac{y' v_o}{\nu}, \quad u = \frac{u'}{U_o} \quad P = \frac{\rho \nu C_p}{k}$$

3.6 $$T = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}, \quad G = \frac{\nu g \beta (T'_\omega - T'_\infty)}{U_o v_o^2}$$

$$M = \frac{\sigma B_o^2 \nu}{\rho v_o^2}, \quad K = \frac{v_o^2 K'}{\nu^2}, \quad E = \frac{U_o^2}{C_p (T'_\omega - T'_\infty)}$$

in equations 3.2 and 3.3 we have respectively,

3.7 $$\frac{d^2 u}{dy^2} + \frac{du}{dy} - (M + \frac{1}{K}) u = -GT$$

3.8 $$\frac{d^2 T}{dy^2} + \frac{P dT}{dy} = -PE (\frac{du}{dy})^2.$$

The boundary conditions 3.4 become in non-dimensional form,

3.9
$$u = 1, \quad T = 1 \quad \text{at } y = 0$$
$$u \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

The coupled non-linear equations 3.7 and 3.8 are not solvable in exact form, so we find the approximate solution. In order to do so, we expand u and T in power of E, the Eckert number assuming that it is very small. This is justified in low speed incompressible flows. So that

3.10
$$u = u_o + Eu_1 + 0\ (E^2)$$
$$T = T_o + ET_1 + 0\ (E^2)$$

where u_o, u_1, T_o and T_1 are functions of y, substituting 3.10 in equations 3.7 and 3.8 and equating the coefficients of different powers of E, neglecting those of E^2 and higher powers of E we have the following set of equations

3.11 $$u''_o + u'_o - (M + \frac{1}{K})\ u_o = -GT_o$$

3.12 $$u''_1 + u'_1 - (M + \frac{1}{K})\ u_1 = -GT_1$$

3.13 $$T_o'' + PT_o' = 0$$

3.14 $$T_1'' + PT_1' = -Pu_o'^2$$

where the primes denote differentiation with respect to y.

The boundary conditions for u_o, u_1, T_o and T_1 are,

$$u_o = 1, \quad u_1 = 0, \quad T_o = 1, \quad T_1 = 0 \quad \text{at } y = 0$$

3.15

$$u_o \rightarrow 0, \quad u_1 \rightarrow 0, \quad T_o \rightarrow 0, \quad T_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

The solutions of equations 3.11 to 3.14 subject to boundary conditions 3.15 are given respectively by

3.16 $$u_o = A_2 e^{-B_1 y} - A_1 e^{-Py}$$

3.17 $$u_1 = L_2 e^{-B_1 y} - C_1 e^{-Py} + C_2 e^{-2B_1 y}$$

$$- C_3 e^{-(B_1+P)y} + C_4 e^{-2Py}$$

3.18 $$T_o = e^{-Py}$$

3.19 $$T_1 = L_1 e^{-Py} - A_3 e^{-2B_1 y} + A_4 e^{-(B_1+P)y} - A_5 e^{-2Py}$$

Hence the expressions for velocity and temperature fields are given by $u = u_o + Eu_1$ and $T = T_o + ET_1$ respectively from 3.10, where u_o, u_1, T_o and T_1 are given by 3.16 to 3.19 respectively.

The skin friction in non dimensional form is given by

3.20 $$\tau = \left.\frac{du}{dy}\right|_{y=0}$$

$$= - A_2 B_1 + A_1 P + E(-L_2 B_1 + C_1 P - 2C_2 B_1 + C_3 (B_1 + P) - 2C_4 P)$$

The different constants are defined in the appendix.

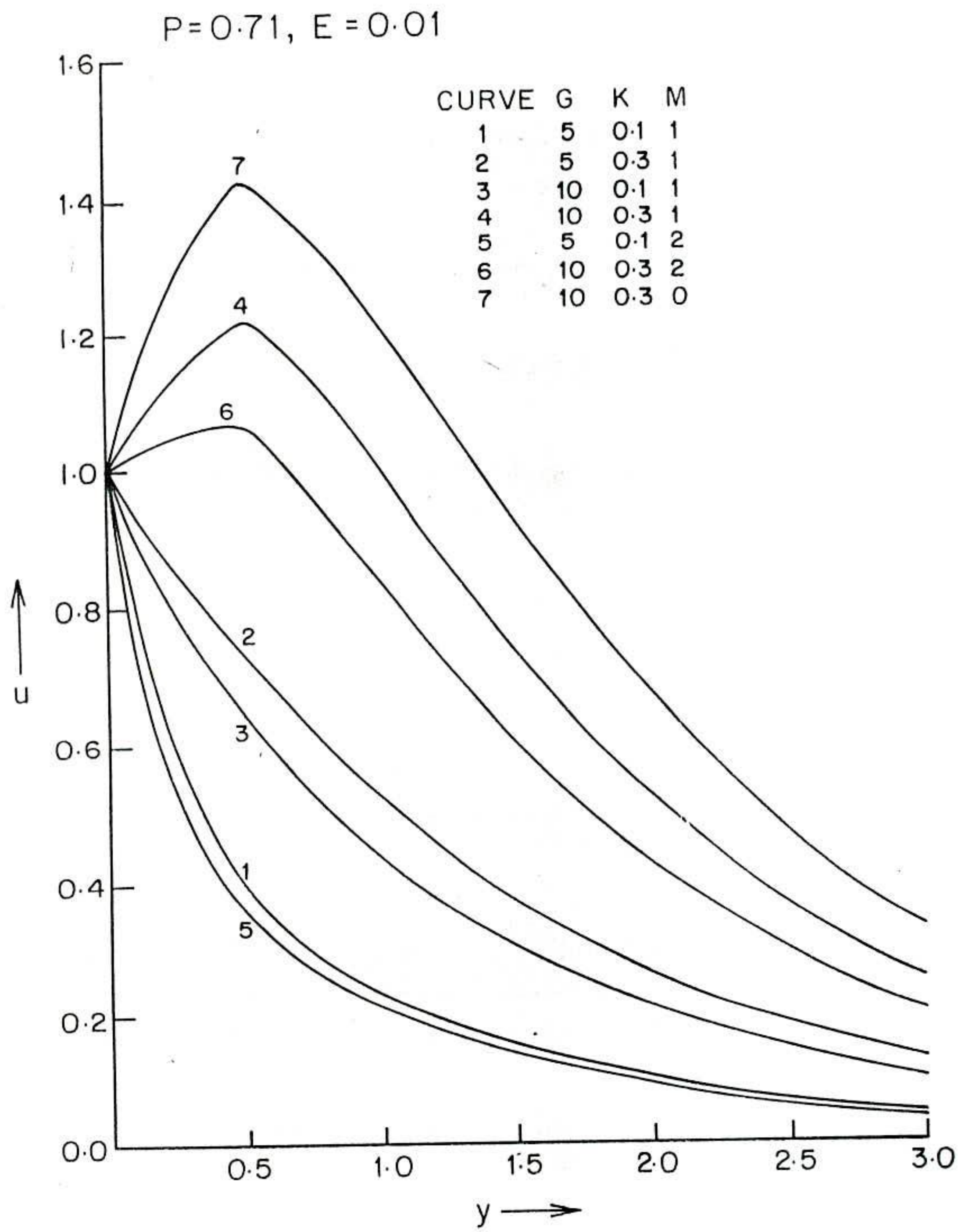

FIG. 3·1 VELOCITY DISTRIBUTION u AGAINST y

Table 3.1: Values of skin friction τ (P=0.71, E=0.01).

M	G	K	τ
0	5	.1	- 2.2348
	10	.1	- 0.7686
	5	.3	- 0.0130
	10	.3	2.3756
1	5	.1	- 2.4501
	10	.1	- 1.0467
	5	.3	- 0.5122
	10	.3	1.6203
2	5	.1	- 2.6512
	10	.1	- 1.3030
	5	.3	- 0.9181
	10	.3	1.0284

Table 3.2: Values of temperature (E=0.01, P=0.71, G=10).

M	Y	K=.1	K=.3
0	0.0	1.0000	1.0000
	0.5	0.7019	0.7031
	1.0	0.4925	0.4946
	1.5	0.3455	0.3479
	2.0	0.2423	0.2446
	2.5	0.1700	0.1718
	3.0	0.1192	0.1206
1	0.0	1.0000	1.0000
	0.5	0.7019	0.7026
	1.0	0.4925	0.4938
	1.5	0.3454	0.3469
	2.0	0.2423	0.2437
	2.5	0.1699	0.1711
	3.0	0.1192	0.1200
2	0.0	1.0000	1.0000
	0.5	0.7019	0.7023
	1.0	0.4924	0.4933
	1.5	0.3454	0.3464
	2.0	0.2422	0.2432
	2.5	0.1699	0.1706
	3.0	0.1191	0.1197

Discussion

For the purpose of discussing the effects of various parameters on the flow behaviour numerical calculations have been carried out for velocity u, temperature T and skin fiction τ corresponsing to different value of magnetic parameter M, Grashoff number G and permeability K. In order to be realistic, the value of the Prandtl number P is zchosen to be 0.71 which corresponds to air. In the entire calculation we have taken E = 0.01.

The velocity profiles u against y have been displayed in figure 3.1. It is clear from the figure that u increases with increase in K and G. The effect of the magnetic parameter M is to decrease the velocity, that is, it decreases with increase in M.

Values of the skin friction are presented in Table 3.1. From the table we conclude that the skin friction decreases with increase in M. The skin friction increases with increase in K and G.

Values of the temperature are displayed in Table 3.2. It is clear from the table that the temperature increases with increase in K. The effect of the magnetic parameter M is to decrease the temperature (the effect being rather small).

The values corresponding to M=0 represent the non magnetic case.

APPENDIX

$$M_1 = M + \frac{1}{K}, \qquad B_1 = \frac{1+(1+4M_1)^{1/2}}{2}$$

$$A_1 = \frac{G}{P^2 - P - M_1}, \qquad A_2 = 1 + A_1,$$

$$A_3 = \frac{PA_2^2 B_1}{4B_1 - 2P}, \qquad A_4 = \frac{2A_1 A_2 P^2}{B_1 + P}$$

$$A_5 = \frac{A_1^2 P}{2}, \qquad L_1 = A_3 - A_4 + A_5,$$

$$C_1 = \frac{GL_1}{P^2 - P - M_1}, \qquad C_2 = \frac{GA_3}{4B_1^2 - 2B_1 - M_1}$$

$$C_3 = \frac{GA_4}{(B_1+P)^2 - (B_1+P) - M_1}, \qquad C_4 = \frac{GA_5}{4P^2 - 2P - M_1},$$

$$L_2 = C_1 - C_2 + C_3 - C_4.$$

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Raptis, A.A., Kafousias, N.G. and Massalas, C.V. (1982), ZAMM 62, 489.

PART 'B'

MHD FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM PAST A STEADILY MOVING PLATE IN ROTATING SYSTEM

Introduction:

The problem considered here is an extension of that of part 'A' of this chapter. In part A we have considered the flow of an electrically conducting viscous incompressible fluid through a porous medium, past a steadily moving porous infinite vertical plate. In this part of the chapter we have considered the same problem, taking into account the effect of rotation on the flow. The whole system is assumed to be in a state of rigid body rotation, due to rotation the flow becomes three dimensional. The magnetic Reynolds number of the flow is assumed small so that the induced magnetic field has been neglected. The influence of the various parameters occuring in the problem on the temperature field, the axial and transverse components of the velocity and skin friction is discussed with the help of tables and graphs.

Mathematical Analysis:

We consider the steady MHD free convective flow of an electrically conducting, viscous and incompressible fluid through a porous medium occupying a semi-infinite region of space bounded by an infinite vertical porous plate at $z'=0$. The whole system is in a state of rigid body rotation with constant angular velocity $\bar{\Omega}'$ about z'-axis, which is normal to the plate. The plate is assumed to be moving with uniform velocity U_o in its own plane in the vertically upward direction, along which x'-axis is chosen, the plate is assumed to be electrically non-conducting. Since the plate is infinite in extent all the physical variables are functions of z' only.

A uniform magnetic field of strength H_o is assumed to be applied transversely to the porous plate. Assuming the magnetic Reynolds number to be small we neglect the induced magnetic field in comparison with the applied magnetic field. In our problem we assume the pressure to be constant, hence neglecting the pressure gradient and centrifugal force terms from the steady MHD equations of motion for free convection flow through porous medium in rotating co-ordinates, viz.:

$$3.21 \quad (\bar{q}'\cdot\nabla)\bar{q}' = \frac{\bar{F}}{\rho} - \frac{1}{\rho}\nabla p' - \bar{\Omega}'\times(\bar{\Omega}'\times\bar{r}') - 2\bar{\Omega}'\times\bar{q}' + \nu\nabla^2\bar{q}'$$

$$+ \frac{1}{\rho}(\bar{J}\times\bar{B}) - \frac{\nu\bar{q}'}{K'}$$

gives

3.22 $$w' \frac{du'}{dz'} - 2\,\Omega' v' = \nu \frac{d^2u'}{dz'^2} + g\beta(T' - T'_\infty) - \frac{\sigma\mu_e^2 H_o^2 u'}{\rho} - \frac{\nu u'}{K'}$$

3.23 $$w' \frac{dv'}{dz'} + 2\,\Omega' u' = \nu \frac{d^2v'}{dz'^2} - \frac{\sigma\mu_e^2 H_o^2}{\rho} v' - \frac{\nu v'}{K'}$$

energy equation is

3.24 $$w' \frac{dT'}{dz'} = \frac{k}{\rho C_p} \frac{d^2T'}{dz'^2} + \frac{\nu}{C_p} \left[\left(\frac{du'}{dz'}\right)^2 + \left(\frac{dv'}{dz'}\right)^2 \right]$$

and equation of continuity is

3.25 $$\frac{dw'}{dz'} = 0$$

where the third and fourth terms of equation 3.21 are respectively the centrifugal and Coriolis forces and F is the body force term per unit volume due to free convection; the other physical quantities have their usual meanings.

The boundary conditions are:

3.26 $$u' = U_o, \quad v' = 0 \quad T' = T'_\omega \qquad \text{at } z' = 0$$

$$u' \longrightarrow 0, \quad v' \longrightarrow 0, \quad T' \longrightarrow T'_\infty \qquad \text{as } z' \longrightarrow \infty$$

Since the fluid is subjected to constant suction velocity at the plate, equation 3.25 integrates to:

3.27 $w' = -w_o \quad (w_o > 0)$

where w_o is the constant normal velocity of suction at the plate,

Introducing the following non-dimensional quantities,

$$z = \frac{z' U_o}{\nu}, \quad S = \frac{w_o}{U_o}, \quad P = \frac{\mu C_p}{k}, \quad u = \frac{u'}{U_o}$$

$$v = \frac{v'}{U_o}, \quad M = \frac{\nu \mu_e^2 H_o^2}{\rho U_o^2}, \quad \Omega = \frac{2\Omega' \nu}{U_o^2}$$

3.28

$$T = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}, \quad G = \frac{\nu g \beta \, (T'_\omega - T'_\infty)}{U_o^3}$$

$$K = \frac{U_o^2 \, K'}{\nu^2}, \quad E = \frac{U_o^2}{C_p (T'_\omega - T'_\infty)}$$

in equations 3.22 — 3.24 we get

3.29 $$-S\frac{du}{dz} - \Omega v = \frac{d^2u}{dz^2} + GT - u \left(M + \frac{1}{K}\right)$$

3.30 $$-S\frac{dv}{dz} + \Omega u = \frac{d^2v}{dz^2} - v \left(M + \frac{1}{K}\right)$$

3.31 $$-S\frac{dT}{dz} = \frac{1}{P}\frac{d^2T}{dz^2} + E\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right]$$

The boundary conditions 3.26 becomes in non-dimensional form

3.32
$$u = 1,\quad v = 0,\quad T = 1 \quad \text{at} \quad z = 0$$

$$u \rightarrow 0,\quad v \rightarrow 0,\quad T \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty$$

Introducing the complex variable $Q = u+iv$ equations 3.29 and 3.30 can be combined into a single equation giving

3.33 $$\frac{d^2Q}{dz^2} + S\frac{dQ}{dz} - \left(M + \frac{1}{K} + i\Omega\right) Q = -GT$$

Equation 3.31 becomes

3.34 $$\frac{d^2T}{dz^2} + SP\frac{dT}{dz} = -PE\left[\frac{dQ}{dz} \cdot \frac{d\bar{Q}}{dz}\right)$$

where $\bar{Q}$ is the complex conjugate of Q. The boundary conditions 3.32 becomes

3.35 $$Q = 1,\quad T = 1, \quad \text{at} \quad z = 0$$

$$Q \rightarrow 0,\quad T \rightarrow 0, \quad \text{as} \quad z \rightarrow \infty$$

Equations 3.33 and 3.24 are coupled equations. In order to solve them let us expand Q and T in terms of E, the Eckert number assuming it to be very small ($E \ll 1$ for incompressible fluid).

3.36
$$Q = q_o + E\,q_1 + 0\,(E^2)$$
$$T = T_o + ET_1 + 0\,(E^2)$$

Substituting 3.36 in equation 3.33 and 3.34 we get the following set of equations

3.37 $$q_o'' + Sq_o' - (M + \frac{1}{K} + i\Omega)q_o = -GT_o$$

3.38 $$q_1'' + Sq_1' - (M + \frac{1}{K} + i\Omega)q_1 = -GT_1$$

3.39 $$T_o'' + SPT_o' = 0$$

3.40 $$T_1'' + SPT_1' = -P\,q_o'\,\bar{q}_o'$$

where a dash represents differentiation with respect to z.

The boundary conditions for q_o, q_1, T_o and T_1 are

3.41
$$q_o = 1,\quad q_1 = 0,\quad T_o = 1,\quad T_1 = 0 \quad \text{at} \quad z = 0$$
$$q_o \rightarrow 0,\ q_1 \rightarrow 0,\quad T_o \rightarrow 0,\ T_1 \rightarrow 0 \quad \text{as } z \rightarrow \infty$$

Solutions of equations 3.37 — 3.40 subject to boundary condition 3.41 are as follows

3.42 $$q_o = A_2 e^{-B_1 z} - A_1 e^{-SPz}$$

3.43 $$q_1 = L_2 e^{-B_1 z} - A_7 e^{-SPz} + A_8 e^{-(B_1+\overline{B}_1)z}$$

$$- A_9 e^{-(SP+B_1)z} - A_{10} e^{-(SP+\overline{B}_1)z} + A_{11} e^{-2SPz}$$

3.44 $$T_o = e^{-SPz}$$

3.45 $$T_1 = L_1 e^{-SPz} - A_3 e^{-(B_1+\overline{B}_1)z}$$

$$+ A_4 e^{-(SP+B_1)z} + A_5 e^{-(SP+\overline{B}_1)z} - A_6 e^{-2SPz}$$

whence from 3.36 we obtain the expression for Q and T.

In the absence of rotation and for S=1 the solutions given by 3.42 — 3.45 reduce to those given by 3.16 — 3.19 in Part A

If t_x and t_y are the axial and transverse components of skin friction we obtain

3.46 $$t_x + it_y = \left.\frac{dQ}{dz}\right|_{z=0}$$

$$= -A_2 B_1 + A_1 SP + E[-L_2 B_1 + A_7 SP$$

$$- A_8 (B_1+\overline{B}_1) + A_9 (SP+B_1) + A_{10} (SP+\overline{B}_1) - 2A_{11} SP]$$

where the different constants are defined in the appendix.

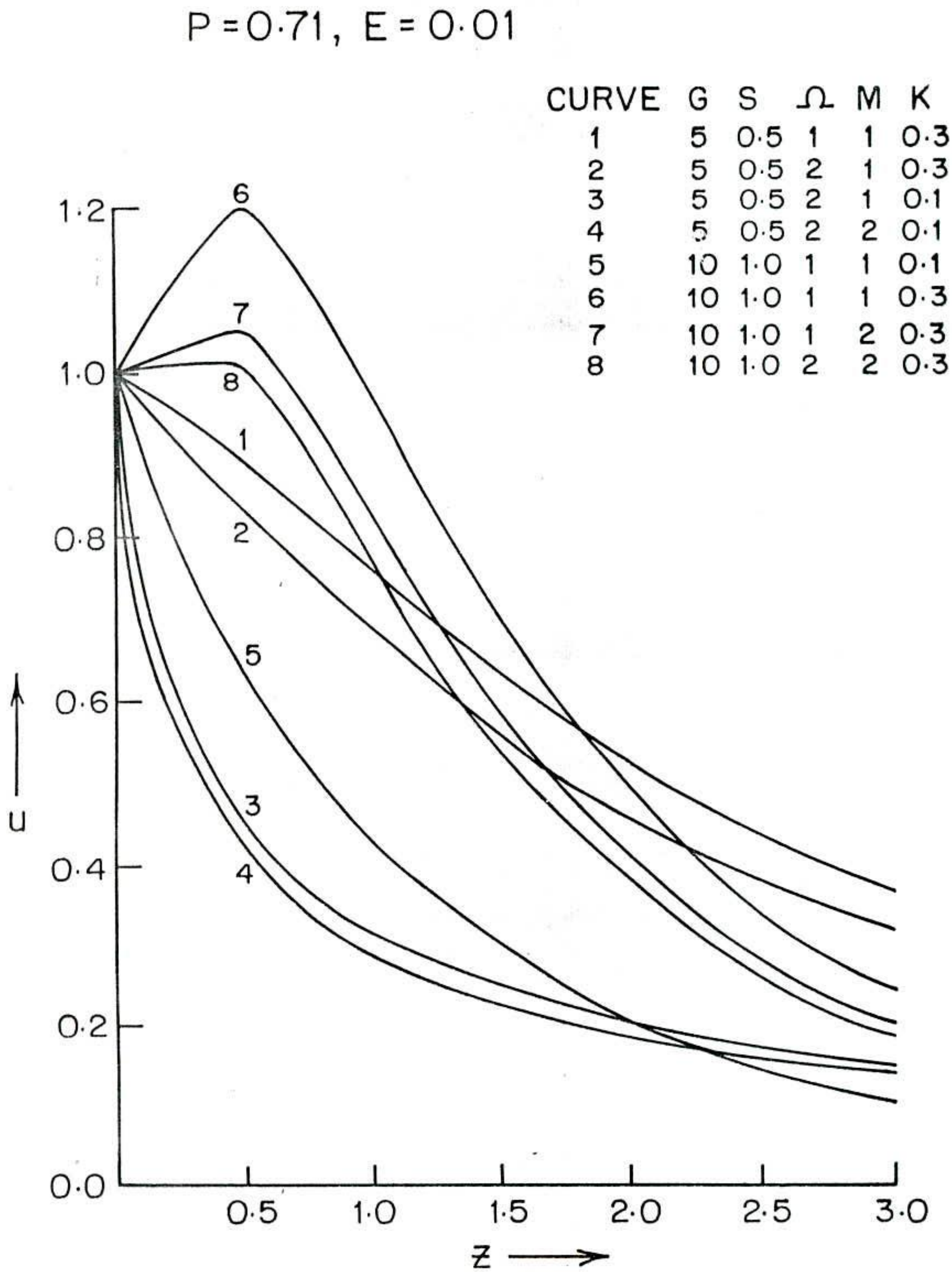

FIG. 3·2 PRIMARY VELOCITY DISTRIBUTION u AGAINST z

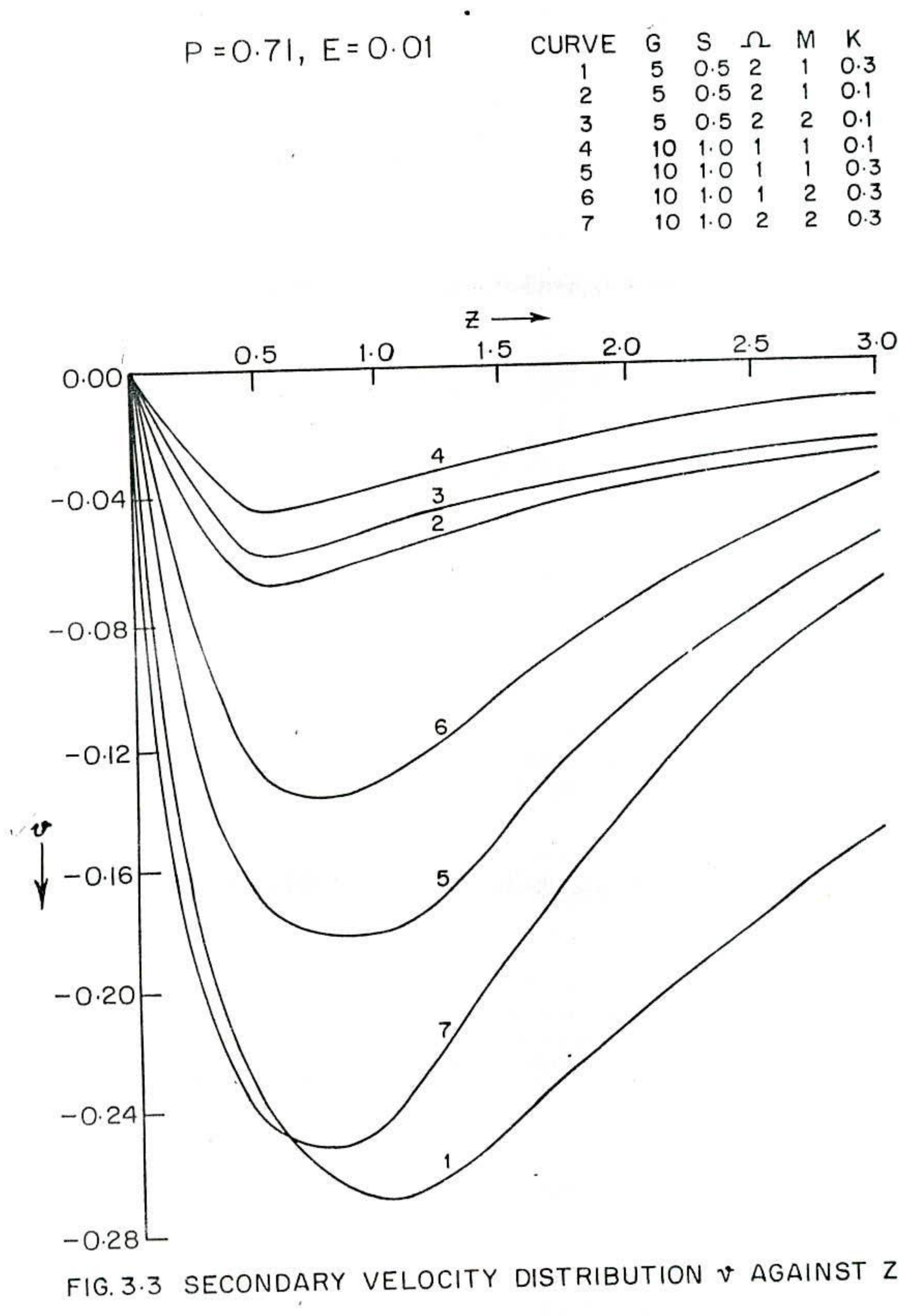

FIG. 3·3 SECONDARY VELOCITY DISTRIBUTION v AGAINST Z

Table 3.3

Numerical values of skin friction components τ_x and τ_y, P= 071, E= 0.01, G=5.

K	S	Ω	M	τ_x	τ_y
.1	0.5	1.0	1	2.1255	0.2138
			2	2.3312	0.1999
		2.0	1	2.1479	0.4248
			2	2.3501	0.3977
	1.0	1.0	1	2.4571	0.2074
			2	2.6571	0.1945
		2.0	1	2.4779	0.4124
			2	2.6748	0.3871
.3	0.5	1.0	1	0.1269	0.4760
			2	0.5465	0.3934
		2.0	1	0.2677	0.9061
			2	0.6408	0.7620
	1.0	1.0	1	0.5563	0.4384
			2	0.9473	0.3680
		2.0	1	0.6760	0.8404
			2	1.0293	0.7156

Discussion of the Results:

In order to study the effects of the various parameters on the primary and secondary velocities the components of skin friction due to primary and secondary flows and the temperature field, numerical calculations have been carried out for different values of the parameters. In order to be realistic the value of the Prandtl number P is chosen to be 0.71 which corresponds to air. In the entire calculation we have taken $E = 0.01$.

The primary velocity profiles u against z have been displayed in figure 3.2. It is clear from the figure that u decreases with increase in rotation parameter Ω and magnetic parameter M, but it increases with the permeability K of the medium. Therefore we conclude that rotation and magnetic field exert a retording influence on the primary velocity.

The secondary velocity profiles v against z are shown in figures 3.3. It is seen from the figure that v takes negative values and it decreases with increase in Ω and K. The effect of the M is to increase the secondary velocity v, that is, it increases with increase in M.

Table 3.3 gives the numerical values of the skin friction components τ_x and τ_y. It is clear from the table that τ_x increases with the increase in magnetic parameter and rotation parameter, but decreases with the permeability K of the medium. τ_y decreases with increase in M but increases with the increase in rotation parameter Ω and K.

Values of the temperature are given in table ?.4. From the table we conclude that the temperature increases with increase in rotation parameter and K whereas it decreases with increase in M. The change in temperature with the various parameters is rather small.

APPENDIX

$$M_1 = M + \frac{1}{K} + iQ, \qquad B_1 = \frac{S + (S^2 + 4M_1)^{1/2}}{2}$$

$$A_1 = \frac{G}{S^2P^2 - S^2P - M_1}, \qquad A_2 = 1 + A_1$$

$$A_3 = \frac{P|A_2|^2 |B_1|^2}{(B_1+\bar{B}_1)^2 - SP(B_1+\bar{B}_1)}, \qquad A_4 = \frac{SP^2 A_2 \bar{A}_1}{B_1 + SP}$$

$$A_5 = \bar{A}_4 \qquad A_6 = \frac{P|A_1|^2}{2}$$

$$L_1 = A_3 - A_4 - A_5 + A_6$$

$$A_7 = \frac{GL_1}{S^2P^2 - S^2P - M_1} \qquad A_8 = \frac{GA_3}{(B_1+\bar{B}_1)^2 - S(B_1+\bar{B}_1) - M_1}$$

$$A_9 = \frac{GA_4}{(SP+B_1)^2 - S(SP+B_1) - M_1}, \qquad A_{10} = \frac{GA_5}{(SP+\bar{B}_1)^2 - S(SP+\bar{B}_1) - M_1}$$

$$A_{11} = \frac{GA_6}{4S^2P^2 - 2S^2P - M_1},$$

$$L_2 = A_7 - A_8 + A_9 + A_{10} - A_{11}.$$

CHAPTER – IV

FREE CONVECTION EFFECTS ON THE HYDROMAGNETIC OSCILLATORY FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH CONSTANT HEAT FLUX

Introduction:

The flow of an incompressible viscous fluid past an impulsively started horizontal plate in its own plane was first studied by Stokes (1851). Georgantopoulos (1979) has discussed the free convection effects of the oscillatory flow in the stokes problem past an infinite porous vertical plate with constant suction. Kafousias et al (1980) have extended the above problem in the presence of a transverse magnetic field without taking into account the induced magnetic field.

In the present chapter we have reviewed the problem of Kafousias et al (1980) where we have considered the effect of constant heat flux at the plate. Hence the object of the present chapter is to study the free convection effects on the flow of an incompressible, viscous and electrically conducting fluid past an impulsively started infinite vertical porous plate in presence of a constant transverse magnetic field; the magnetic Reynolds number of the flow is assumed to be small so that the induced magnetic field has been neglected. The free stream oscillates in time about a constant mean value. The flow is subjected to constant suction through the porous plate and there is constant heat flux at the plate. Approximate solutions to the coupled non-linear equations relevant to the problem have been obtained. The effects of the various parameters on the mean flow and transient flow are discussed with the help of tables and graphs.

Mathematical Analysis:

We consider the 2-dimensional unsteady free convection flow of an electrically conducting, incompressible and viscous fluid past an infinite vertical porous plate. Initially the porous plate is at rest but at time $t'>0$ it starts moving impulsively in its own plane with a constant velocity U_o and heat is also being supplied to the plate at a constant rate. The flow is assumed to be in the x'-direction which is taken along the vertical plate in the upward direction and y'-axis is taken normal to the plate. A uniform magnetic field of strength H_o is acting normal to the plate; assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison with the applied magnetic field. The free stream is assumed to oscillate in time about a constant mean.

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term.

The equations of motion relevant to the present problem are,

$$4.1 \qquad \frac{\partial v'}{\partial y'} = 0$$

$$4.2 \qquad \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g\,\beta\,(T' - T'_\infty) + \nu \frac{\partial u'}{\partial y'^2} + \frac{\sigma \mu_e^2 H_o^2}{\rho}(U' - u')$$

$$4.3 \qquad \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p} \left(\frac{\partial u'}{\partial y'} \right)^2$$

The boundary conditions are,

$$t' \leq 0: \; u'(y', t') = 0, \quad T'(y', t') = T'_\infty$$

$$4.4 \qquad t' > 0: \; u'(0, t) = U_o, \; \frac{\partial T'}{\partial y'} = \frac{-q'}{k}$$

$$u'(\infty, t') = U'(t'), \; T'(\infty, t') = T'_\infty$$

Equation 4.1 integrates to

$$4.5 \qquad v' = -v_o \; (v_o > 0)$$

where v_o is the constant suction velocity at the plate, the negative sign indicates that there is suction at the plate.

Introducing the following non-dimensional quantities

$$y = \frac{y' U_o}{\nu}, \quad t = t' U_o^2/\nu, \quad u = u'/U_o, \quad S = \frac{v_o}{U_o}$$

4.6 $$T = \frac{T' - T'_\infty}{\frac{q' \nu}{k U_o}}, \quad U = \frac{U'}{U_o}, \quad G = \frac{\nu^2 g \beta q'}{k\, U_o^4}$$

$$P = \frac{\rho \nu c_p}{k}, \quad E = \frac{k\, U_o^{\,3}}{q' \nu c_p}$$

$$n = \frac{n' \nu}{U_o^{\,2}}, \quad M = \frac{\sigma \mu_e^2\, H_o^2 \nu}{\rho\, U_o^2}$$

in equations 4.2 and 4.3, we get

4.7 $$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y} = \frac{dU}{dt} + GT + \frac{\partial^2 u}{\partial y^2} + M(U-u)$$

4.8 $$P \frac{\partial T}{\partial t} - SP \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + PE \left(\frac{\partial u}{\partial y} \right)^2$$

and the corresponding boundary conditions assume the form

$$u = 1, \quad \frac{\partial T}{\partial y} = -1 \quad \text{at} \quad y = 0$$

4.9 $t>0$:

$$u = U(t), \; T = 0 \quad \text{at} \quad y = \infty$$

To solve these coupled non-linear equations, we assume following Lighthill (1954) that the unsteady flow is superimposed on the mean steady flow. Hence we write in the neighbourhood of the plate,

4.10

$$u(y,t) = u_o(y) + \varepsilon u_1(y) e^{int}$$

$$T(y,t) = T_o(y) + \varepsilon T_1(y) e^{int}$$

and the free stream is given by,

4.11 $$U = 1 + \varepsilon \, e^{int}$$

where ε is a positive constant ($\varepsilon \ll 1$), and u_o and T_o are the mean and u_1 and T_1 are the corresponding unsteady components of oscillatory motion.

Substituting 4.10 and 4.11 in equations 4.7 and 4.8 we obtain the governing differential equations for u_o, T_o, u_1 and T_1 to be

4.12 $$u_o'' + Su_o' - M \, u_o = - M - GT_o$$

4.13 $$T_o'' + SP\ T_o' = -PE\ u_o'^2$$

4.14 $$u_1'' + Su_1' - (M+in)\ u_1 = -in - M - G\ T_1$$

4.15 $$T_1'' + SPT_1' - in\ PT_1 = -2PE\ u_o'\ u_1'$$

where primes denote differentiation with respect to y.

The boundary conditions are

4.16
$$u_o = 1,\quad u_1 = 0,\quad T_o' = -1,\quad T_1' = 0 \quad \text{at} \quad y = 0$$
$$u_o = 1,\quad u_1 = 1,\quad T_o = 0,\quad T_1 = 0 \quad \text{at} \quad y = \infty$$

The system of equations 4.12 to 4.15 is still coupled and non-linear and in order to solve them we expand u_o, u_1, T_o and T_1 as

4.17
$$u_o = u_{o1} + Eu_{o2} + 0(E^2)$$
$$u_1 = u_{11} + Eu_{12} + 0(E^2)$$
$$T_o = T_{01} + ET_{o2} + 0(E^2)$$
$$T_1 = T_{11} + ET_{12} + 0(E^2)$$

where E the Eckert number is very small for incompressible fluids ($E \ll 1$).

Introducing equations 4.17 into equations 4.12 to 4.15 we obtaine the following system of equations 4.18 — 4.21 and 4.22 — 4.25 which govern the mean steady flow and the oscillatory one.

4.18 $$u''_{o1} + Su'_{o1} - Mu_{o1} = -M - GT_{o1}$$

4.19 $$u''_{o2} + Su'_{o2} - Mu_{o2} = -G\,T_{o2}$$

4.20 $$T''_{o1} + SPT'_{o1} = 0$$

4.21 $$T''_{o2} + SPT'_{02} = -Pu'^{2}_{o1}$$

4.22 $$u''_{11} + Su'_{11} - (M+in)u_{11} = -(M+in) - GT_{11}$$

4.23 $$u''_{12} + Su'_{12} - (M+in)u_{12} = -GT_{12}$$

4.24 $$T''_{11} + SPT'_{11} - in\,PT_{11} = 0$$

4.25 $$T''_{12} + SPT'_{12} - in\,PT_{12} = -2P(u'_{o1}\,u'_{11})$$

subject to boundary conditions

$$u_{o1} = 1,\ u_{o2} = 0,\ T'_{o1} = -1,\ T'_{o2} = 0 \quad \text{at} \quad y = 0$$

4.26

$$u_{o1} = 1,\ u_{o2} = 0,\ T_{o1} = 0,\ T_{o2} = 0 \quad \text{at} \quad y = \infty$$

For the mean steady flow, and

$$u_{11} = 0,\ u_{12} = 0,\ T'_{11} = 0,\ T'_{12} = 0 \quad \text{at} \quad y = 0$$

4.27

$$u_{11} = 1,\ u_{12} = 0,\ T_{11} = 0,\ T_{12} = 0 \quad \text{at} \quad y = \infty$$

for the oscillatory flow.

First we proceed to obtain the solution for the mean flow, while the solution of the unsteady flow field will be presented later.

The solutions of equations 4.18 — 4.21 under boundary condition 4.26 are given by

4.28 $$u_{o1} = A_1 e^{-B_1 y} - A_1 e^{-SPy} + 1$$

4.29 $$u_{o2} = L_2 e^{-B_1 y} - F_1 e^{-SPy} + F_2 e^{-2B_1 y}$$

$$+ F_3 e^{-2SPy} - F_4 e^{-(B_1 + SP)y}$$

4.30 $$T_{o1} = \frac{1}{SP} e^{-SPy}$$

4.31 $$T_{o2} = L_1 e^{-SPy} - D_1 e^{-2B_1 y} - D_2 e^{-2SPy} + D_3 e^{-(B_1+SP)y}$$

where the different constants are defined in the appendix at the end of the chapter.

Hence the velocity and temperature fields for the mean steady flow are given respectively by,

$$u_o = u_{o1} + E\, u_{o2}$$

and

$$T_o = T_{o1} + E\, T_{o2} \text{ from } 4.17$$

where u_{o1}, u_{o2}, T_{o1} and T_{o2} are given by 4.28 – 4.31,

Knowing the mean velocity we now calculate the mean skin friction τ_o due to mean steady flow. In non-dimensional form it is given by

4.32 $$\tau_o = \left.\frac{\partial u_o}{\partial y}\right|_{y=0}$$

$$= -A_1B_1 + A_1SP + E\left[-L_2B_1 + F_1SP - 2B_1F_2 - 2SPF_3 + F_4(B_1+SP)\right]$$

Now we proceed to obtain the solution for the unsteady part of the flow field. The unsteady flow field is described by the equations 4.22 – 4.25, viz.

4.22 $$u''_{11} + Su'_{11} - (M+in)u_{11} = -(M+in) - GT_{11}$$

4.23 $$u''_{12} + Su'_{12} - (M+in)u_{12} = -GT_{12}$$

4.24 $$T''_{11} + SPT'_{11} - in\ P\ T_{11} = 0$$

4.25 $$T''_{12} + SPT'_{12} - in\ P\ T_{12} = -2P(u'_{01}\ u'_{11})$$

under the boundary conditions.

4.27 $$u_{11} = 0,\ u_{12} = 0,\ T'_{11} = 0,\ T'_{12} = 0 \quad \text{at } y = 0$$
$$u_{11} = 1,\ u_{12} = 0,\ T_{11} = 0,\ T_{12} = 0 \quad \text{at } y = \infty$$

The solution of the above equations of the unsteady steady flow under their boundary conditions are given by,

4.33 $$u_1(y) = u_{11}(y) + Eu_{12}(y)$$

$$= 1 - e^{-P_1 y} + E[X_6 e^{-P_1 y} - X_7 e^{-P_2 y}$$

$$- X_4 e^{-(B_1+P_1)y} + X_5 e^{-(SP+P_1)y}]$$

4.34 $$T_1(y) = T_{11}(y) + ET_{12}(y)$$

$$= E [Ke^{-P_2 y} + X_1 e^{-(B_1+P_1)y} - X_2 e^{-(SP+P_1)y}]$$

where all the constants are defined in the appendix at the end of the chapter.

Now, since we know u_o, u_1, T_o and T_1 we obtain the expression for u and T from 4.10, viz.

$$u(y,t) = u_o(y) + \varepsilon\, e^{int} u_1(y)$$

4.10

$$T(y,t) = T_o(y) + \varepsilon\, e^{int} T_1(y)$$

The expressions for u and T may be written as

4.35 $$u(y,t) = u_o(y) + \varepsilon\, [(M_r \cos nt - M_i \sin nt)$$

$$+ i(M_i \cos nt + M_r \sin nt)]$$

and

4.36 $$T(y,t) = T_o(y) + \varepsilon\, [(T_r \cos nt - T_i \sin nt)$$

$$+ i\,(T_i \cos nt + T_r \sin nt)]$$

where $u_1 = M_r + i\, M_i$

4.37 $$T_1 = T_r + i\, T_i$$

From the expressions 4.35 and 4.36 we can obtain the expressions for transient velocity and transient temperature respectively for $nt = \frac{\pi}{2}$ as

4.38 $$u(y, \frac{\pi}{2n}) = u_o - \varepsilon M_1$$

and,

4.39 $$T(y, \frac{\pi}{2n}) = T_o - \varepsilon T_1$$

(neglecting the imaginary part)

The skin friction τ is given by

4.40 $$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \left.\frac{\partial u_o}{\partial y}\right|_{y=0} + \varepsilon e^{int} \left.\frac{\partial u_1}{\partial y}\right|_{y=0}$$

$$= \tau_o + \varepsilon e^{int}[P_1 + E(-X_6 P_1 + X_3 P_2 + X_4(B_1 + P_1)$$

$$- X_5 (SP + P_1)]$$

where,

$$\tau_o = \left.\frac{\partial u_o}{\partial y}\right|_{y=0}$$

The skin friction for $nt = \frac{\pi}{2}$ is given by

4.41 $$\tau = \tau_o - \varepsilon B_i$$

(neglecting the imaginary part).

4.42 where $$B_r + i B_i = \left.\frac{\partial u_1}{\partial y}\right|_{y=0}.$$

$P = 0.71, \; E = 0.001$

CURVE	G	S	M
1	5	1	0
2	5	1	4
3	10	1	4
4	10	1	8
5	10	2	4

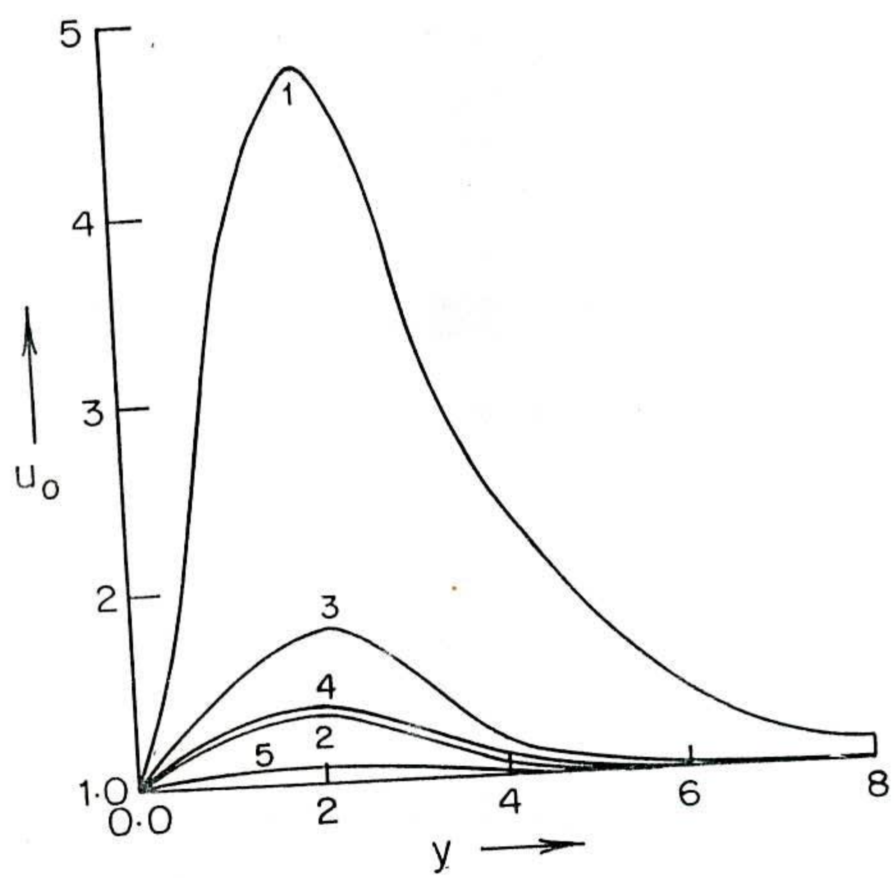

FIG. 4·1 MEAN VELOCITY DISTRIBUTION u_0 AGAINST y

Table 4.1

Values of mean temperature T_o (P=0.71, E=0.001).

S = 1

y	G = 5 M=0	G = 5 M=4	G = 5 M=8	G = 10 M=0	G = 10 M=4	G = 10 M=8
0	1.43721	1.40992	1.40909	1.52351	1.41432	1.41100
2	0.35127	0.34091	0.34063	0.38378	0.34222	0.34121
4	0.08595	0.08241	0.08233	0.09696	0.08278	0.08248
6	0.02090	0.01992	0.01990	0.02395	0.02001	0.01993
8	0.00506	0.00481	0.00481	0.00583	0.00483	0.00481

S = 2

y	G = 5 M=0	G = 5 M=4	G = 5 M=8	G = 10 M=0	G = 10 M=4	G = 10 M=8
0	0.70467	0.70431	0.70427	0.70602	0.70460	0.70442
2	0.04120	0.04115	0.04114	0.04137	0.04118	0.04116
4	0.00240	0.00240	0.00240	0.00240	0.00240	0.00240
6	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 4.2

Values of mean skin-friction τ_0 (P=0.71, E=0.001)

G = 5

M	S=1	S=2
0	10.20536	2.48190
4	3.10378	1.32590
8	2.28587	1.02965
	G = 10	
0	22.13085	4.97725
4	6.22907	2.65308
8	4.57846	2.05979

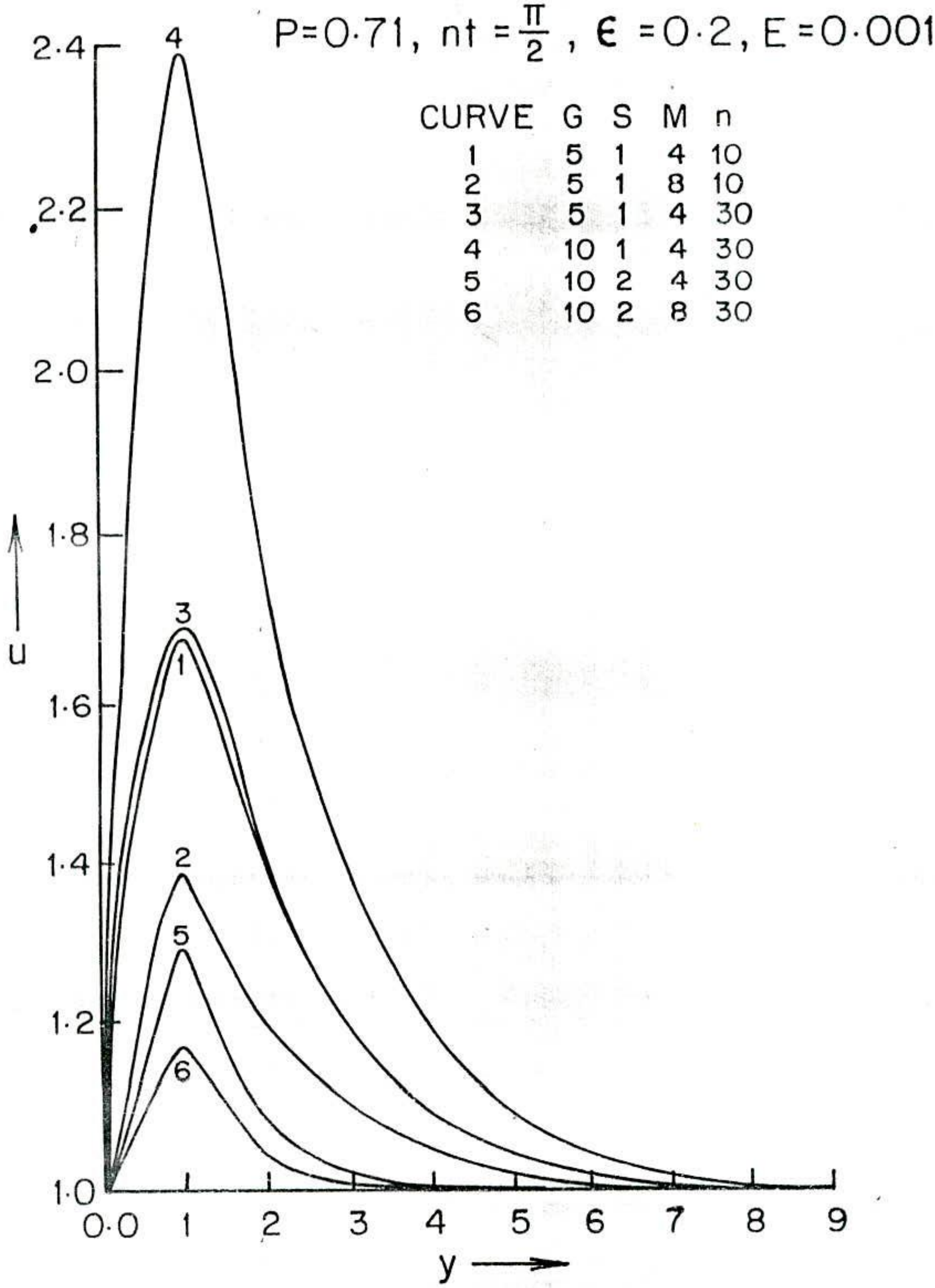

FIG. 4·2 TRANSIENT VELOCITY DISTRIBUTION u AGAINST

Table 4.3

Variation of transient temperature T in air ($P=0.71$, $nt = \frac{\pi}{2}$, $\xi=0.2$, $\varepsilon= 0.001$)

	S	M	n	y	0	1	2	3	4	5	6	7	8	9
5	1	0	10	T	1.43747	0.71082	0.35127	0.17385	0.08595	0.04242	0.02090	0.10292	0.00506	0.00249
5	1	4	10		1.40998	0.69333	0.34091	0.16762	0.08241	0.04051	0.01992	0.00979	0.00481	0.00236
5	1	8	10		1.40914	0.69283	0.34063	0.16747	0.08233	0.04048	0.01990	0.00978	0.00481	0.00236
5	2	8	10		0.70429	0.17023	0.04114	0.00994	0.00240	0.00058	0.00014	0.00003	0.00000	0.00000
5	1	4	30		1.40996	0.69331	0.34091	0.16762	0.08241	0.04051	0.01992	0.00979	0.00481	0.00236
0	1	4	10		1.41446	0.69592	0.34231	0.16835	0.08278	0.04070	0.02001	0.00983	0.00483	0.00237
0	1	4	30		1.41442	0.69589	0.34232	0.16835	0.08278	0.04070	0.02001	0.00983	0.00483	0.00237
0	2	4	30		0.70463	0.17035	0.04118	0.00995	0.00240	0.00058	0.00014	0.00003	0.00000	0.00000
0	2	8	30		0.70444	0.17028	0.04116	0.00994	0.00240	0.00058	0.00014	0.00003	0.00000	0.00000
0	2	8	10		0.70445	0.17029	0.04116	0.00994	0.00240	0.00058	0.00014	0.00003	0.00000	0.00000

Table 4.4

Values of skin friction τ P=0.71, G=5,

E=0.001, ϵ=0.2, nt= $\frac{\pi}{2}$,

	n = 10		n = 30	
M	S=1	S = 2	S = 1	S = 2
0	9.7642	2.0565	9.4341	1.7201
4	2.7401	0.9743	2.3820	0.6129
8	1.9789	0.7312	1.6096	0.3615

<u>**Discussion of the Results:**</u>

In order to get physical insight into the problem, numerical calculations have been carried out for u_o, T_o, τ_o, the transient velocity, the transient temperature and skin friction at $nt = \frac{\pi}{2}$ corresponding to different values of the parameters. In the entire calculation we have taken E=0.001 and the value of the Prandtl number P is chosen to be 0.71 which corresponds to air.

The mean velocity profiles u_o against y have been displayed in Fig. 4.1. From the figure we conclude that u_o increases with increase in G. The fluid mean velocity is greater in the hydrodynamic case (M=0) than in the hydro-magnetic case. The mean velocity decreases with increase in the magnetic parameter M and suction parameter S.

Values of the mean temperature T_o is given in Table 4.1. It is clear from the table that T_o decreases with increase in M and S. It increases with increase in G. Values of the mean skin friction τ_o is given in Table 4.2. From the table we conclude that the mean skin friction in-crease with increase in G but decreases with increase in M and S.

Transient velocity profiles $u(y, \frac{\pi}{2n})$ against y have been displayed in Fig. 4.2. From the figure it is clear that the transient velocity decreases with increase in suction parameter S and magnetic parameter M, but increases with increase in Grashof number G. The transient velocity increases with increase in the frequency parameter n, near to the porous plate, but away from the plate the influence of n is insignificant.

Table 4.3 shows the values of the transient temperature $T(y,\frac{\pi}{2n})$. The effect of M and S is to decrease the transient temperature whereas rise in G causes a rise in $T(y,\frac{\pi}{2n})$. The transient temperature decreases with increase in n near the plate, but away from the plate the effect of n is insignificant.

Values of skin friction τ for $nt=\frac{\pi}{2}$ are given in Table 4.4. From the table we conclude that the skin friction decreases with increase in M, and S and n.

APPENDIX

$$A_1 = \frac{G}{SP(S^2P^2 - S^2P - M)}, \qquad B_1 = \frac{S + (S^2 + 4M)^{1/2}}{2}$$

$$C_1 = \frac{A_1^2\ B_1^2}{S(2B_1 - SP)}, \qquad C_2 = A_1^2\ P$$

$$L_1 = C_1 - C_2$$

$$D_1 = \frac{SPC_1}{2B_1}, \qquad D_2 = \frac{C_2}{2}, \qquad D_3 = \frac{2SPC_2}{B_1 + SP}$$

$$F_1 = \frac{GL_1}{S^2P^2 - S^2P - M}, \qquad F_2 = \frac{GD_1}{4B_1^2 - 2B_1S - M}$$

$$F_3 = \frac{GD_2}{4S^2P^2 - 2S^2P - M}, \qquad F_4 = \frac{GD_3}{(B_1 + SP)^2 - S(B_1 + SP) - M}$$

$$L_2 = F_1 - F_2 - F_3 + F_4$$

$$M_1 = M + in$$

$$P_1 = \frac{S + (S^2 + 4M_1)^{1/2}}{2}$$

$$P_2 = \frac{SP + (B^2P^2 + 4in\ P)^{1/2}}{2}$$

$$K_1 = \frac{2PP_1\ A_1B_1(B_1+P_1)}{P_2[(B_1+P)^2 - SP(B_1+P_1) - in\ P}$$

$$K_2 = \frac{2SP^2\ A_1P_1(SP+P_1)}{P_2[(SP+P_1)^2 - SP(SP+P_1) - in\ P]}$$

$$K = K_2 - K_1$$

$$X_1 = \frac{K_1\ P_2}{B_1 + P_1}, \qquad X_2 = \frac{K_2\ P_2}{SP + P_1}$$

$$X_3 = \frac{GK}{P_2^2 - SP_2 - M_1} \qquad X_4 = \frac{GX_1}{(B_1+P_1)^2 - S(B_1+P_1) - M_1}$$

$$X_5 = \frac{GX_2}{(SP+P_1)^2 - S(SP+P_1) - M_1} \qquad X_6 = X_3 + X_4 - X_5.$$

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CHAPTER — V

HALL EFFECTS ON MHD FLOW PAST AN INFINITE VERTICAL POROUS PLATE WHEN PLATE TEMPERATURE OSCILLATES IN TIME ABOUT A CONSTANT MEAN

Introduction:

The unsteady free convection flow of an incompressible and viscous fluid past an infinite vertical unmoving porous plate, with constant suction has been studied by Soundalgekar (1972). The plate temperature was considered to oscillate in time about a constant mean. Soundalgekar and Wavre (1977) have extended the above problem, taking into account the effects of mass transfer. However, the flow past plates started impulsively from rest plays an important role. These are particularly important in the design of space ships, solar energy collectors etc. On the other hand the effects of a magnetic field on the flow of an electrically conducting fluid have many technical applications e.g. in the boundary layer flow of high speed air-craft, in the region between the surface of blunt body and its shock wave, etc. However, if the strength of the magnetic field is strong, one can not neglect the effects of Hall currents.

Hence, the object of the present chapter is to study the effects of Hall currents on the MHD free convective flow past an impulsively started infinite, vertical porous plate in the presence of a strong transverse magnetic field; the plate temperature is considered to oscillate in time about a constant mean. The flow is subjected to constant suction through the porous plate. Hall currents give rise to a cross flow making the flow three-dimensional; the magnetic Reynolds number of the flow is taken to be small enough so the induced magnetic field is negligible. Approximate solutions to the coupled non-linear equations, occuring in the problem have been obtained. The effects of the various parameters on the mean flow and transient flow have been discussed with the help of tables and graphs.

Mathematical Analysis:

We consider the unsteady free convective flow of an electrically conducting, incompressible and viscous fluid past an infinite vertical porous plate. The x'-axis is taken along the plate in the vertical upward direction and y'-axis is normal to the plate. Initially the fluid and the plate are at rest but at time $t' > 0$ the plate starts moving impulsively in its own plane with constant velocity U_o. A uniform magnetic field of strength H_o is acting transverse to the plate. The plate temperature is considered to oscillate in time about a constant mean. In the present problem the pressure is assumed to be constant.

Since the plate is infinite in extent all physical quantities are functions of y' and t' only. The fluid is subjected to constant suction at the plate and hence if $\overline{V} = (u', v', w')$ the equation of continuity gives $v' = -v_o$ where v_o is the constant suction velocity. Using the relation $\nabla.\overline{H} = 0$ for the magnetic field $\overline{H} = (H_x, H_y, H_z)$ we obtain $H_y = H_o$ everywhere in the fluid (H_o is the constant externally applied magnetic field). If $\overline{J} = (J_x, J_y, J_z)$ is the current density, from the relation $\nabla.\overline{J} = 0$ we have J_y = constant. Since the plate is non-conducting $J_y = 0$ at the plate and hence zero everywhere. Assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison with the applied magnetic field. The generalized Ohm's law, taking Hall current into account, (Cowling (1957)) in the absence of electric field is of the form,

$$5.1 \qquad \overline{J} + \frac{w_e \tau_e}{H_o} \overline{J} \times \overline{H} = \sigma (\mu_e \overline{V} \times \overline{H} + \frac{1}{en_e} \nabla p_e)$$

Under the usual assumption that the electron pressure (for a weakly ionized gas), the thermoelectric pressure and ion slip are negligible we have from 5.1.

$$5.2 \qquad J_x - w_e \tau_e J_z = -\sigma \mu_e H_o w'$$

$$5.3 \qquad J_z + w_e \tau_e J_x = \sigma \mu_e H_o u'$$

from which we get

5.4 $$J_x = \frac{\sigma\mu_e H_o}{1+m^2}(mu' - w')$$

5.5 $$J_z = \frac{\sigma\mu_e H_o}{1+m^2}(u' + mw')$$

where,

σ	—	the electric conductivity
μ_e	—	the magnetic permeability
w_e	—	the cyclotron frequency
τ_e	—	the electron collision time
e	—	the electric charge
n_e	—	the number density of electron
p_e	—	the electron pressure
$m = w_e\tau_e$	—	the Hall parameter

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term. The basic equations relevant to the problem are, 5.6 $$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = g\beta (T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^2 H_o^2}{\rho (1+m^2)} (u' + mw')$$

5.7 $$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} = \nu \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma \mu_e^2 H_o^2}{\rho (1+m^2)} (mu' - w')$$

5.8 $$\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p} \left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2 \right]$$

where all the physical quantities have their usual meanings. The initial and boundary conditions are:

5.9
$$t' \leq 0: \quad u' = 0, \quad w' = 0, \quad T' = T'_\infty \quad \forall \quad y'$$

$$t' > 0: \quad u' = U_o, \; w' = 0, \quad T' = T_w' \, (1 + \varepsilon e^{in't'}) - T'_\infty \varepsilon e^{in't'} \quad \text{at } y' = 0$$

$$u' = 0, \quad w' = 0, \quad T' = T'_\infty \quad \text{at } y' = \infty$$

Introducing the following non-dimensional quantities.

$$y = \frac{y' U_o}{\nu}, \quad t = \frac{t' U_o^2}{\nu}, \quad n = \frac{n' \nu}{U_o^2}, \quad u = \frac{u'}{U_o}$$

$$5.10 \quad M = \frac{\sigma\mu_e^2 H_o^2 \nu}{\rho U_o^2}, \quad S = \frac{v_o}{U_o}, \quad w = \frac{w'}{U_o}, \quad P = \frac{\mu C_p}{k}$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad G = \frac{\nu g\beta (T'_w - T'_\infty)}{U_o^3}, \quad E = \frac{U_o^2}{C_p (T'_w - T'_\infty)}$$

in equations 5.6 to 5.8 we get

$$5.11 \qquad \frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} - \delta \; (u + mw)$$

$$5.12 \qquad \frac{\partial w}{\partial t} - S \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \delta \; (mu - w)$$

$$5.13 \qquad P \frac{\partial T}{\partial t} - SP \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + PE \left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right]$$

where, $\delta = \dfrac{M}{1+m^2}$

Boundary conditions 5.9 become in non-dimensional form,

$$u = 1, \quad w = 0, \quad T = 1 + \varepsilon e^{int} \quad \text{at } y = 0$$

5.14 $t > 0$:

$$u = 0, \quad w = 0, \quad T = 0 \quad \text{at } y = \infty$$

The task of solving equations 5.11 – 5.13 under boundary conditions 5.14 is quite complicated. To simplify the mathematical part, we introduce a complex variable defined a

5.15 $$Q = u + iw$$

which enables us to combine equations 5.11 and 5.12 into a single equation of the form

5.16 $$\frac{\partial^2 Q}{\partial y^2} + S\frac{\partial Q}{\partial y} - \delta\,(1-im)Q - \frac{\partial Q}{\partial t} = -\,GT$$

equation 5.13 with the help of 5.15 becomes

5.17 $$\frac{\partial^2 T}{\partial y^2} + SP\frac{\partial T}{\partial y} - P\frac{\partial T}{\partial t} = -\,PE\left(\frac{\partial Q}{\partial y} \times \frac{\partial \bar{Q}}{\partial y}\right)$$

The corresponding boundary conditions assume the form

5.18 $t>0$:

$$Q = 1, \quad T = 1 + \varepsilon\, e^{int} \quad \text{at } y = 0$$

$$Q = 0, \quad T = 0 \quad \text{at } y = \infty$$

Equations 5.16 and 5.17 are coupled and non-linear. In order to solve them we can represent the velocity and temperature in the neighbourhood of the plate as follows (assuming small amplitude of oscillation)

5.19

$$Q(y,t) = q_o(y) + \varepsilon\, q_1(y)\, e^{int}$$

$$T(y,t) = T_o(y) + \varepsilon\, T_1(y)\, e^{int}$$

Substituting 5.19 in equations 5.16 and 5.17 and equating coefficients of different powers of ε neglecting those of ε^2 and higher powers of ε we obtains the following set of equations:

5.20 $$q_o'' + Sq_o' - M_1 q_o = - GT_o$$

5.21 $$q_1'' + Sq_1' - (M_1 + in)\, q_1 = - GT_1$$

5.22 $$T_o'' + SPT_o' = - PE(q_o' \bar{q}_o')$$

5.23 $$T_1'' + SPT_1' - in\, PT_1 = - PE(q_1' \bar{q}_o' + \bar{q}_1' q_o')$$

where $M_1 = \delta(1 - im)$ and primes denote differentiation with respect to y.

The corresponding boundary conditions are,

5.24
$$q_o = 1, \quad q_1 = 0, \quad T_o = 1, \quad T_1 = 1 \quad \text{at} \quad y=0$$
$$q_o = 0, \quad q_1 = 0, \quad T_o = 0, \quad T_1 = 0 \quad \text{at} \quad y=\infty$$

The equations 5.20 to 5.23 are still coupled and non-linear and hence difficult to solve analytically. In order to solve them we expand q_o, q_1, T_o and T_1 in powers of E the Eckert number assuming it to be very small as follows ($E \ll 1$ for incompressible fluids).

5.25
$$q_o(y) = q_{01}(y) + Eq_{02}(y) + 0(E^2)$$
$$q_1(y) = q_{11}(y) + Eq_{12}(y) + 0(E^2)$$
$$T_o(y) = T_{01}(y) + ET_{02}(y) + 0(E^2)$$
$$T_1(y) = T_{11}(y) + ET_{12}(y) + 0(E^2)$$

Substituting 5.25 in equations 5.20 to 5.23 we obtain the following system of equations 5.26 to 5.29 and 5.30 to 5.33 which govern the mean steady flow and the unsteady one.

5.26
$$q''_{01} + Sq'_{01} - M_1 q_{01} = - GT_{01}$$

5.27
$$q''_{02} + Sq'_{02} - M_1 q_{02} = - GT_{02}$$

5.28
$$T''_{01} + SPT'_{01} = 0$$

5.29
$$T''_{02} + SPT'_{02} = - P(q'_{01}\ \bar{q}'_{01})$$

5.30 $$q''_{11} + Sq'_{11} - q_{11}(M_1 + in) = -GT_{11}$$

5.31 $$q''_{12} + Sq'_{12} - q_{12}(M_1 + in) = -GT_{12}$$

5.32 $$T''_{11} + SPT'_{11} - in\,P\,T_{11} = 0$$

5.33 $$T''_{12} + SP\,T'_{12} - in\,P\,T_{12} = -P(q'_{11}\bar{q}'_{01} + \bar{q}'_{11}q'_{01})$$

subject to the boundary conditions

5.34 $$q_{01} = 1, \quad q_{02} = 0, \quad T_{01} = 1, \quad T_{02} = 0 \quad \text{at } y=0$$

$$q_{01} = 0, \quad q_{02} = 0, \quad T_{01} = 0, \quad T_{02} = 0 \quad \text{at } y=\infty$$

for the mean steady flow and

5.35 $$q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 1, \quad T_{12} = 0 \quad \text{at } y=0$$

$$q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 0, \quad T_{12} = 0 \quad \text{at } y=\infty$$

for the unsteady flow

First, we proceed to obtain the solution for the mean steady flow while the solution for the unsteady flow will be obtained later.

The solutions of equations 5.26 to 5.29 subject to boundary conditions 5.34 are given by

5.36 $$q_{01} = A_2 e^{-B_1 y} - A_1 e^{-SPy}$$

5.37 $$q_{02} = L_2 e^{-B_1 y} - A_7 e^{-SPy} + A_8 e^{-(B_1+\bar{B}_1)y}$$

$$- A_9 e^{-(SP+B_1)y} - A_{10} e^{-(SP+\bar{B}_1)y}$$

$$+ A_{11} e^{-2SPy}$$

5.38 $$T_{01} = e^{-SPy}$$

5.39 $$T_{02} = L_1 e^{-SPy} - A_3 e^{-(B_1+\bar{B}_1)y} + A_4 e^{-(SP+B_1)y}$$

$$+ A_5\, e^{-(SP+\bar{B}_1)y} - A_6\, e^{-2SPy}$$

The expression for mean steady velocity and temperature are given from 5.25 as

$$q_o = q_{01} + E\, q_{02}$$

$$T_o = T_{01} + E\, T_{02}$$

where q_{01}, q_{02}, T_{01} and T_{02} are given by 5.36 to 5.39.

If τ_{mu} and τ_{mw} are the components of mean skin friction τ_o at the plate due to mean primary velocity u_o and mean secondary velocity w_o we have

5.40 $$\tau_o = \tau_{mu} + i\,\tau_{mw} = \left.\frac{dq_o}{dy}\right|_{y=0}$$

$$= -A_2B_1 + A_1SP + E\,[-L_2B_1 + A_7SP - A_8(B_1+\bar{B}_1) + A_9(SP+B_1) + A_{10}(SP+\bar{B}_1) - 2A_{11}SP)$$

where the different constant are defined in the appendix at the end of the chapter.

Now we proceed to solve the unsteady part of the flow field which are characterised by equations 5.30 to 5.33, viz:

5.30 $$q''_{11} + Sq'_{11} - q_{11}(M_1 + in) = -GT_{11}$$

5.31 $$q''_{12} + Sq'_{12} - q_{12}(M_1 + in) = -GT_{12}$$

5.32 $$T''_{11} + SPT'_{11} - in\,PT_{11} = 0$$

5.33 $$T''_{12} + SP\, T'_{12} - in\, PT_{12} = -P(q'_{11}\, \bar{q}'_{01} + \bar{q}'_{11}\, q'_{01})$$

subject to the boundary conditions

$$q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 1, \quad T_{12} = 0 \quad \text{at} \quad y=0$$

5.35

$$q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 0, \quad T_{12} = 0 \quad \text{at} \quad y=\infty$$

The solution of the equations 5.30 to 5.33 of the unsteady flow field under their boundary condition 5.35 are given by,

5.41 $$q_1 = q_{11} + E\, q_{12}$$

$$= D_1 e^{-h_2 y} - D_1 e^{-h_1 y} + E[X_2 e^{-h_2 y} - D_{10}\, e^{-h_1 y}$$

$$+ D_{11} e^{-(h_2+\bar{B}_1)y} - D_{12} e^{-(h_2+SP)y}$$

$$- D_{13}\, e^{-(h_1+\bar{B}_1)y} + D_{14}\, e^{-(h_1+SP)y}$$

$$+ D_{15} e^{-(\bar{h}_2+B_1)y} - D_{16}\, e^{-(\bar{h}_2+SP)y}$$

$$- D_{17} e^{-(\bar{h}_1+B_1)y} + D_{18} e^{-(\bar{h}_1+SP)y}\ ]$$

and

5.42
$$T_1 = T_{11} + ET_{12}$$

$$= e^{-h_1 y} + E[X_1 e^{-h_1 y} - D_2 e^{-(h_2 + \bar{B}_1)y}$$

$$+ D_3 e^{-(h_2 + SP)y} + D_4 e^{-(h_1 + \bar{B}_1)y}$$

$$- D_5 e^{-(h_1 + SP)y} - D_6 e^{-(\bar{h}_2 + B_1)y}$$

$$+ D_7 e^{-(\bar{h}_2 + SP)y} + D_8 e^{-(\bar{h}_1 + B_1)y}$$

$$- D_9 e^{-(\bar{h}_1 + SP)y}]$$

where the constants appearing in the solution are defined in the appendix at the end of this chapter.

Since, now we know q_o, q_1, T_o and T_1 we obtain the expression for Q and T from 5.19 viz-:

$$Q(y,t) = q_o(y) + \varepsilon\, q_1(y)\, e^{int}$$

5.19

and

$$T(y,t) = T_o(y) + \varepsilon\, T_1(y)\, e^{int}$$

The expression for $Q(y,t)$ may be written as, 5.43 $$Q(y,t) = q_o + \varepsilon\,[(M_r \cos nt - M_i \sin nt) + i(M_i \cos nt + M_r \sin nt)]$$

where,

5.44 $$q_1 = M_r + i\,M_i$$

similarly,

5.45 $$T(y,t) = T_o + \varepsilon\,[(T_r \cos nt - T_i \sin t) + i\,(T_i \cos nt + T_r \sin nt)]$$

where,

5.46 $$T_1 = T_r + i\,T_i$$

From the expression 5.43 we can obtain the expression of the transient primary velocity and transient secondary velocity and from expression 5.45 we obtain the expression for transient temperature at $nt = \frac{\pi}{2}$ as

5.47 $$u(y, \tfrac{\pi}{2n}) = u_o(y) - \varepsilon\, M_i$$

5.48 $$w(y, \tfrac{\pi}{2n}) = w_o(y) + \varepsilon\, M_r$$

where,

5.49 $$q_o = u_o + i\, w_o$$

and

5.50 $$T(y, \frac{\pi}{2n}) = T_o(y) - \varepsilon\, T_i$$

neglecting the imaginary part,

where u_o, w_o and T_o are the mean primary velocity, mean secondary velocity and mean temperature respectively.

The skin friction is given by

5.51 $$\tau = \tau_x + i\, \tau_z$$

$$= \left.\frac{\partial Q}{\partial y}\right|_{y=0}$$

$$= \left.\frac{\partial q_o}{\partial y}\right|_{y=0} + \varepsilon\, e^{int} \left.\frac{\partial q_1}{\partial y}\right|_{y=0}$$

$$= \tau_o + \varepsilon\, e^{int}[-D_1 h_2 + D_1 h_1 + E \{ -X_2 h_2$$

$$+D_{10} h_1 - D_{11}(h_2 + \bar{B}_1) + D_{12}(h_2 + SP)$$

$$+D_{13}(h_1 + \bar{B}_1) - D_{14}(h_1 + SP) - D_{15}(\bar{h}_2 + B_1)$$

$$+ D_{16}(\bar{h}_2 + SP) + D_{17}(\bar{h}_1 + B_1) - D_{18}(\bar{h}_1 + SP) \}]$$

where, $\tau_o = \left.\frac{\partial q_o}{\partial y}\right|_{y=0}$ is the mean skin friction.

Table 5.1: Values of mean primary velocity u_o and mean secondary velocity w_o (P=0.71, E=0.005).

M	m	S	y→	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
2	1.0	0.5	u_o	1.0000	1.6225	1.6719	1.5089	1.2912	1.0810	0.8981	0.7451	0.6190	0.5153
			w_o	0.0000	0.6582	0.9620	1.0423	1.0015	0.9050	0.7906	0.6774	0.5741	0.4836
2	0.5	0.5	u_o	1.0000	1.4826	1.5076	1.3684	1.1863	1.0078	0.8487	0.7120	0.5964	0.4994
			w_o	0.0000	0.3613	0.5094	0.5380	0.5080	0.4541	0.3942	0.3367	0.2850	0.2401
4	0.5	0.5	u_o	1.0000	1.0013	0.8731	0.7349	0.6138	0.5124	0.4282	0.3582	0.2998	0.2510
			w_o	0.0000	0.2850	0.3749	0.3312	0.2912	0.2485	0.2095	0.1759	0.1474	0.1235
4	0.5	1.0	u_o	1.0000	0.8194	0.5958	0.4196	0.2933	0.2050	0.1435	0.1005	0.0704	0.0494
			w_o	0.0000	0.2175	0.2239	0.1793	0.1324	0.0947	0.0670	0.0471	0.0331	0.0232
4	0.5	0.5	u_o	1.0000	1.6633	1.6380	1.4386	1.2208	1.0253	0.8588	0.7190	0.6020	0.5041
			w_o	0.0000	0.4960	0.6461	0.6381	0.5726	0.4938	0.4187	0.3524	0.2958	0.2479
4	1.0	0.5	u_o	1.0000	1.8309	1.7981	1.5478	1.2859	1.0623	0.8804	0.7327	0.6116	0.5115
			w_o	0.0000	0.9236	1.2522	1.2655	1.1484	0.9946	0.8438	0.7098	0.5951	0.4985
2	1.0	0.5	u_o	1.0000	2.7717	3.1423	2.9493	2.5756	2.1812	1.8244	1.5200	1.2661	1.0558
			w_o	0.0000	1.2173	1.8351	2.0310	1.9807	1.8093	1.5929	1.3726	1.1681	0.9870
2	0.5	0.5	u_o	1.0000	2.5249	2.8323	2.6697	2.3561	2.0205	1.7103	1.4393	1.2080	1.0128
			w_o	0.0000	0.6578	0.9634	1.0424	1.0000	0.9033	0.7897	0.6778	0.5757	0.4860

Table 5.2

Values of mean temperature T_o. (P=0.71, E=0.003, M=4.0).

	G = 5				G = 10			
	m= 0.5		m= 1.0		m= 0.5		m= 1.0	
y	S=0.5	S=1.0	S=0.5	S=1.0	S=0.5	S=1.0	S=0.5	S=1.0
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	0.8376	0.7015	0.8379	0.7016	0.8384	0.7018	0.8395	0.7023
1.0	0.7016	0.4920	0.7019	0.4922	0.7025	0.4926	0.7038	0.4932
1.5	0.5876	0.3451	0.5880	0.3452	0.5887	0.3457	0.5901	0.3463
2.0	0.4921	0.2420	0.4925	0.2422	0.4933	0.2426	0.4947	0.2431
2.5	0.4121	0.1697	0.4125	0.1698	0.4134	0.1702	0.4147	0.1706
3.0	0.3452	0.1190	0.3455	0.1191	0.3463	0.1194	0.3475	0.1197

Table 5.3

Values of τ_{mu} and τ_{mw} (P=0.71, E=0.003).

G	m	M	S	τ_{mu}	τ_{mw}
5	0.5	2	0.5	1.7776	1.0110
			1.0	1.1386	0.8327
		4	0.5	0.3250	0.9650
			1.0	-0.1286	0.8724
	1.0	2	0.5	2.2324	1.7549
			1.0	1.5803	1.3967
		4	0.5	0.7378	1.6280
			1.0	0.3312	1.4427
10	0.5	2	0.5	5.1534	1.7285
			1.0	4.1795	1.3818
		4	0.5	2.7608	1.5006
			1.0	2.1495	1.3256
	1.0	2	0.5	5.8812	3.0925
			1.0	4.8754	2.3868
		4	0.5	3.4046	2.6248
			1.0	2.7890	2.2721

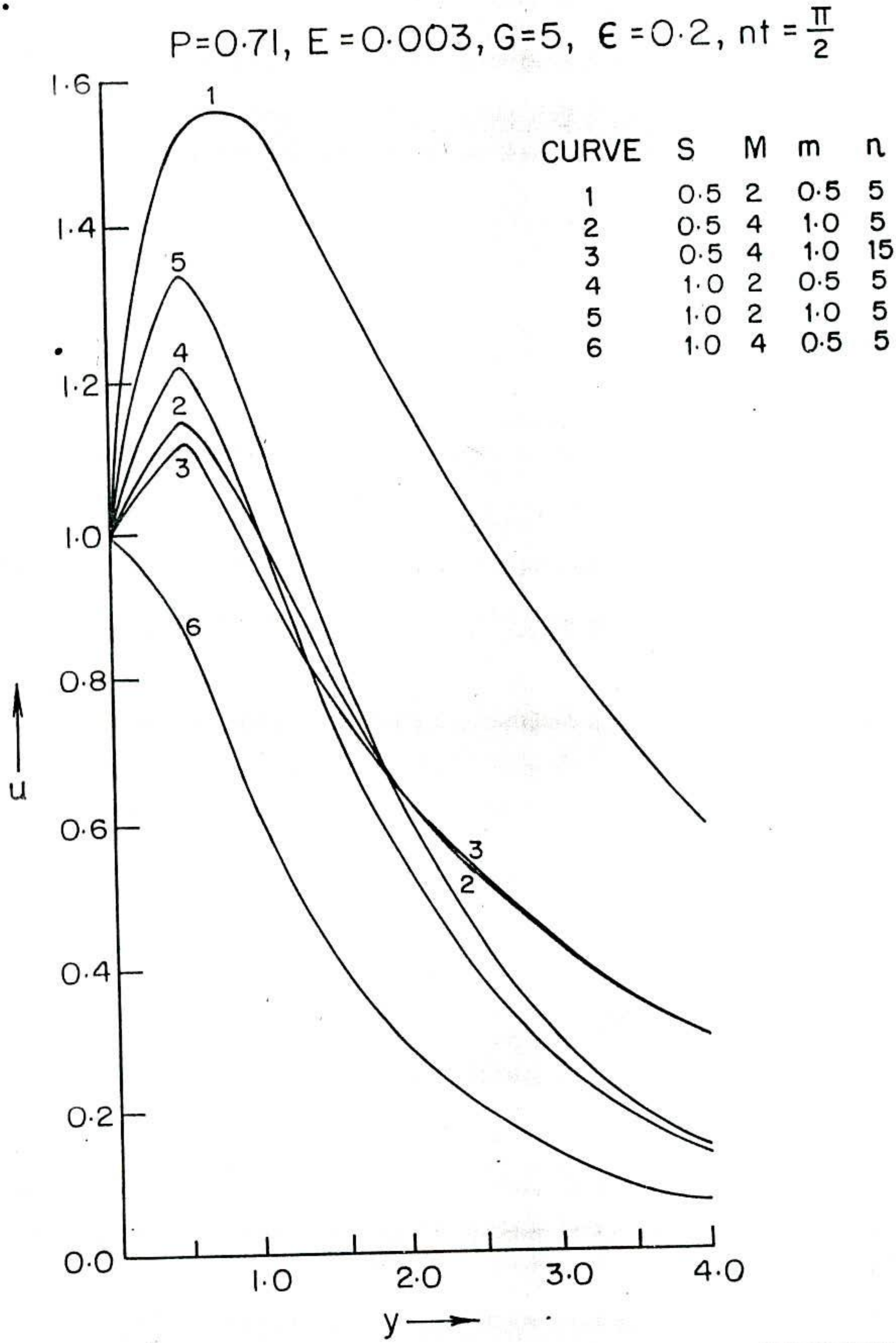

FIG. 5·1 TRANSIENT PRIMARY VELOCITY DISTRIBUTION u AGAINST y

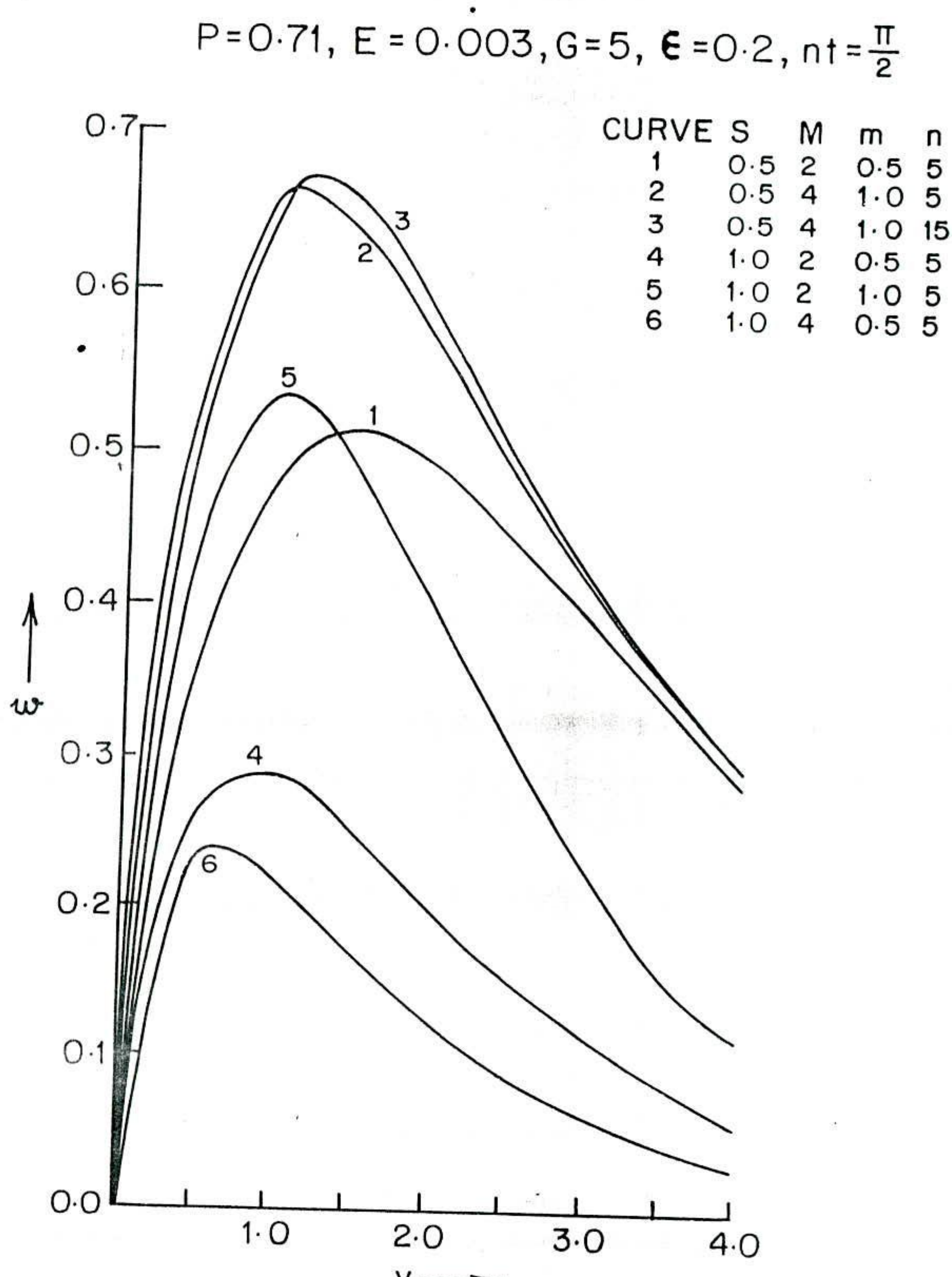

FIG. 5·2 TRANSIENT SECONDARY VELOCITY DISTRIBUTION w AGAINST y

Table 5.4

Variation of transient temperature $T(y,\frac{\pi}{2n})$ in air ($P = 0.71$, $E=0.003$, $G=5$, $\varepsilon=0.2$, $nt=\frac{\pi}{2}$)

S	M	m	n	y→ 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
.5	2	0.5	5	1.0000	0.8957	0.7447	0.6071	0.4974	0.4122	0.3444	0.2888	0.2424
.5	2	1.0	5	1.0000	0.8964	0.7456	0.6080	0.4984	0.4131	0.3453	0.2897	0.2431
.5	4	1.0	5	1.0000	0.8956	0.7445	0.6067	0.4970	0.4117	0.3439	0.2883	0.2419
.5	4	1.0	15	1.0000	0.8905	0.7142	0.5864	0.4911	0.4123	0.3455	0.2893	0.2423
.0	2	0.5	5	1.0000	0.7533	0.5271	0.3595	0.2455	0.1696	0.1184	0.0831	0.0584
.0	2	1.0	5	1.0000	0.7535	0.5274	0.3599	0.2458	0.1698	0.1186	0.0832	0.0585
.0	4	0.5	5	1.0000	0.7532	0.5269	0.3592	0.2452	0.1693	0.1181	0.0829	0.0583
.0	4	0.5	15	1.0000	0.7492	0.5023	0.3440	0.2410	0.1695	0.1190	0.0834	0.0585

Table 5.5

Values of skin friction components τ_x and τ_z at $nt=\frac{\pi}{2}$

($P= 0.71$, $E= 0.003$, $G= 5$, $\varepsilon= 0.2$)

S	M	m	n	τ_x	τ_z
0.5	2	0.5	5	1.9402	1.2061
			15	1.8762	1.1146
		1.0	5	2.4050	1.9485
			15	2.3327	1.8580
	4	0.5	5	0.4665	1.1770
			15	0.4213	1.0730
		1.0	5	0.9502	1.8446
			15	0.8876	1.7350
1.0	2	0.5	5	1.3020	1.0307
			15	1.2381	0.9369
		1.0	5	1.7538	1.5935
			15	1.6815	1.5003
	4	0.5	5	0.0130	1.0869
			15	- 0.0314	0.9810
		1.0	5	0.4936	1.6625
			15	0.4319	1.5504

RESULTS AND DISCUSSION:

In order to get physical insight into the problem numerical calculations have been carried out for mean flow and transient flow corresponding to different values of the Grashoff number G, suction parameter S, Hall parameter m, magnetic parameter M and frequency parameter n. In order to be realistic the value of the Prandtl number P is chosen to be 0.71 which corresponds to air. In the entire calculation we have taken E=0.003 and $\varepsilon = 0.2$.

Values of the mean primary velocity u_o and mean secondary velocity w_o are given in Table 5.1. It is seen from the table that u_o increase with increase in m and G. It decreases with increase in S and M. From the same table we conclude that the effects of the various parameters on w_o are similar to their effects on u_o.

Table 5.2 shows the variations of the mean temperature T_o in air (P=0.71). It is clear from the table that the temperature increases with increase in m and G and decreases with increase in S.

Table 5.3 gives the values of the mean skin friction components τ_{mu} and τ_{mw}. From the table we observe that τ_{mu} the mean skin friction component due to mean primary flow increases with increase in m but decreases with increase in S and M. τ_{mw} the mean skin friction component due to mean secondary flow increases with increase in m but decreases with increase in S. τ_{mw} in general, decreases with increase in M, but increases with increase in M for G=5 and S=1. Both the components of skin friction increase with increase in G. The transient primary velocity profiles $u(y, \frac{\pi}{2n})$ have been displayed in Figure 5.1. It is clear from the figure that the transient primary velocity decreases with increase in M and S, but increases with increase in m. We also observe from the figure that near to the porous plate $u(y, \frac{\pi}{2n})$ decreases with increase in n, but away from the plate it increases with increase in n.

The transient secondary velocity profiles $w(y, \frac{\pi}{2n})$ are shown in Figure 5.2. From the figure we conclude that the effects of m, M and S on $w(y,\frac{\pi}{2n})$ are similar to their effects on $u(y, \frac{\pi}{2n})$. As for the effect of n we see that near to the porous plate the transient secondary velocity decreases with increase in n, but away from the plate it increases with increase in n; further away from the plate the influence of n is insignificant.

Table 5.4 displays the values of transient temperature $T(y,\frac{\pi}{2n})$ of air. We observe from the table that the transient temperature increases with increase in m whereas rise in M and S causes a fall in $T(y,\frac{\pi}{2n})$. It decreases with increase in n, near the plate but increases with increase in n away from the plate.

Table 5.5 shows the values of the skin friction components τ_x and τ_z at $nt=\frac{\pi}{2}$. τ_x decreases with increase in n, M and S and increases with increase in m. τ_z decreases with increase in n and S, but increase with increase in m. The effect of M on τ_z depends on S. For $S = 0.5$, τ_z decreases with increase in M but for $S=1.0$ τ_z increases with increase in M.

APPENDIX

$$A_1 = \frac{G}{P^2S^2 - S^2P - M_1}, \qquad B_1 = \frac{S + (S^2 + 4M_1)^{1/2}}{2}, \quad A_2 = 1 + A_1$$

$$A_3 = \frac{P|A_2|^2|B_1|^2}{(B_1+\bar{B}_1)^2 - SP(B_1+\bar{B}_1)}, \qquad A_4 = \frac{SP^2 A_2 \bar{A}_1}{B_1 + SP}$$

$$A_5 = \frac{SP^2 A_1 \bar{A}_2}{\bar{B}_1 + SP}, \qquad A_6 = \frac{P|A_1|^2}{2}$$

$$= \bar{A}_4$$

$$L_1 = A_3 - A_4 - A_5 + A_6$$

$$A_7 = \frac{GL_1}{S^2P^2 - S^2P - M_1}, \qquad A_8 = \frac{GA_3}{(B_1+\bar{B}_1)^2 - S(B_1+\bar{B}_1) - M_1}$$

$$A_9 = \frac{G\,A_4}{(S)+B_1)^2 - S(SP+B_1) - M_1}, \qquad A_{10} = \frac{G\,A_5}{(SP+\bar{B}_1)^2 - S(SP+\bar{B}_1) - M_1}$$

$$A_{11} = \frac{G\,A_6}{4S^2P^2 - 2S^2P - M_1}, \qquad L_2 = A_7 - A_8 + A_9 + A_{10} - A_{11}$$

$$h_1 = \frac{SP + (S^2P^2 + 4P\ in)^{1/2}}{2}, \qquad M_2 = M_1 + in$$

$$h_2 = \frac{S+(S^2+4M_2)^{1/2}}{2}, \qquad D_1 = \frac{G}{h_1^2 - Sh_1 - M_2}$$

$$C_1 = \bar{A}_2 \bar{B}_1 D_1 h_2 P, \qquad C_2 = \bar{A}_1 D_1 h_2 SP^2$$

$$C_3 = \bar{A}_2 \bar{B}_1 D_1 h_1 P, \qquad C_4 = \bar{A}_1 D_1 h_1 SP^2$$

$$D_2 = \frac{C_1}{(h_2+\bar{B}_1)^2 - SP(h_2+\bar{B}_1) - \text{in } P}$$

$$D_3 = \frac{C_2}{(h_2+SP)^2 - SP(h_2+SP) - \text{in } P}$$

$$D_4 = \frac{C_3}{(h_1+\bar{B}_1) - SP(h_1+\bar{B}_1) - \text{in } P}$$

$$D_5 = \frac{C_4}{(h_1+SP)^2 - SP(h_1+SP) - \text{in } P}$$

$$D_6 = \frac{\bar{C}_1}{(\bar{h}_2+B_1)^2 - SP(\bar{h}_2+B_1) - \text{in } P}$$

$$D_7 = \frac{\bar{C}_2}{(\bar{h}_2+SP)^2 - SP(\bar{h}_2+SP) - \text{in } P},$$

$$D_8 = \frac{\bar{C}_3}{(\bar{h}+B_1)^2 - SP(\bar{h}_1+B_1) - \text{in } P}$$

$$D_9 = \frac{\bar{C}_4}{(\bar{h}_1+SP)^2 - SP(\bar{h}_1+SP) - \text{in } P}$$

$$X_1 = D_2 - D_3 - D_4 + D_5 + D_6 - D_7 - D_8 + D_9$$

$$D_{10} = \frac{GX_1}{h_1^2 - Sh_1 - M_2}, \qquad D_{11} = \frac{GD_2}{(h_2+\bar{B}_1)^2 - S(h_2+\bar{B}_1) - M_2}$$

$$D_{12} = \frac{GD_3}{(h_2+SP)^2 - S(h_2+SP) - M_2} \qquad D_{13} = \frac{GD_4}{(h_1+\bar{B}_1)^2 - S(h_1+\bar{B}_1) - M_2}$$

$$D_{14} = \frac{GD_5}{(h_1+SP)^2 - S(h_1+SP) - M_2} \qquad D_{15} = \frac{GD_6}{(\bar{h}_2+B_1)^2 - S(\bar{h}_2+B_1) - M_2}$$

$$D_{16} = \frac{GD_7}{(\bar{h}_2+SP)^2 - S(\bar{h}_2+SP) - M_2} \qquad D_{17} = \frac{GD_8}{(\bar{h}_1+B_1)^2 - S(\bar{h}_1+B_1) - M_2}$$

$$D_{18} = \frac{GD_9}{(\bar{h}_1+SP)^2 - S(\bar{h}_1+SP) - M_2}$$

$$X_2 = D_{10} - D_{11} + D_{12} + D_{13} - D_{14} - D_{15} + D_{16} + D_{17} - D_{18}.$$

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CHAPTER - VI

HALL EFFECTS ON UNSTEADY MHD FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM IN ROTATING FLUID WITH CONSTANT HEAT FLUX

Introduction:

The effect of Hall currents on unsteady MHD free convective flow of a viscous incompressible and electrically conducting fluid past an infinite vertical porous plate has been studied by Agrawal et al (1983). The whole system is assumed to be in a state of rigid body rotation and in addition, the free stream is considered to oscillate in time about a constant mean. The effects of Hall currents and rotation on steady free convection and mass transfer flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux at the plate has been carried out by Raptis and Ram (1984). The flow is subjected to constant suction velocity at the plate.

In the present chapter we have modified the work of Raptis and Ram (1984), in the sense that we have considered the unsteady case and the plate is assumed to oscillate in time. The effect of mass transfer has been neglected. Approximate solutions to the coupled non linear equations occurring in the problem have been obtained. The effect of the various parameters on the mean and transient flows are discussed with the help of graphs and tables.

Mathematical Analysis:

We consider the unsteady free convection flow of an electrically conducting, viscous and incompressible fluid through a porous medium occupying a semi-infinite region of space bounded by an infinite vertical, non-conducting porous plate. The x'-axis is taken along the plate in the vertical upward direction and y'-axis is taken normal to the plate. Initially the fluid as well as the plate are at rest but at time $t' > 0$ the whole system begins to rotate with constant angular velocity $\bar{\Omega}'$ about y'-axis and in addition the plate is assumed to oscillate with frequency n. A uniform strong magnetic field is acting transverse to the plate. The pressure gradient is assumed to be constant. Since the plate is infinite in extent all physical quantities are functions of y' and t' only. The fluid is subjected to constant suction at the plate and hence if $\bar{V} = (u', v', w')$ the equation of continuity gives $v' = -v_o$ where v_o is the constant suction velocity. Using the relation $\nabla . \bar{H} = 0$ for the magnetic field $\bar{H} = (H_x, H_y, H_z)$ we obtain $H_y = H_o$ (H_o is the constant externally applied magnetic field). If $\overline{J} = (J_x, J_y, J_z)$ is the current density from the relation $\nabla.\overline{J} = 0$ we have J_y= constant. Since the plate is non-conducting, J_y= 0 at the plate and hence zero everywhere. Assuming the magnetic Reynolds to be small we neglect the induced magnetic field in comparison with the applied magnetic field.

The generalized Ohm's law taking Hall current into account (Cowling (1957)) in the absence of electric field, is of the form

$$6.1 \qquad \overline{J} + \frac{w_e \tau_e}{H_o} \overline{J} \times \overline{H} = \sigma(\mu_e \overline{V} \times \overline{H} + \frac{1}{en_e} \nabla p_e)$$

By the same reasoning as in chapter V, we have from 6.1

$$6.2 \qquad J_x = \frac{\sigma \mu_e H_o}{1+m^2} (mu' - w')$$

$$6.3 \qquad J_z = \frac{\sigma \mu_e H_o}{1 + m^2} (u' + mw')$$

where $m = w_e \tau_e$ is the Hall parameter. The other quantities are defined in chapter V.

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term. The basic equations relevant to the problem are

6.4
$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} + 2\Omega' w' = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho(1+m^2)}(u' + mw') - \frac{\nu u'}{K'}$$

6.5
$$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} - 2\Omega' u' = \nu \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma B_o^2}{\rho(1+m^2)}(mu' - w') - \frac{\nu w'}{K'}$$

6.6
$$\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right]$$

where K' is the permeability of the medium, the other physical quantities have their usual meanings. The initial and boundary conditions are

6.7
$$t' \le 0: \quad u' = 0, \quad w' = 0, \quad T' = T'_\infty$$

$$t' > 0: \quad u' = U_o(1 + \varepsilon e^{in't'}), \quad w' = 0, \quad \frac{\partial T'}{\partial y'} = -\frac{q'}{k'} \quad \text{at } y' = 0$$

$$u' \to 0, \quad w' \to 0, \quad T' \to T'_\infty \quad \text{as } y' \to \infty$$

Introducing the following non-dimensional quantites

$$u = \frac{u'}{U_o}, \quad y = \frac{y'U_o}{\nu}, \quad t = \frac{t'U_o^2}{\nu}, \quad n = \frac{n'\nu}{U_o^2}, \quad S = \frac{V_o}{U_o}$$

6.8

$$w = \frac{w'}{U_o}, \quad P = \frac{\mu C_p}{k}, \quad T = \frac{T' - T'_\infty}{\frac{q'\nu}{kU_o}}, \quad G = \frac{\nu^2 g\beta q'}{kU_o^4}$$

$$E = \frac{k\,U_o^3}{q'\nu C_p}, \quad M = \frac{\sigma B_o^2 \nu}{\rho U_o^2}, \quad K = \frac{K'U_o^2}{\nu^2}, \quad \Omega = \frac{2\,\Omega'\nu}{U_o^2}$$

in equations 6.4 to 6.6 we get

6.9 $$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} + \Omega\, w = GT + \frac{\partial^2 u}{\partial y^2} - \mathcal{S}\,(u + mw) - \frac{u}{K}$$

6.10 $$\frac{\partial w}{\partial t} - S\frac{\partial w}{\partial y} - \Omega\, u = \frac{\partial^2 w}{\partial y^2} + \mathcal{S}\,(mu - w) - \frac{w}{K}$$

6.11 $$\frac{\partial T}{\partial t} - S\,\frac{\partial T}{\partial y} = \frac{1}{P}\,\frac{\partial^2 T}{\partial y^2} + E\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

The boundary condition 6.7 becomes in non-dimensional form:

6.12

$$t>0 : u = 1 + \varepsilon\, e^{int}, \quad w = 0, \quad \frac{\partial T}{\partial y} = -1 \quad \text{at } y = 0$$

$$u \to 0, \quad w \to 0, \quad T \to 0 \quad \text{as } y \to \infty$$

where,

$$\mathcal{S} = \frac{M}{1+m^2} \quad .$$

The task of solving equations 6.9 to 6.11 under boundary condition 6.12 is quite complicated. To simplify the mathematical part we introduce a complex variable defined as

6.13 $$Q = u + iw$$

which enable us to combine equations 6.9 and 6.10 into a single equation of the form

6.14 $$\frac{\partial^2 Q}{\partial y^2} + S \frac{\partial Q}{\partial y} - \left[S + \frac{1}{K} - i(S m + \Omega) \right] Q - \frac{\partial Q}{\partial t} = -GT$$

Equation 6.11 with the help of 6.13 becomes

6.15 $$\frac{\partial^2 T}{\partial y^2} + SP \frac{\partial T}{\partial y} - P \frac{\partial T}{\partial t} = - PE\left(\frac{\partial Q}{\partial y} \times \frac{\partial \bar{Q}}{\partial y} \right)$$

The corresponding boundary conditions assume the form,

$$Q = 1 + \varepsilon \; e^{int}, \quad \frac{\partial T}{\partial y} = -1 \qquad \text{at } y = 0$$

6.16

$$Q \rightarrow 0, \quad T \rightarrow 0 \qquad \text{as } y \rightarrow \infty$$

Equations 6.14 and 6.15 are coupled non-linear equations, in order to solve them we can represent the velocity and temperature in the neighbourhood of the plate as follows (assuming small amplitude of oscillation)

$$Q(y,t) = q_o(y) + \varepsilon\, q_1(y)\, e^{int}$$

6.17

$$T(y,t) = T_o(y) + \varepsilon\, T_1(y)\, e^{int}$$

substituting 6.17 in equations 6.14 and 6.15 and equating coefficients of different powers of ε, neglecting those of ε^2 and higher powers of ε we obtain the following set of equations.

6.18 $$q_o'' + Sq_o' - M_1 q_o = - GT_o$$

6.19 $$q_1'' + Sq_1' - (M_1 + in)\, q_1 = -GT_1$$

6.20 $$T_o'' + SPT_o' = -PE\, q_o' \overline{q}_o'$$

6.20 $$T_1'' + SPT_1' - in\, PT_1 = -PE(q_1' \overline{q}_o' + \overline{q}_1' q_o')$$

where $M_1 = \delta + \frac{1}{K} - i\,(\delta\, m + \Omega)$

and primes denote differentiation with respect to y. The corresponding boundary conditions are

$$q_o = 1,\quad q_1 = 1,\quad T_o' = -1,\quad T_1' = 0 \qquad \text{at } y=0$$

6.22

$$q_o \to 0,\quad q_1 \to 0,\quad T_o \to 0,\quad T_1 \to 0 \qquad \text{as } y \to \infty$$

The equations 6.18 to 6.21 are still coupled and non linear and hence difficult to solve analytically. In order to solve them we expand q_o, q_1, T_o and T_1 in powers of E the Eckert number assuming it to be very small as follows ($E \ll 1$ for incompressible fluids).

$$q_o(y) = q_{01}(y) + Eq_{02}(y) + 0(E^2)$$

$$q_1(y) = q_{11}(y) + Eq_{12}(y) + 0(E^2)$$

6.23

$$T_o(y) = T_{01}(y) + ET_{02}(y) + 0(E^2)$$

$$T_1(y) = T_{11}(y) + ET_{12}(y) + 0(E^2)$$

substituting 6.23 in equations 6.18 to 6.21 we obtain the following system of equations 6.24 to 6.27 and 6.28 to 6.31 which govern the mean steady flow and the unsteady one respectively.

6.24 $$q''_{01} + Sq'_{01} - M_1 q_{01} = - GT_{01}$$

6.25 $$q''_{02} + S q'_{02} - M_1 q_{02} = - GT_{02}$$

6.26 $$T''_{01} + S PT'_{01} = 0$$

6.28 $$q''_{11} + Sq'_{11} - q_{11}(M_1 + in) = -GT_{11}$$

6.29 $$q''_{12} + Sq'_{12} - q_{12}(M_1 + in) = -G\,T_{12}^{*}.$$

6.30 $$T''_{11} + SP\,T'_{11} - in\,PT_{11} = 0$$

6.31 $$T''_{12} + SPT'_{12} - in\,PT_{12} = -P(q'_{11}\bar{q}'_{01} + \bar{q}'_{11}q'_{01})$$

subject to the boundary conditions

6.32
$$q_{01} = 1,\quad q_{02} = 0,\quad T'_{01} = -1,\quad T'_{02} = 0 \qquad \text{at } y = 0$$
$$q_{01} \to 0,\quad q_{02} \to 0,\quad T_{01} \to 0,\quad T_{02} \to 0 \qquad \text{as } y \to \infty$$

for the mean steady flow and

6.33
$$q_{11} = 1,\quad q_{12} = 0,\quad T'_{11} = 0,\quad T'_{12} = 0 \qquad \text{at } y = 0$$
$$q_{11} \to 0,\quad q_{12} \to 0,\quad T_{11} \to 0,\quad T_{12} \to 0 \qquad \text{as } y \to \infty$$

for the unsteady flow.

First we obtain the solution for the mean steady flow while the solution for the unsteady flow will be obtained later.

The solutions of equations 6.24 to 6.27 subject to boundary conditions 6.32 are given by

6.34 $$q_{01} = A_2 e^{-B_1 y} - A_1 e^{-SPy}$$

6.35 $$q_{02} = L_2 e^{-B_1 y} - F_1 e^{-SPy} + F_2 e^{-(B_1+\bar{B}_1)y} + F_3 e^{-2SPy} - F_4 e^{-(SP+\bar{B}_1)y} - F_5 e^{-(SP+B_1)y}$$

6.36 $$T_{01} = \frac{1}{SP} e^{-SPy}$$

6.37 $$T_{02} = L_1 e^{-SPy} - C_1 e^{-(B_1+\bar{B}_1)y} - C_2 e^{-2SPy} + C_3 e^{-(SP+\bar{B}_1)y} + C_4 e^{-(SP+B_1)y}.$$

The expression for the mean steady velocity q_0 and mean temperature T_o are given from 6.23 as

$$q_0 = q_{01} + E\, q_{02}$$

$$T_o = T_{01} + ET_{02}$$

respectively, where q_{01}, q_{02}, T_{01} and T_{02} are given by equations 6.34 to 6.37 respectively.

If τ_{mu} and τ_{mw} are the components of mean skin friction τ_o at the plate due to mean primary velocity u_o and mean secondary velocity w_o we have,

6.38 $$\tau_o = \tau_{mu} + i\,\tau_{mw} = \frac{dq_o}{dy}\bigg|_{y=0}$$

$$= -A_2B_1 + A_1SP + E[-L_2B_1 + F_1SP - F_2(B_1 + \bar{B}_1) - 2SP\,F_3 + F_4(SP + \bar{B}_1) + F_5(SP + B_1)]$$

where the different constants are given in the appendix.

Now we proceed to solve the unsteady part of the flow field which are characterised by equations 6.28 to 6.31 viz.

6.28 $$q''_{11} + S\,q'_{11} - q_{11}(M_1 + in) = -GT_{11}$$

6.29 $$q''_{12} + S\,q'_{12} - q_{12}(M_1 + in) = -GT_{12}$$

6.30 $$T''_{11} + SPT'_{11} - in\,PT_{11} = 0$$

6.31 $$T''_{12} + SPT'_{12} - in\,PT_{12} = -P(q'_{11}\,\bar{q}'_{01} + \bar{q}'_{11}\,q'_{01})$$

Subject to the boundary conditions

6.33

$$q_{11} = 1,\quad q_{12} = 0,\quad T'_{11} = 0,\quad T'_{12} = 0 \qquad \text{at } y = 0$$

$$q_{11} \to 0,\quad q_{12} \to 0,\quad T_{11} \to 0,\quad T_{12} \to 0 \qquad \text{as } y \to \infty$$

The solution of the equations 6.28 to 6.33 of the unsteady flow field under their boundary conditions 6.33 are given by

6.39

$$q_1 = q_{11} + E\, q_{12}$$

$$= e^{-h_1 y} + E[L_4 e^{-h_1 y} - P_9 e^{-h_2 y} + P_{10} e^{-(h_1+\bar{B}_1)y} - P_{11} e^{-(SP+h_1)y} + P_{12} e^{-(\bar{h}_1+B_1)y} - P_{13} e^{-(SP+\bar{h}_1)y}]$$

and

6.40

$$T_1 = T_{11} + ET_{12}$$

$$= E[L_3 e^{-h_2 y} - P_1 e^{-(h_1+\bar{B}_1)y} + P_2 e^{-(SP+h_1)y} - P_3 e^{-(\bar{h}_1+B_1)y} + P_4 e^{-(SP+\bar{h}_1)y}]$$

where all the constants appearing in the solution are defined in the appendix at the end of this chapter.

Since now we know q_o, q_1, T_o and T_1 we obtain the expression for Q and T from 6.17 viz.

6.17

$$Q(y,t) = q_o(y) + \varepsilon \; q_1(y) \; e^{int}$$

$$T(y,t) = T_o(y) + \varepsilon \; T_1(y) \; e^{int}$$

The expression for $Q(y,t)$ may be written as

6.41 $$Q(y,t) = q_o + \varepsilon \left[M_r \cos nt - M_i \sin nt \right) + i \left(M_i \cos nt + M_r \sin nt \right) \right]$$

6.42 where $q_1 = M_r + i\, M_i$

similarly,

6.43 $$T(y,t) = T_o + \varepsilon \; (T_r \cos nt - T_i \sin nt) + i \; (T_i \cos nt + T_r \sin nt)$$

6.44 where, $T_1 = T_r + i\, T_i$

From the expression 6.41 we can obtain the expressions of the transient primary velocity and transient secondary velocity and from expression 6.43 we obtain the expression for transient temperature at $nt = \frac{\pi}{2}$ as

6.45 $$u(y, \frac{\pi}{2n}) = u_o(y) - \varepsilon \; M_i$$

6.46 $$w(y, \frac{\pi}{2n}) = w_o(y) + \varepsilon \; M_r$$

6.47 where $q_o = u_o + i \; w_o$

and

6.48 $$T(y, \frac{\pi}{2n}) = T_o(y) - \varepsilon \; T_i$$

where

u_o, w_o and T_o are the mean primary velocity, mean secondary velocity and mean temperature respectively,

The skin friction is given by

6.49 $$\tau = \tau_x + i \, \tau_z$$

$$= \left. \frac{\partial Q}{\partial y} \right|_{y=0}$$

$$= \left. \frac{\partial q_o}{\partial y} \right|_{y=0} + \varepsilon \, e^{int} \left. \frac{\partial q_1}{\partial y} \right|_{y=0}$$

$$= \tau_o + \varepsilon \; e^{int} \left[-h_1 + E \; \{ -h_1 L_4 + h_2 \; P_9 \right.$$

$$\left. - (h_1 + \bar{B}_1) P_{10} + (SP + h_1) P_{11} - (\bar{h}_1 + B_1) P_{12} + (SP + \bar{h}_1) P_{13} \} \; \right]$$

where $\tau_o = \frac{\partial q_o}{\partial y}\Big|_{y=0}$ is the mean skin friction. τ_x and τ_z are the component of skin friction along x, and z directions respectively.

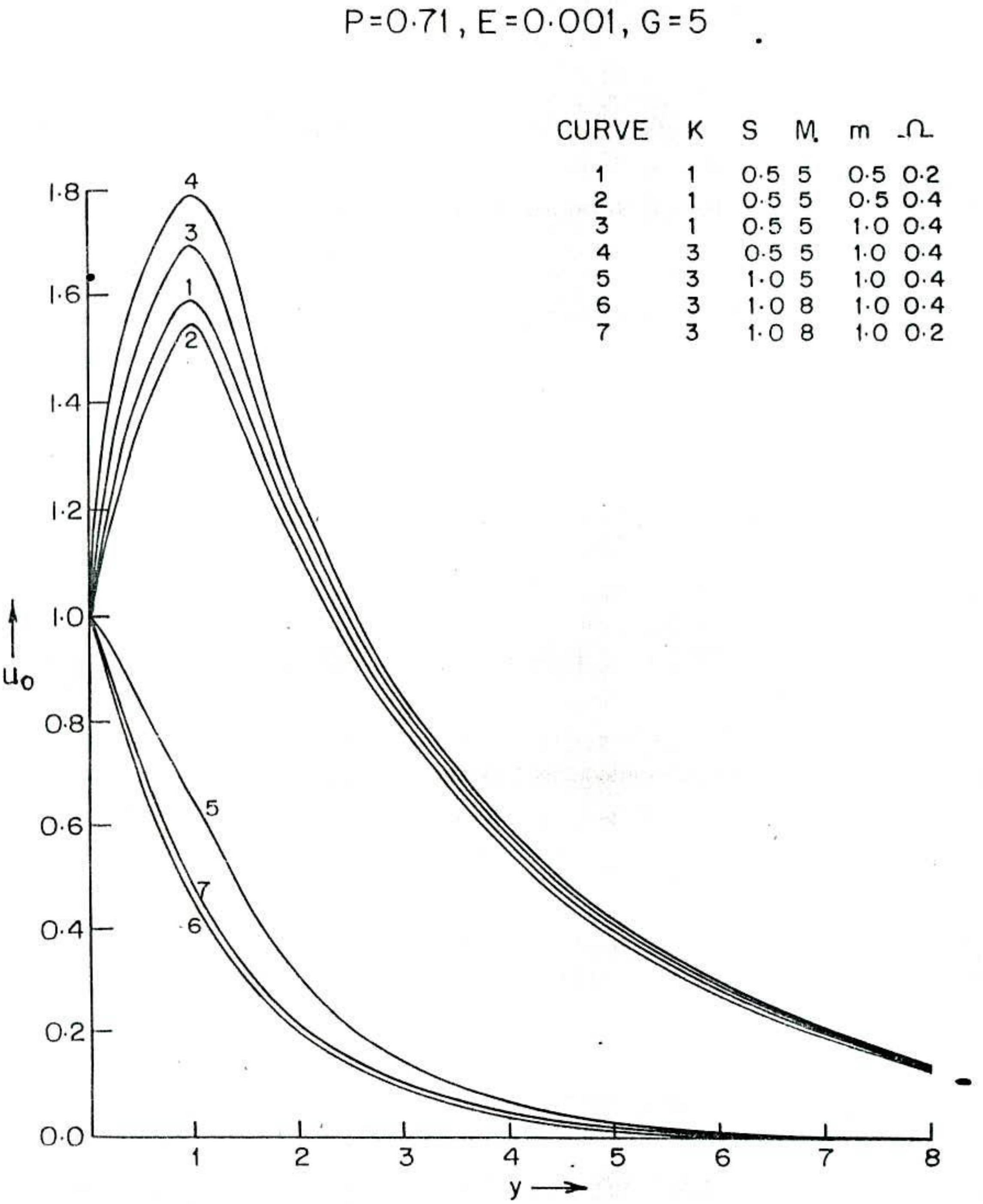

FIG. 6·1 MEAN PRIMARY VELOCITY DISTRIBUTION u_0 AGAINST y

P=0.71, E=0.001, G=5

CURVE	K	S	M	m	Ω
1	1	0.5	5	0.5	0.2
2	1	0.5	5	0.5	0.4
3	1	0.5	5	1.0	0.4
4	3	0.5	5	1.0	0.4
5	3	1.0	5	1.0	0.4
6	3	1.0	8	1.0	0.4
7	1	1.0	8	1.0	0.2
8	1	1.0	8	0.5	0.2

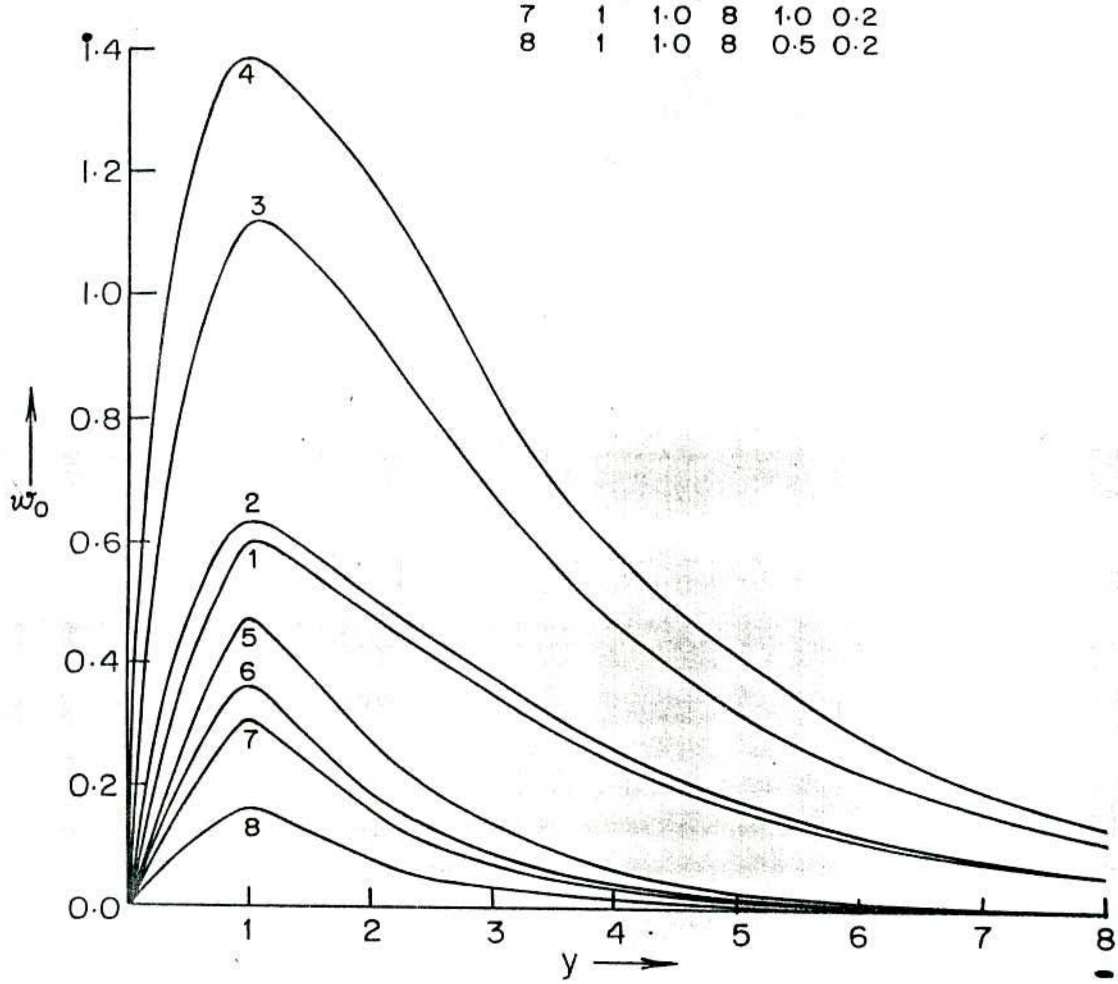

FIG. 6.2 MEAN SECONDARY VELOCITY DISTRIBUTION w_0 AGAINST y

Table 6.1: Values of mean temperature T_0. P=0.71, E=0.001, G=5

K	S	M	m	Ω	y	0.0	1	2	3	4	5	6	7	8	9
1	0.5	5	0.5	0.2	T	2.8215	1.9788	1.3877	0.9731	0.6824	0.4785	0.3355	0.2353	0.1650	0.1157
1	0.5	5	0.5	0.4		2.8215	1.9788	1.3877	0.9731	0.6824	0.4785	0.3355	0.2353	0.1650	0.1157
1	0.5	5	1.0	0.4		2.8256	1.9820	1.3900	0.9748	0.6836	0.4794	0.3361	0.2357	0.1653	0.1159
3	0.5	5	1.0	0.4		2.8286	1.9843	1.3918	0.9761	0.6845	0.4800	0.3366	0.2360	0.1655	0.1160
3	1.0	5	1.0	0.4		1.4094	0.6931	0.3408	0.1676	0.0824	0.0405	0.0199	0.0098	0.0048	0.0024
3	1.0	8	1.0	0.4		1.4093	0.6930	0.3408	0.1675	0.0824	0.0405	0.0199	0.0098	0.0048	0.0024
3	1.0	8	1.0	0.2		1.4093	0.6930	0.3407	0.1675	0.0824	0.0405	0.0199	0.0098	0.0048	0.0024
1	1.0	8	1.0	0.2		1.4092	0.6930	0.3407	0.1675	0.0824	0.0405	0.0199	0.0098	0.0048	0.0024
1	1.0	8	0.5	0.2		1.4091	0.6928	0.3406	0.1675	0.0823	0.0405	0.0199	0.0098	0.0048	0.0024

Table 6.[illegible]: Values of mean skin friction components τ_{mu} and τ_{mw} P=0.71, E=0.001, G=5, M=8

K	S	m	Ω	τ_{mu}	τ_{mw}
1	0.5	0.5	0.2	1.6023	1.5805
			0.4	1.5615	1.6590
		1.0	0.2	2.3433	2.6014
			0.4	2.2645	2.6773
	1.0	0.5	0.2	– 1.1000	1.0413
			0.4	– 1.1218	1.0955
		1.0	0.2	– 0.5451	1.6342
			0.4	– 0.5865	1.6896
3	0.5	0.5	0.2	1.8755	1.7266
			0.4	1.8273	1.8094
		1.0	0.2	2.6029	2.8887
			0.4	2.5101	2.9626
	1.0	0.5	0.2	– 0.9130	1.1187
			0.4	– 0.9385	1.1756
		1.0	0.2	– 0.3564	1.7838
			0.4	– 0.4048	1.8393

ble 6.3: Values of transient primary velocity $u(y, \frac{\pi}{2n})$ and transient secondary velocity $w(y, \frac{\pi}{2n})$.

$P = 0.71,\ E = 0.001,\ \varepsilon = 0.2,\ G = 5,\ S = 0.5$

M	m	Ω	n	y	0	1	2	3	4	5	6	7	8
5	0.5	0.2	10	u	1.0000	1.6002	1.1550	0.8100	0.5677	0.3981	0.2791	0.19574	0.1372
				w	0.2000	0.5989	0.4919	0.3518	0.2472	0.1733	0.1215	0.0852	0.0597
5	0.5	0.2	30	u	1.0000	1.5889	1.1549	0.8100	0.5677	0.3981	0.2791	0.1957	0.1372
				w	0.2000	0.5951	0.4925	0.3519	0.2472	0.1733	0.1215	0.0852	0.0597
5	0.5	0.4	30	u	1.0000	1.5529	1.1223	0.7863	0.5510	0.3864	0.2709	0.1899	0.1332
				w	0.2000	0.6347	0.5225	0.3727	0.2617	0.1835	0.1287	0.0902	0.0632
5	1.0	0.2	10	u	1.0000	1.7829	1.2572	0.8724	0.6102	0.4277	0.2999	0.2103	0.1474
				w	0.2000	1.0883	0.9208	0.6606	0.4638	0.3252	0.2280	0.1598	0.1121
8	1.0	0.2	10	u	1.0000	1.2133	0.8198	0.5698	0.3993	0.2800	0.1963	0.1376	0.0965
				w	0.2000	0.8714	0.6726	0.4737	0.3320	0.2328	0.1632	0.1144	0.0802
5	0.5	0.4	30	u	1.0000	1.6880	1.2227	0.8554	0.5992	0.4201	0.2946	0.2065	0.1448
				w	0.2000	0.7763	0.6516	0.4667	0.3279	0.2299	0.1612	0.1130	0.0792
8	1.0	0.4	10	u	1.0000	1.1750	0.7886	0.5479	0.3840	0.2692	0.1888	0.1323	0.0928
				w	0.2000	0.8850	0.6786	0.4772	0.3345	0.2345	0.1641	0.1153	0.0808
5	0.5	0.4	10	u	1.0000	1.7007	1.2228	0.8554	0.5992	0.4201	0.2946	0.2065	0.1448
				w	0.2000	0.7801	0.6508	0.4666	0.3279	0.2299	0.1612	0.1130	0.0792
8	0.5	0.4	10	u	1.0000	1.0834	0.7556	0.5288	0.3707	0.2599	0.1823	0.1278	0.0896
				w	0.2000	0.4803	0.3625	0.2554	0.1791	0.1256	0.0880	0.0617	0.0433

Table 6.4: Values of transient temperature $T(y, \frac{\pi}{2n})$

$P = 0.71,\ E = 0.001,\ \varepsilon = 0.2,\ G = 5,\ S = 0.5$

K	M	m	Ω	n	y	0.0	1	2	3	4	5	6	7	8
1	5	0.5	0.2	10	T	2.82136	1.97875	1.38768	0.97312	0.68238	0.47850	0.33552	0.23527	0.16497
1	5	0.5	0.2	30		2.82139	1.97877	1.38768	0.97312	0.68238	0.47850	0.33552	0.23527	0.16497
1	5	0.5	0.4	30		2.82139	1.97877	1.38767	0.97311	0.68238	0.47849	0.33552	0.23526	0.16496
1	5	1.0	0.2	10		2.82557	1.98205	1.39012	0.97489	0.68366	0.47941	0.33617	0.23572	0.16529
1	8	1.0	0.2	10		2.82144	1.97877	1.38764	0.97306	0.68232	0.47845	0.33548	0.23524	0.16494
3	5	0.5	0.4	30		2.82294	1.97999	1.38858	0.97378	0.68286	0.47884	0.33576	0.23544	0.16509
1	8	1.0	0.4	10		2.82146	1.97877	1.38763	0.97305	0.68232	0.47844	0.33548	0.23523	0.16494
3	5	0.5	0.4	10		2.82292	1.97997	1.38859	0.97378	0.68286	0.47884	0.33576	0.23544	0.16509
1	8	0.5	0.4	10		2.81877	1.97669	1.38611	0.97195	0.68153	0.47789	0.33509	0.23496	0.16479

Table 6.5: Values of skin friction components τ_x and τ_z, $nt = \frac{\pi}{2}$

$P = 0.71,\ E = 0.001,\ \varepsilon = 0.2,\ G = 5,\ M = 5,\ S = 0.5$

K	m	Ω	n	τ_x	τ_z
1	0.5	0.2	10	3.38344	1.01422
			30	3.77354	0.73385
		0.4	10	3.31115	1.13912
			30	3.70415	0.85803
	1.0	0.2	10	4.12775	2.08239
			30	4.51822	1.77726
		0.4	10	3.99594	2.20620
			30	4.38953	1.89986
3	0.5	0.2	10	3.83173	1.28100
			30	4.21911	0.99099
		0.4	10	3.73977	1.41717
			30	4.13008	1.12624
	1.0	0.2	10	4.56297	2.58629
			30	4.94903	2.26728
		0.4	10	4.39475	2.70264
			30	4.78391	2.38578

Discussion of the results

In order to get physical insight into the problem, numerical calculations have been carried out for mean primary velocity u_0, mean secondary velocity w_0, mean temperature T_0, components of mean skin friction τ_{mu}, τ_{mw}, transient primary velocity $u(y, \frac{\pi}{2n})$, transient secondary velocity $w(y, \frac{\pi}{2n})$, components of skin friction τ_x and τ_z at $nt = \frac{\pi}{2}$ corresponding to different values of the parameters. In order to be realistic, the value of the Prandtl number is chosen to be 0.71 which corresponds to air. In the entire calculation we have taken $E = 0.001$, $\varepsilon = 0.2$ and $G = 5$.

Variations of the mean primary velocity profiles are illustrated in figure 6.1. It is clear from the figure that u_0 increases with increase in the Hall parameter m and permeability K. A rise in the rotation parameter Ω, magnetic parameter M and suction parameter S causes a fall in the mean primary velocity u_0.

Figure 6.2 display the mean secondary velocity profiles. From the figure we conclude that mean secondary velocity w_0 increases with increase in Ω, K and m but decreases with increase in M and S.

Table 6.1 shows the values of the mean temperature T_0, we conclude from the table that T_0 decreases with increase in S. The effect of Ω on T_0 is insignificant. T_0 increases with increase in K and m and decreases with increase in M(the changes being rather small).

Table 6.2 gives the values of the mean skin friction components τ_{mu} and τ_{mw}. τ_{mu} increases with increase in m and k, while it decreases with increase in Ω and S. τ_{mw} increases with increase in Ω, m and K and decreases with increase in S.

Values of the transient primary velocity $u(y, \frac{\pi}{2n})$ are given in table 6.3. We conclude from the table that the transient primary velocity decreases with increase in and M, but increases with increase in m and K. $u(y, \frac{\pi}{2n})$ decreases with increase in the frequency parameter n near the plate, but away from the plate the influence of n is insignificant.

Values of the transient secondary velocity - $w(y, \frac{\pi}{2n})$ are also given in table 6.3. It is clear from the table. The transient secondary velocity increases with increase in m, Ω and K. It decreases with increase in M. $w(y, \frac{\pi}{2n})$ decreases with increase in n near the plate, but away from the plate it increases with increase in n. Further away from the plate the influence of n is insignificant.

Table 6.4 shows the values of the transient temperature $T(y,\frac{\pi}{2n})$. The transient temperature rises with rise in m and K. It falls with a rise in M. Near the plate $T(y,\frac{\pi}{2n})$ increases with increase in n; away from the plate the influence of n is insignificant.

Values of skin friction components τ_x and τ_z at $nt = \frac{\pi}{2}$ are given in table 6.5. Increase in m, n and K leads to increase in τ_x. τ_x decreases with increase in Ω. τ_z increases with increase in Ω, m and K, but decreases with increase in n.

APPENDIX

$$A_1 = \frac{G}{SP(S^2P^2-S^2P-M_1)} , \quad A_2 = 1 + A_1, \quad B_1 = \frac{S + (S^2+4M_1)^{\frac{1}{2}}}{2}$$

$$C_1 = \frac{P\ |A_2|^2\ |B_1|^2}{(B_1 + \bar{B}_1)^2 - SP(B_1 + \bar{B}_1)} , \quad C_2 = \frac{P\ |A_1|^2}{2}$$

$$C_3 = \frac{P^2 A_1 \bar{A}_2 S}{SP + \bar{B}_1} , \quad C_4 = \frac{SP^2 \bar{A}_1 A_2}{SP + B_1}$$

$$D_1 = \frac{(B_1 + \bar{B}_1)C_1}{SP} , \quad D_2 = 2C_2 , \quad D_3 = \frac{(SP + \bar{B}_1)C_3}{SP}$$

$$D_4 = \frac{(SP + B_1)C_4}{SP} , \quad L_1 = D_1 + D_2 - D_3 - D_4$$

$$F_1 = \frac{GL_1}{S^2P^2 - S^2P - M_1} , \quad F_2 = \frac{GC_1}{(B_1+\bar{B}_1)^2 - S(B_1+\bar{B}_1) - M_1}$$

$$F_3 = \frac{GC_2}{4S^2P^2 - 2S^2P - M_1} , \quad F_4 = \frac{GC_3}{(SP+\bar{B}_1)^2 - S(SP+\bar{B}_1) - M_1}$$

$$F_5 = \frac{GC_4}{(SP+B_1)^2 - S(SP+B_1) - M_1}$$

$$L_2 = F_1 - F_2 - F_3 + F_4 + F_5$$

$$M_2 = M_1 + in , \quad h_1 = \frac{S + (S^2+4M_2)^{\frac{1}{2}}}{2}$$

$$h_2 = \frac{SP(S^2P^2+4inP)^{\frac{1}{2}}}{2}$$

$$P_1 = \frac{\bar{A}_2\bar{B}_1\ h_1P}{(h_1+\bar{B}_1)^2 - SP(h_1+\bar{B}_1) - inP}$$

$$P_2 = \frac{\bar{A}_1 SP^2 h_1}{(SP+h_1)^2 - SP(SP+h_1) - inP}$$

$$P_3 = \frac{A_2B_1\bar{h}_1P}{(\bar{h}_1+B_1)^2 - SP(\bar{h}_1+B_1) - inP}, \quad P_4 = \frac{A_1SP^2\bar{h}_1}{(SP+\bar{h}_1)^2 - SP(SP+\bar{h}_1) - in}$$

$$P_5 = \frac{(h_1+\bar{B}_1)P_1}{h_2}, \quad P_6 = \frac{(SP + h_1)P_2}{h_2}, \quad P_7 = \frac{P_3(\bar{h}_1 + B_1)}{h_2}$$

$$P_8 = \frac{P_4(SP + \bar{h}_1)}{h_2}, \quad L_3 = P_5 - P_6 + P_7 - P_8$$

$$P_9 = \frac{GL_3}{h_2^2 - Sh_2 - M_2}, \quad P_{10} = \frac{GP_1}{(h_1+\bar{B}_1)^2 - S(h_1+\bar{B}_1) - M_2}$$

$$P_{11} = \frac{GP_2}{(SP+h_1)^2 - S(SP+h_1) - M_2}, \quad P_{12} = \frac{GP_3}{(\bar{h}_1+B_1)^2 - S(\bar{h}_1+B_1) - }$$

$$P_{13} = \frac{GP_4}{(SP+\bar{h}_1)^2 - S(SP+\bar{h}_1) - M_2}$$

$$L_4 = P_9 - P_{10} + P_{11} - P_{12} + P_{13}$$

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